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Solving Fredholm Integral Equations by Numerical Methods Adomian Decomposition Method and Variational Iteration Method

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ABSTRACT
In this research paper, numerical methods Adomian Decomposition Method and Variational Iteration Method is used to solve integral equations .Numerical examples illustrate the accuracy of the Adomian Decomposition Method and Variational Iteration Method.
KEYWORDS: **Integral Equations, Fredholm integral equation, Adomian Decomposition Method, Variational Iteration Method**

1. ADOMIAN DECOMPOSITION METHOD

The Adomian Decomposition method (ADM) is very powerful method which considers the approximate solution of a non-linear equation as an infinite series which actually converges to the exact solution in this paper, ADM is proposed to solve some first order, second order and third order differential equations and integral equations . The Adomian Decomposition method (ADM) was firstly introduced by George Adomian in 1981. This method has been applied to solve differential equations and integral equations of linear and non-linear problem in Mathematics, Physics, Biology and Chemistry up to know a large number of research paper have been published to show the feasibility of the decomposition method.

1.1 Adomian Decomposition Method for Solving Fredholm Integral Equations

The Adomian decomposition method includes decomposing the unknown function $y(z)$ of any equation into a sum of an infinite number of constituents well-defined by the decomposition series

$$y(z) = \sum_{n=0}^{\infty} y_n(z)$$

Equivalently

$$y(z) = y_0(z) + y_1(z) + y_2(z) + y_3(z) + \cdots$$

When the components $y_n(z), n \geq 0$ will be resolved. The Adomian decomposition method concerns itself with discover the constituents

$$y_0(z), y_1(z), y_2(z) \dots$$

From above we can found recurrence relation as

$$\sum_{n=0}^{\infty} y_n(z) = f(z) + \int_c^d K(z,t)(\sum_{n=0}^{\infty} y_n(t))dt$$

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Or equivalently

$$y_0(z) + y_1(z) + y_2(z) + \cdots = f(z) + \int_c^d K(z,t)[y_0(t) + y_1\,(t) + \cdots]dt$$

The zeroth component $y_0(z)$ is acknowledged by all terms that are not comprised under the integral sign. This means that the constituents $y_n(z), n \geq 0$ of the unknown function $y(z)$are totally resolved by the recurrence relation

$$y_0(z) = F(z), \qquad y_{n+1}(z) \ = \int_a^b K(z,t) y_n\,(t)dt, \qquad n \geq 0$$

Equivalently

$$y_0(z) = f(z),$$

$$y_1(z) \ = \int_c^d K(z,t) y_0(t)dt,$$

$$y_2(z) = \int_c^d K(z,t) y_1\,(t)dt,$$

$$y_3(z) = \int_c^d K(z,t) y_2\,(t)dt,$$

and so on are the other constituents. Thus, the constituents $y_0(z), y_1(z), y_2(z)$, ... are resolved totally.

2. VARIATIONAL ITERATION METHOD

Variational iteration method is a strong and power device for solving various kinds of linear and non-linear functional equations. It was presented by Ji-Huan He in1998 and has been used by many mathematicians and engineers to resolve many kinds of functional equations, for instance wave equation, hyperbolic differential equations, Telegraph equation, non-linear chemistry problems, Cauchy reaction diffusion problem, and several other equations. Variational iteration method (VIM) is used to resolve integral equations. By this method we can resolve many non-linear equations. The following general non-linear system is measured to demonstrate the basic idea of this method

$$L[U(t)] + N[U(t)] = f(t)$$

L is a linear operator, N is a non-linear operator and $f(t)$ is a continuous function. The elementary character of the method is to shape a correction functional for the system, which states

$$U_{k+1}(t) = \ U_k(t) + \int_0^z \gamma(s)[LU_k(s) + \ \overline{N}\, U_k(s) - f(t)]ds$$

$\gamma\ is\ a$ Langragian multiplier which can be acknowledged optimally via Varaitional theory, U_k is the nth approximate solution and U_k signifies the limited difference

3. NUMERICAL EXAMPLES

Example 1. Consider the Fredholm integral equation given as

$$u(x) = x + \int_0^1 (tx^2 + xt^2)u(t)dt$$

BY ADOMIAN DECOMPOSITIOIN METHOD

$$u_0(x) = x$$
$$u_1(x) = \int_0^1 (tx^2 + xt^2)tdt$$
$$= \int_0^1 t^2x^2 + xt^3$$
$$= \frac{x^2}{3} + \frac{x}{4}$$
$$u_2(x) = \int_0^1 [(tx^2 + xt^2)\frac{t^2}{3} + \frac{t}{4}]dt$$
$$= \frac{x^2}{6} + \frac{31x}{240}$$
$$u_3(x) = \int_0^1 [(tx^2 + xt^2)\frac{t^2}{6} + \frac{31t}{240}]dt$$
$$= \int_0^1 [\frac{x^2t^3}{6} + \frac{31x^2t^2}{240} + \frac{xt^4}{6} + \frac{31xt^3}{240}]dt$$
$$= \frac{x^2 1464}{17280} + \frac{1890x}{28800}$$

And so on

$$u_n(x) = 1.44479167x + 0.584722189x^2$$

BY VARIATIONAL ITERATION METHOD

$$u_0(x) = x$$
$$u_1(x) = x + \int_0^1 (tx^2 + xt^2)tdt$$
$$= x + \frac{x^2}{3} + \frac{x}{4}$$
$$= \frac{x^2}{3} + \frac{5x}{4}$$
$$u_2(x) = x + \int_0^1 (tx^2 + xt^2)\frac{t^2}{3} + \frac{5t}{4}dt$$
$$= x + \int_0^1 (\frac{t^3x^2}{3} + \frac{5t^2x^2}{4} + \frac{xt^4}{3} + \frac{5xt^3}{4}dt$$
$$= \frac{x^2}{2} + \frac{331x}{240}$$
$$u_3(x) = x + \int_0^1 (tx^2 + xt^2)\frac{t^2}{2} + \frac{331t}{240}dt$$
$$= x + \int_0^1 [\frac{331x^2t^2}{240} + \frac{331xt^3}{240} + \frac{xt^4}{2} + \frac{x^2t^3}{2}]dt$$
$$= \frac{3368x^2}{5760} + \frac{1387x}{960}$$

There are various noise terms appearing in the iteration and we will obtain the solution given as

$$u_n(x) = 0.584722222x^2 + 1.44479167x$$

Values	0.1	0.2	0.3	0.4	0.5
ADM	0.150326389	0.312347222	0.486062498	0.671472218	0.868576382
VIM	0.150326389	0.312347223	0.486062501	0.671472218	0.868576391

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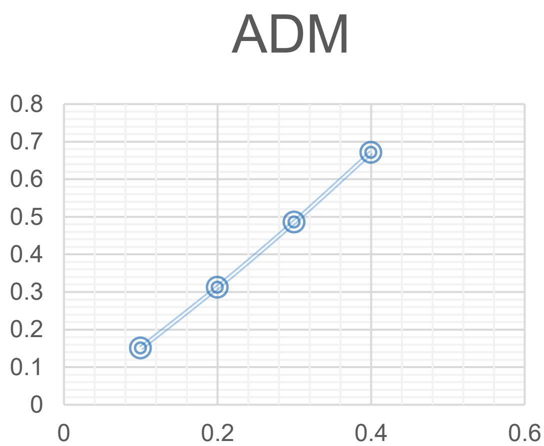

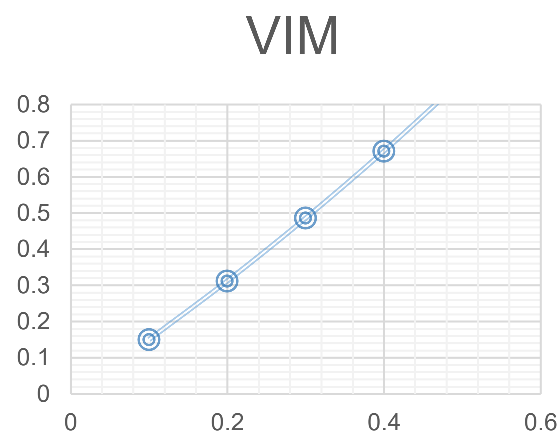

Example 2. Consider the Fredholm integral equation given as

$$u(x) = 1 + \int_{-1}^{1}(tx + x^2t^2)u(t)dt \quad -1 \leq x \leq 1$$

BY ADOMIAN DECOMPOSITION METHOD

$$u_0(x) = 1$$
$$u_1(x) = \int_{-1}^{1}(tx + x^2t^2)1dt$$
$$= x\int_{-1}^{1}(tdt + x^2\int_{-1}^{1}t^2\,dt)$$
$$= \frac{2x^2}{3}$$
$$u_2(x) = \int_{-1}^{1}(tx + x^2t^2)u_1(t)dt$$
$$= \int_{-1}^{1}(tx + x^2t^2)\frac{2t^2}{3}dt$$
$$= \int_{-1}^{1}\left(\frac{2xt^3}{3} + \frac{2x^2t^4}{3}\right)dt$$
$$= \frac{4x^2}{15}$$
$$u_3(x) = \int_{-1}^{1}(tx + x^2t^2)u_2(t)dt$$
$$= \int_{-1}^{1}(tx + x^2t^2)\frac{4t^2}{15}dt$$
$$= \int_{-1}^{1}\left(\frac{4xt^3}{15} + \frac{4x^2t^4}{15}\right)dt$$
$$= \frac{8x^2}{75}$$

And so on

$$u_n(x) = 1 + 1.04x^2$$

BY VARIATIONAL ITERATION METHOD

$$u_0(x) = 1$$
$$u_1(x) = 1 + \int_{-1}^{1}(tx + x^2t^2)1dt$$
$$= 1 + \frac{2x^2}{3}$$

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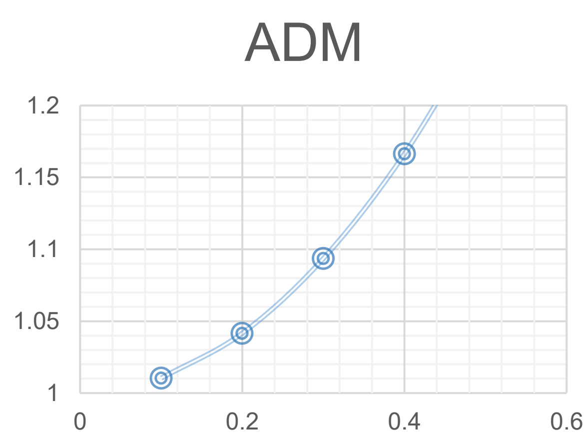
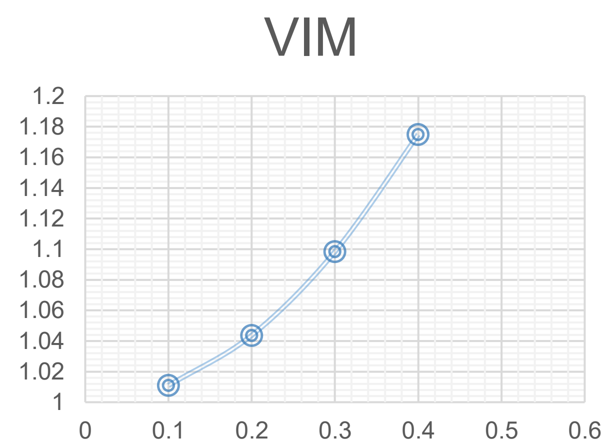
$$u_2(x) = 1 + \int_{-1}^{1} (tx + x^2t^2)1dt$$
$$= 1 + \int_{-1}^{1} (tx + \frac{2xt^2}{3} + x^2t^2 + \frac{2x^2t^4}{3})1dt$$
$$= 1 + \frac{14x^2}{15}$$
$$u_3(x) = 1 + \int_{-1}^{1} (tx + x^2t^2)1 + \frac{42t^2}{45}dt$$
$$= 1 + \int_{-1}^{1} \left(tx + \frac{42xt^3}{45} + x^2t^2 + \frac{42x^2t^4}{45}\right) dt$$
$$= 1 + \frac{82x^2}{75}$$

There are various noise terms appearing in the iteration and we will obtain the solution given as

$$u_n(x) = 1 + 1.09333333x^2$$

Values	0.1	0.2	0.3	0.4	0.5
ADM	1.0104	1.0416	1.0936	1.1664	1.26
VIM	1.01093333	1.0437332	1.09839997	1.17493328	1.2733325





4. CONCLUSION

In this paper, ADM and VIM were successfully applied to solve the Fredholm integral equations. VIM and ADM are very powerful and operative method for finding the solution for wide classes of problems. It is worth noting that these numerical methods are quick and converge approximately at one point.

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