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Some Investigation Of Incompressible Inviscid Steady Flow In Engineering Systems: A Case Study Of The Bernoulli's Equation With Its Application To Leakage In Tanks

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Abstract

During fluid spill incidents involving damaged tanks, the amount of the product released may be uncertain. Many accidents occur under adverse conditions, so determining the volume lost by sounding the tanks may not be practical. In the first few hours, initial volume estimates often are based on visual observations of the resulting levels, a notorious unreliable approach. This study has described a computer based model by Bernoulli's equation that will help in responding to leakages in tanks.

1. Introduction

Fluid storage tanks can be damaged in the course of their life and a substantial amount of the fluid lost through leakage. There is need therefore to calculate the rate at which the tank leaks in order to adequately respond to the situation. The volume of a fluid escaping through a hole in the side of a storage tank has been calculated by the Bernoulli's equation which has been adapted to a streamline from the surface to the orifice.

1.1 Objectives

The objectives of the study are:

i. To determine the velocity of the fluid through the opening in the tank.
ii. To determine the volume flow rate through the opening.
iii. To determine the mass flow rate through the opening.

1.2 Justification

Fluid storage tanks e.g. water reservoirs & petroleum oil tanks are commonly used in our society today. This storage tanks are prone to accidents which when not adequately responded to, great losses will be incurred in terms of money and also pollution to the environment. The volume of a fluid escaping through a hole in the side of a storage tank is calculated by the Bernoulli's equation calculator the results of which are useful for developing the intuitively skills of responders and planners in spill releases.

1.3 Literature Review

Castelli and Tonicelli (1600) were first to state that the velocity through a hole in a tank varies as square root of water level above the hole. They also stated that the volume flow rate through the hole is proportional to the open area. It was almost another century later that a Swiss physicist Daniel Bernoulli (1738) developed an equation that defined the relationship of forces due to the line pressure to energy of the moving fluid and earth's gravitational forces on the fluid. Bernoulli's theorem has since been the basis for flow equation of flow meters that expresses flow rate to differential pressure between two reference points. Since the differential pressure can also be expressed in terms of height or head of liquid above a reference plane. An Italian scientist, Giovanni Venturi (1797), demonstrated that the differential pressure across an orifice plate is a square root function of the flow rate through the pipe. This is the first known use of an orifice for measuring flow rate through a pipe. Prior to Giovanni's experimental demonstration, the only accepted flow measurement method was by filling a bucket of known volume and counting the number of buckets being filled. The use of orifice plates as a continuous flow rate measuring device has a history of over two hundred years. Subsequent experiments, Industry standards online in their demonstration showed that liquids flowing from a tank through an orifice close to the bottom are governed by the Bernoulli's equation which can be adapted to a streamline from the surface to the orifice. They evaluated three cases whereby in a vented tank, the velocity out from the tank is equal to the speed of a freely body falling the distance h -also known as Torricelli's Theorem. In a pressurized tank, the tank is pressurized so that product of gravity and height (gh) is much lesser than the pressure difference divided by the density. The velocity out from the tank depends mostly on the pressure difference. Due to friction the real velocity will be somewhat lower than these theoretic examples. On leaking tank models, a recent literature review reveals numerous papers describing formulas for calculating discharges of non-volatile liquids from tanks Burgreen (1960); Dodge and Bowls(1982); Elder and Sommerfeld (1974); Fthenakis and Rohatgi (1999); Hart and Sommerfeld (1993); Koehler(1984); Lee and Sommerfeld (1984); Shoaei and Sommerfeld (1984); Simecek-Beatty et al. There are also computer models available in ship salvage operations. These types of models estimate the hydrostatic, stability, and strength characteristic of a vessel using limited data. The models require the user to have an understanding of basic salvage and architectural principles, skills not typical of most spill responders or contingency planners. A simple computer model, requiring input of readily available data, would be useful for developing the intuitively skills of responders and planners in spill releases. Mathematical formulas have been developed to accurately describe releases for light and heavy oils. Dodge and Bowles (1982); Fthenakis and Rohatgi (1999); Simecek-Beatty et al. (1997). However, whether any of these models can describe the unique characteristics of fluids accurately is not known. Observational data of these types of releases are needed to test existing models, and probably modify them in the case of a given fluid. J. Irrig and Drain. Engrg (August 2010) carried out experiments under different orifice diameters and water heads. The dependence of the discharge coefficient on the orifice diameter and water head was analysed, and then an empirical relation was developed by using a dimensional analysis and a regression analysis. The results showed that the larger orifice diameter or higher water head have a smaller discharge coefficient and the orifice diameter plays more significant influence on the discharge coefficient than the water head does. The discharge coefficient of water flow through a bottom orifice is larger than that through a sidewall orifice under the similar conditions of the water head, orifice diameter, and hopper size.

In this research work a simple computer model, based on Bernoulli's equation requiring input of readily available data, is used to investigate leakage from vessels or tanks: the case of two pipeline companies (Oil refineries and Kenya pipeline co.) in the Coast province of Kenya. The results are of vital importance in responding to leakages to minimize losses.

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2. Governing Equations

2. 1 Overview

The equations governing the flow of an incompressible inviscid steady fluid through an orifice in a tank are presented in this chapter. This chapter first considers the assumptions and approximations made in this particular flow problem and the consequences arising due to these assumptions. The conservation equations of mass, momentum, energy to be considered in the study are stated, Bernoulli's equation derived followed by a model description of the fluid flow under consideration.

Finally a simple computer model, requiring input of readily available data is built which will be used in investigating flow rate in chapter three.

2.2 Assumptions and approximations

The following assumptions have been made.

1. Fluid is incompressible i.e. density ρ is a constant and $\frac{\partial \rho}{\partial t} = 0$
2. Fluid is inviscid i.e. the fluid under investigation is not viscous.
3. Flow is steady in that is if F is a flow variable defined as $F(x, y, t)$ then $\frac{\partial F}{\partial t} = 0$
4. Flow is along a streamline.
5. The fluid speed is sufficiently subsonic $(V < mach\ 0.3)$
6. Discharge coefficient C value is typically between 0.90 and 0.98.

2.3 Consequences as a result of the assumptions

The fact that as the water issues out of the orifice from the tank, the level in it changes and the flow is, strictly speaking, not steady. However, if $A_1 \geq A_2$, this effect (as measured by the ratio of the rate of change in the level and velocity V_2), is so small that the error made is insignificant.

The assumption of incompressibity would not hold at varying temperatures for a fluid like water which has differing densities at different temperatures. However this is taken care of by our model which allows the user to enter determined values of densities at various temperatures.

2.4 The Governing Equations

2.4.1 Equation of conservation of mass

Consider the fluid element of volume v fixed in space surrounded by a smooth surface S. If the density of the fluid is ρ and dv is the volume element of v at a point p in v, then the total mass of the fluid in v is given by

$$\int_v \rho dv \qquad (1)$$

We let δs be an element of the surface S. Then if the fluid flows outwards through δs, and after time δt this fluid is displaced a distance equal to δx. Then the rate at which mass is leaving the volume element is given by

$$\lim_{\delta t \to 0} \rho \delta s \frac{\delta x}{\delta t} = \rho \delta s \lim_{\delta t \to 0} \frac{\delta x}{\delta t} \qquad (2)$$

$\frac{\delta x}{\delta t}$ is taken to be speed.

If $\hat{n}$ is the unit normal vector to δs, then

$$\lim_{\delta t \to 0} \frac{\delta x}{\delta t} \hat{n} = \vec{f} \qquad (\vec{f} \text{ denotes velocity}).$$

Thus

$$\lim_{\delta t \to 0} \rho \delta s \frac{\delta x}{\delta t} = \rho \delta s . f\hat{n} \qquad (3)$$

This gives the rate of mass flux flowing out of v.

From (3), the total mass flux flowing out of v through S is given by

$$\int_s \rho f \hat{n} \delta s \qquad (4)$$

From Gauss theorem, equation (4) can be written as

$$\int_s \rho f \hat{n} \delta s = \int_v \left(\nabla \rho \vec{f}\right) dv \qquad (5)$$

From equation (1) the rate at which the mass is changing in volume v is

$$\frac{\partial}{\partial t} \int_v \rho dv \qquad (6)$$

Applying the law of conservation of mass, we have,

$$\int_v \left(\nabla \rho \vec{f}\right) dv = -\frac{\partial}{\partial t} \int_v \rho dv = -\int_v \frac{\partial \rho}{\partial t} dv$$

Or

$$\int_v \left(\nabla \rho \vec{f}\right) dv + \int_v \frac{\partial \rho}{\partial t} dv = 0$$

$$\int_v \left[\nabla\left(\rho \vec{f}\right) + \frac{\partial \rho}{\partial t} \right] dv = 0 \qquad (7)$$

Since v is an arbitrary volume, then for equation (7) to hold, we have

$$\nabla\left(\rho \vec{f}\right) + \frac{\partial \rho}{\partial t} = 0 \qquad (8)$$

And this is the general equation of mass conservation.

If ρ is a constant, then equation (8) becomes

$$\rho \nabla \vec{f} = 0 \qquad (9)$$

This is the equation of continuity of an incompressible fluid flow.

On the other hand, if the flow is steady i.e. the flow variables don't depend on time, and then equation (8) reduces to

$$\nabla \rho \vec{f} = 0 \qquad (10)$$

This is the equation of continuity of a steady fluid flow.

2.4.2 Equation of conservation of energy

Consider the fluid element of $\Delta v = dxdydz$ as shown below.

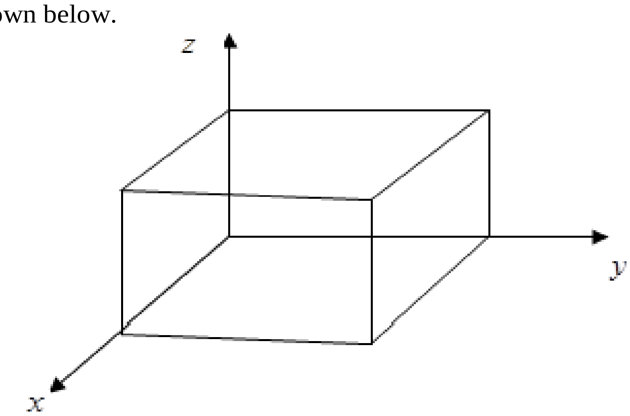

Figure 1: Fluid Element

By Fourier's law heat transferred through the face perpendicular to the x- axis by conduction is given by $-K \frac{\partial T}{\partial x} dydz$

(11)

The element of heat which leaves the volume element along the x-axis is given by

$$\left[K \frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x} \right) dx \right] dydz \qquad (12)$$

Amount added to ΔV through conduction along x –axis =

$$\left[-K\frac{\partial T}{\partial x}+\left(K\frac{\partial T}{\partial x}+\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dx\right)\right]dzdy$$
$$=\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)dxdzdy \quad (13)$$

Similarly the amount added to Δv along the y-axis and z- axis through conduction is $\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial x}\right)dxdydz$ and $\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial x}\right)dxdydz$.

Total heat added to Δv by conduction

$$\frac{\partial Q}{\partial t}=\left[\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)\right]dxdydz \quad (14)$$

(Q is purely due to heat conduction)

If we let e be the internal energy per unit mass, then total internal energy of the fluid element Δv i.e. $E=\rho\nabla v.e=\rho edxdydz$ (15)

From this we have

$$\frac{de}{dt}=\rho\Delta v\frac{de}{dt}=\rho\frac{de}{dt}dxdydz \quad (16)$$

So

$$\left[\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)\right]dxdydz$$
$$=\rho\frac{de}{dt}dxdydz+\frac{dw}{dt}$$

or

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)=\rho\frac{de}{dt}+\frac{1}{\Delta v}\frac{dw}{dt} \quad (17)$$

If the fluid is incompressible then $dw=0$ for incompressible fluids equation (17) reduces to

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)=\rho\frac{de}{dt} \quad (18)$$

Adding heat generation by frictional force to equation (18) we have,

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)=\rho\frac{de}{dt}+\varphi\phi \quad (19)$$

where ϕ is known as the dissipation function given by

$$\phi=2\left[\begin{array}{l}\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\\+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2\\+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2-\frac{2}{3}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)^2\end{array}\right] \quad (20)$$

(19) is the general equation of energy of the fluid flow whose velocity components are u, v and w if ϕ is as defined by (20)

It is noted that if the fluid is incompressible K = a constant and the internal energy E is a function of t alone, then equation (19) reduces to

$$K\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho\frac{df(T)}{dt}+\varphi\phi$$

(T=temperature)

$$K\nabla^2 T=\rho\frac{df(T)}{dt}+\varphi\phi \quad (21)$$

If e is a function of T alone then $e=C_V T\Rightarrow f(T)=C_V T$ (22)

substituting this in (21) we have

$$K\nabla^2 T=\rho C_V\frac{\partial T}{\partial t}+\varphi\phi \quad (23)$$

(23) is the equation of energy of a non- viscous incompressible fluid with constant coefficient of conductivity.

2.43 Equation of motion for compressible fluids

Consider a control volume ABCD (of unit depth) in a 2-D flow field with its centre located at (x, y). If the state of stress at (x, y) in this 2-D flow field is represented by $\sigma_{xx}, \sigma_{yy}, \tau_{xy}$ and τ_{yx}, then the surface forces on the four faces of the CV can be written in terms of these stresses and their derivatives using Taylor series. A few of these are shown in the figure below.

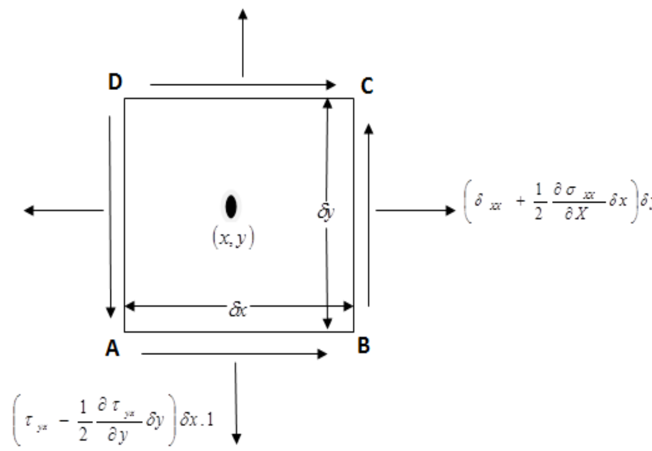

Figure 2: Control Volume

The net surfaces forces acting on this element in the x- and y-directions are easily seen to be

$$\delta F_{surface,x}=\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}\right).\delta x.\delta y.1 \quad (24)$$

And

$$\delta F_{surface,y}=\left(\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}\right).\delta x.\delta y.1 \quad (25)$$

If f_x and f_y are the components of the body force per unit mass, then

$$\delta F_{body,x}=\rho f_x.(\delta x.\delta y.1) \quad (26)$$

And

$$\delta F_{body,y}=\rho f_y.(\delta x.\delta y.1) \quad (27)$$

By Newton's law of motion

$$\rho(\delta x.\delta y.1)\frac{DV_x}{Dt}=\rho f_x\delta x.\delta y.1 + \left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}\right)\delta x.\delta y.1$$

So that

$$\rho\frac{DV_x}{Dt}=\rho f_x+\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y} \quad (28)$$

And similarly

$$\rho\frac{DV_y}{Dt}=\rho f_y+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x} \quad (29)$$

DV_x/Dt and DV_y/Dt may be expressed in terms of the field derivatives using the Euler acceleration formula which gives

$$\rho\left[\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}\right]=\rho f_x+\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y} \quad (30)$$

With similar equations obtained from (30), we can now use the expression for

$$\tau_{yx}=\tau_{xy}=\mu\overset{\circ}{\gamma}_{xy}=\mu\left(\frac{\partial V_y}{\partial x}+\frac{\partial V_x}{\partial y}\right) \text{ and}$$

$$\sigma_{xx}=-p+2\mu\frac{\partial V_x}{\partial x}$$

(From Fluid Mechanics and its Applications by Vijay Gupta and Santosh K Gupta page 150), we obtain,

$$\rho\left[\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}\right]=\rho f_x-\frac{\partial p}{\partial x}+\mu\left[\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}\right] \quad (31)$$

And

$$\rho\left[\frac{\partial V_y}{\partial t}+V_x\frac{\partial V_y}{\partial x}+V_y\frac{\partial V_y}{\partial y}\right]=\rho f_y-\frac{\partial p}{\partial y}+\mu\left[\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}\right] \quad (32)$$

For a general 3-D flow these components can be written in the vector form

$$\rho\left[\frac{\partial V}{\partial t}+(V.\nabla)V\right]=\rho f-\nabla p+\mu\nabla^2 V \quad (33)$$

Known as Navier-Stokes equation. This contains three equations for V_x, V_y, V_z. The LHS of equation (33) represents the fluid acceleration (local and convective) and is referred to as the inertial term. The term on the right represents the body force, the pressure and the viscous forces respectively.

In many flows of engineering interest, the viscous terms are much smaller than the inertial terms and thus can be neglected, giving

$$\rho\left[\frac{\partial V}{\partial t}+(V.\nabla)V\right]=\rho f-\nabla p$$

This is known as the Euler equation which is applicable to non-viscous flows and plays an important role in the study of fluid motion.

2.5 Bernoulli's Equation

Bernoulli's equation states that the total mechanical energy (consisting of kinetic, potential and flow energy) is constant along a streamline.

It is a statement of the conservation of energy in a form useful for solving problems involving fluids. For a non-viscous, incompressible fluid in steady flow, the sum of pressure, potential and kinetic energies per unit volume is constant at any point.

The form of Bernoulli's Equation we have used arises from the fact that in steady flow the particles of fluid move along fixed streamlines, as on rails, and are accelerated and decelerated by the forces acting tangent to the streamlines.

Bernoulli's equation has been obtained by directly integrating the equation of motion of an inviscid fluid.

Thus equation (34) is the Bernoulli's equation

$$\left(\frac{V^2}{2}+gz+\frac{p}{\rho}\right)_i=\left(\frac{V^2}{2}+gz+\frac{p}{\rho}\right)_0 \quad (34)$$

2.6 Illustration of the fluid flow

Fluids flow from a tank or container through a hole/orifice close to the bottom. The Bernoulli equation derived can be adapted to a streamline from the surface of the fluid to the orifice.

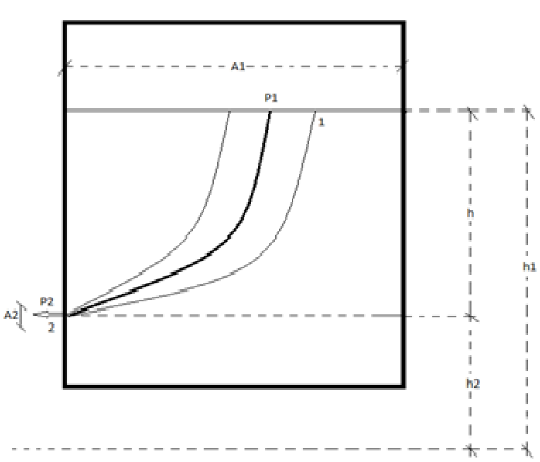

Figure 3: Flow from a Tank

$$g.h_1+\frac{P_1}{\rho}+\frac{V_1^{\,2}}{2}=g.h_2+\frac{P_2}{\rho}+\frac{V_2^{\,2}}{2} \quad (35)$$

Where

$$h=h_1-h_2 \quad (36)$$

Since (1) and (2)'s heights from a common reference are related as (36), and the equation of continuity can be expressed as (37), it is possible to transform (35) to (38).

$$V_1=\left(\frac{A_2}{A_1}\right).V_2 \quad (37)$$

$$V_2=\sqrt{\frac{2}{\left(1-\frac{A_2^{\,2}}{A_1^{\,2}}\right)}\left(\frac{P_1-P_2}{\rho}+g.h\right)} \quad (38)$$

A special case of interest for equation (38) is when the orifice area is much lesser than the surface area and when the pressure inside and outside the tank is the same- when the tank has an open surface or "vented" to the atmosphere. At this situation (38) can be transformed to (39)

$$V_2=\sqrt{2.g.h} \quad (39a)$$

Due to friction, the real velocity will be somewhat lower than the theoretic velocity in (39). The actual velocity profile at A2 will depend on the nature of the orifice. A sharp-edged orifice will result in much 2-D effects than will a smooth one. In engineering practice, these non-ideal effects are usually taken care of by introducing an experimentally obtained correction factor termed the discharge coefficient C_d. Thus (39a) can be expressed as (39b). The coefficient of discharge can be determined experimentally. For a sharp edged opening it may be as low as low as 0.6. For smooth orifices it may be between 0.95 and 1.

$$V_2=C_d\sqrt{2.g.h} \quad (39b)$$

If the tank is pressurized so that the product of gravity and height (g.h) is much lesser than the pressure difference divided by the density, (38) can be transformed to (40). Thus the velocity out from the tank depends mostly on the pressure difference. However in our study, we have thus concentrated on the vented tank. The pressurized tank will be dealt with in our subsequent studies.

$$V_2=\sqrt{\frac{2.(P_1-P_2)}{\rho}} \quad (40)$$

In the case of the vented tank, the discharge rate is given by

$$\overset{\circ}{Q}=C_d A_2\sqrt{2.g.h} \quad (41)$$

The source of error in the equation is the fact that as the water issues out of the orifice from the tank, the level in it changes and the flow is, strictly speaking, not steady. However, if $A_1 \geq A_2$, this effect (as measured by the ratio of the rate of change in the level and velocity V_2), is so small that the error made is insignificant.

2.7 Water flowing (discharging) from a vented tank, pond, reservoir containing water or other liquid

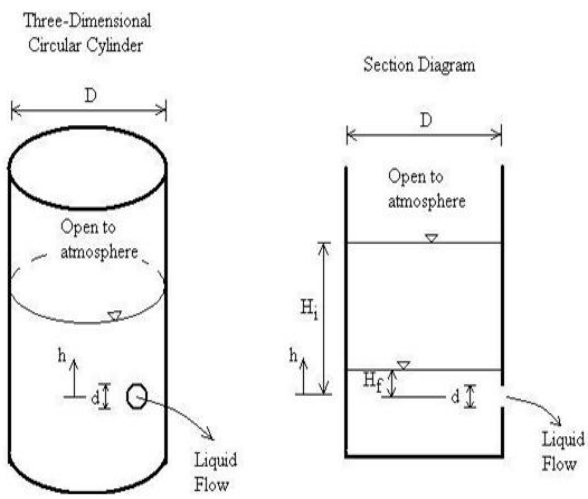

Figure 4: Water discharging from a vented tank

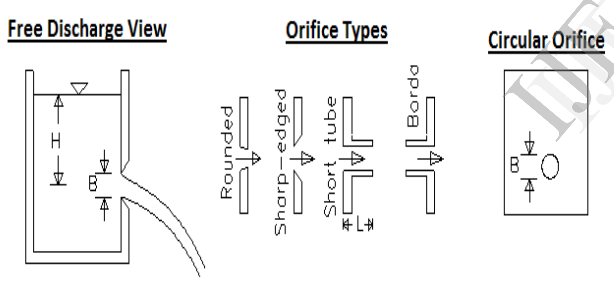

Figure 5: Free discharge view

2.8 Design of Calculator Application

In this study, the free discharge orifice has been modelled. We have studied on circular orifice geometry where B is orifice diameter. A pull-down menu allows you to select an orifice type. Discharge coefficients for the four orifice types are built into the calculation.

Built-in values for orifice discharge coefficients are:

Rounded	Sharp-edged	Short-tube	Borda
0.98	0.6076	0.8	0.5096

The Borda type is also known as a re-entrant since it juts into the tank.C values were obtained from Dally et al. (1993) for circular orifices.

Water (or other liquid) draining out of a tank, reservoir, or pond is a common situation. Our calculation allows you to compute for final liquid depth given the time the tank has discharged and the initial liquid depth of the fluid. This will be convenient for a tank that we know the initial liquid depth of the fluid but we don't know the final liquid depth. The final liquid depth will give us the spout depth. (This works for high tanks that one can't climb to measure the fluid depth from the top of the tank).

Alternatively, the user can compute the time needed to lower the water from one depth to a lower depth or to empty the tank.

The tank (or pond or reservoir) is open to the atmosphere and it can be cylindrical or other cross-section but must have the same cross-section for its entire height. The orifice can be circular or non-circular, but for our study, we have modelled the circular orifice. *Hi*, *Hf*, and *h* are measured vertically from the centerline of the orifice.

Hydrostatic pressure will impart a velocity to an exiting fluid jet. The velocity and flow rate of the jet depend on the depth of the fluid.

Given the time the tank has discharged, the initial liquid depth of the fluid, tank diameter/side dimensions (which will give us tank area), orifice diameter (will give us orifice area) and discharge coefficient we will compute for final liquid depth which will give us the spout depth.

To calculate the jet velocity and flow rate (both volume and mass), enter the parameters specified on the calculator application. (Any interaction of the fluid jet with air is ignored.)

2.9 Equations used in the Calculation

$$V_2 = C_d \sqrt{2.g.h}$$

$$Q = A_{jet} V_{jet} = C A_{spout} V_{jet}$$

Or $$\overset{\circ}{Q} = C_d A_2 \sqrt{2.g.h}$$

Note: The above equations are valid if both the tank and orifice are at the same pressure, even if the pressure is not atmospheric

For a tank with a constant cross-sectional geometry A in the plan view (i.e. as you look down on it), substitute:

$$Q = -A \frac{dh}{dt}$$

Integrate h from Hi to Hf and integrate t from 0 to t, then solve for time t, which is the time required for the liquid to fall from Hi to Hf:

$$t = \frac{A}{aC}\left(\sqrt{H_i} - \sqrt{H_f}\right)\sqrt{\frac{2}{g}}$$

If the tank is circular in plan view (i.e. looking down on it):

$$A = \frac{\pi D^2}{4}$$

If the orifice is circular:

$$a = \frac{\pi d^2}{4}$$

Our calculation allows you to solve for any of the variables: *a, A, Hf,* or *t*. The orifice and tank can be either circular or non-circular. If non-circular, then the diameter dimension is not used in the calculation.

3 Results AND Discussion

3.1 Overview

This section details the results of our work in form of graphs together with discussion of the results obtained.

3.2 Effect of Spout Depth on Exit velocity

Table 1: Effect of Spout Depth on Exit velocity

Spout Depth	26.332158	40.771996	15.036392	8.751358	1.877070
Exit velocity	11.581022	14.4106854	8.751358	5.921695	3.092031

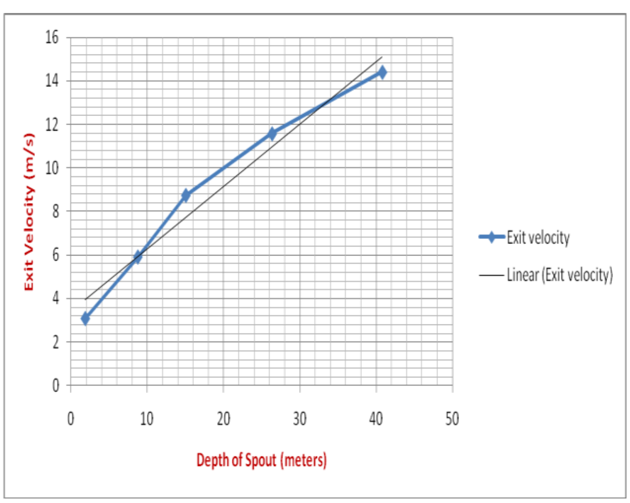

Figure 3a: Effect of Spout Depth on Exit velocity

Spout exit velocity decreases with decrease in the depth of the spout. This is due to hydrostatic pressure which imparts a velocity to an exiting fluid jet but this pressure decreases when the fluid level drops. Thus the velocity of the jet depends on the depth of the fluid.

3.3 Effect of Discharge coefficient on Exit velocity

Table 2: Effect of Discharge coefficient on Exit velocity

Discharge coefficient	0.98	0.8	0.6076	0.5096
Exit velocity	62.45577	38.43185	18.99865	11.58102

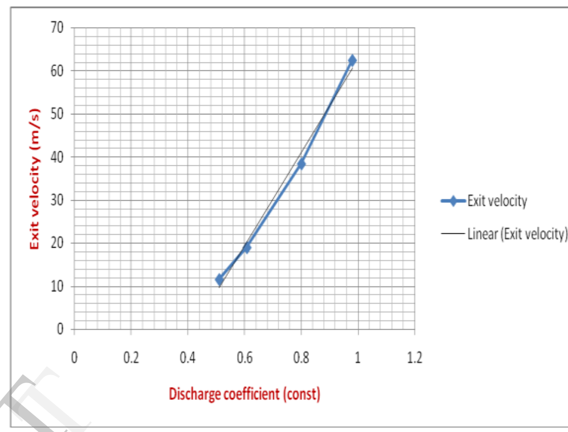

Figure 3b: Effect of Discharge coefficient on Exit velocity

Orifices with higher orifice coefficients give higher exit velocities. This implies that the type of orifice will affect the orifice exit velocity. In our case rounded orifice type will give the highest exit velocity followed by short-tube, sharp-edged then finally borda in decreasing velocities.

3.4 Effect of spout depth on discharge rate

Table 3: Effect of spout depth on discharge rate

Spout Depth	20	18.89763	17.82652	16.78664	15.77802
Discharge	0.024394	0.023712	0.02303	0.022349	0.021667

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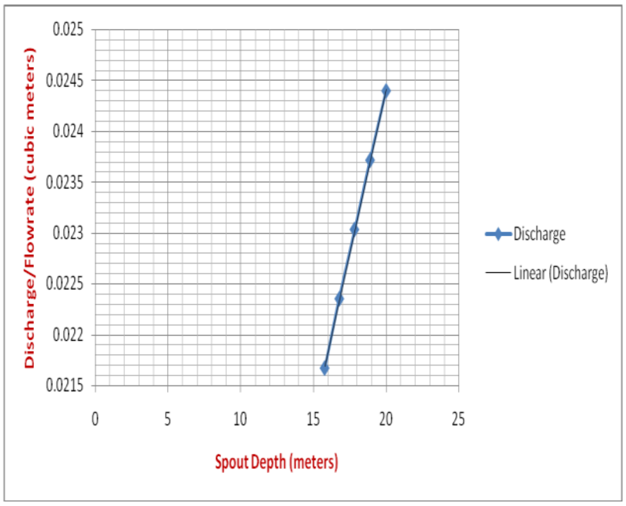

Figure 3c: Effect of spout depth on discharge rate

The highest depth of spout gives the highest discharge. Decrease in the depth of the spout reduces the rate of discharge. Hydrostatic pressure reduces on reduction of fluid level imparts a velocity to an exiting fluid jet. Thus the velocity and flow rate of the jet depend on the depth of the fluid.

3.5 Effects of Orifice area on Discharge

Table 4: Effects of Orifice area on Discharge

Orifice area	0.001257	0.0028278	0.005027	0.007855	0.011311
Discharge	0.024394	0.0548863	0.097576	0.152462	0.219545

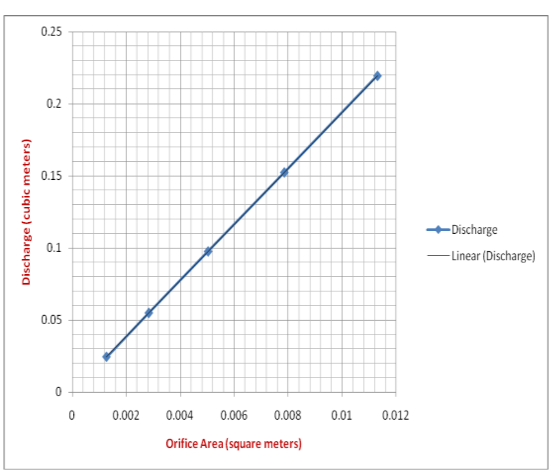

Figure 3d: Effects of Orifice area on Discharge

The larger the orifice area the larger the rate of discharge and vice versa. However, increasing orifice size beyond the carrying capacity of the system has no effect on the flow rate. (From the paper Flow Rate as a Function of Orifice Diameter by Ginger Hepler, Jeff Simon & David Bednarczyk)

3.6 Effects of Discharge coefficient on discharge/flow rate

Table 5: Effects of Discharge coefficient on discharge/flow rate

Discharge coefficient	0.98	0.8	0.6076	0.5096
Flow rate	0.024394	0.0199134	0.015124	0.012685

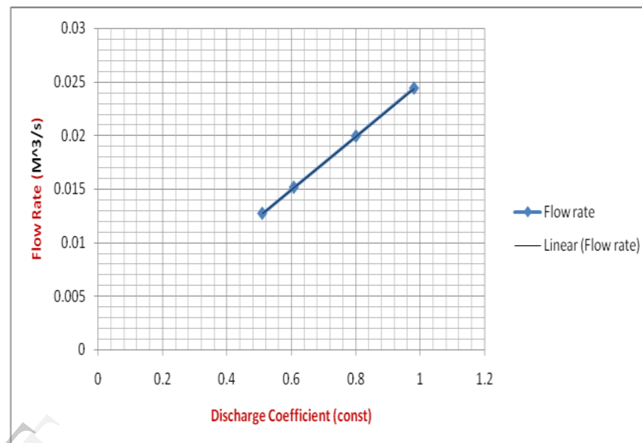

Figure 3e: Effects of Discharge coefficient on discharge/flow rate

Orifices with higher orifice coefficients give higher discharge rates. This implies that the type of orifice will affect the discharge rate. In our case rounded orifice type will give the highest discharge rate followed by short-tube, sharp-edged then finally borda in decreasing discharge rate.

3.7 Effects of density on mass flow rate

Table 6: Effects of density on mass flow rate

Fluid density	1000	850	737.22
Mass Flow rate	24.39391	20.7348262	17.98368

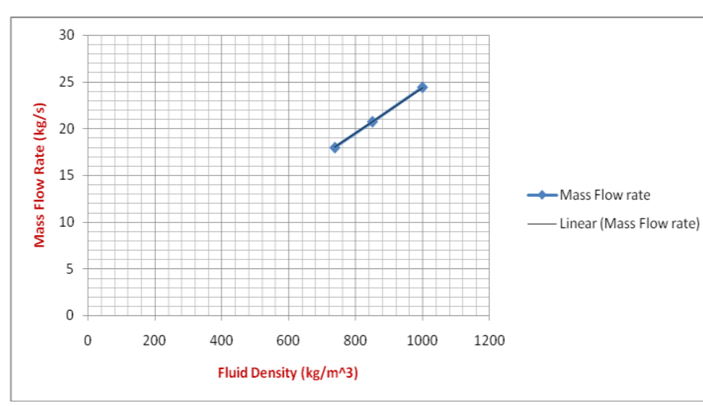

Figure 3f: Effects of density on mass flow rate

Higher fluid densities cause higher mass flow rates. Thus the rate of mass flow of a fluid from the tank is directly proportional to the fluid density

4. Conclusions

Many phenomena regarding the flow of liquids and gases can be analyzed by simply using the Bernoulli equation. However, due to its simplicity, the Bernoulli equation may not provide an accurate enough answer for many situations, but it is a good place to start. It can certainly provide a first estimate of parameter values.

We analyzed the effects of spout depth, orifice area, friction coefficient on the fluid flow rate in a vented tank.

The Bernoulli's equation that modelled the fluid flow was obtained by directly integrating the equation of motion of an inviscid fluid. The effect of a certain parameter value on another was investigated by varying the parameter and observing the behavior of the other.

In this study the pressure is constant that is both the tank and orifice are at the same pressure, (even if the pressure is not atmospheric), in that the exiting fluid jet experiences the same pressure as the free surface.

The major analysis is made on orifice geometry more than on tank geometry. A change in orifice geometry in terms of spout depth and type affected the flow rate. Knowing the duration the tank has leaked enables one to determine the current fluid depth in cases where tanks are too large and the depth can't be determined physically. This way will know whether the tank is empty already or not.

In conclusion, the exit velocity of the fluid is affected by the significant changes in spout depth, orifice area discharge coefficient. The mass flow rate is only affected by change in the fluid density.

4.2 Recommendations

For one to respond to any leak scenario, one has to first establish that a leakage really exists. This is done by various leak detecting techniques e.g. the leak-measurement computer LTC 602 offers virtually unlimited possibilities for all kinds of leak measurement tests. The model built requires that the user has prior knowledge of the time the tank has leaked. Thus this model will be of use practically when used together with a leak detecting model which establishes the time the leak commenced. Some of the areas that need further research include:

i. A pressurized tank, where the tank is pressurized so that the velocity out from the tank depends mostly on the pressure difference.
ii. An application model that establishes that a leakage really exists and then calculate for the discharge rate.
iii. Fluid flow for unsteady, compressible fluids

ACKNOWLEDGEMENT

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APPENDIX

Program code for calculator application in visual basic

```
Public Class frmCalc


    Private Sub
ComboBox6_SelectedIndexChanged(ByV
al sender As System.Object, ByVal
e As System.EventArgs) Handles
cmbOrificeType.SelectedIndexChange
d

        If
cmbOrificeType.SelectedItem =
"Rounded" Then

            txtcoeff.Text = "0.98"

        ElseIf
cmbOrificeType.SelectedItem =
"Sharp-edged" Then

            txtcoeff.Text =
"0.6076"

        ElseIf
cmbOrificeType.SelectedItem =
"Short-tube" Then

            txtcoeff.Text = "0.8"

        ElseIf
cmbOrificeType.SelectedItem =
"Borda" Then

            txtcoeff.Text =
"0.5096"

        End If


        txtTArea.ReadOnly = True

        txtTArea.Text = "Will be
calculated"

        txtTArea.BackColor =
Color.AntiqueWhite

        txtunitEVel.BackColor =
Color.AntiqueWhite

        txtOArea.ReadOnly = True

        txtOArea.Text = "Will be
calculated"

        txtOArea.BackColor =
Color.AntiqueWhite


        txtspout.ReadOnly = True

        txtspout.Text = "Will be
calculated"

        txtspout.BackColor =
Color.AntiqueWhite


        txtMassFlow.ReadOnly =
True

        txtMassFlow.Text = "Will
be calculated"

        txtMassFlow.BackColor =
Color.AntiqueWhite



        txtEVel.ReadOnly = True

        txtEVel.Text = "Will be
calculated"

        txtEVel.BackColor =
Color.AntiqueWhite

        txtVolFlow.ReadOnly = True

        txtVolFlow.Text = "Will be
calculated"

        txtVolFlow.BackColor =
Color.AntiqueWhite



    End Sub
```

```
    Private Sub
btncalc_Click(ByVal sender As
System.Object, ByVal e As
System.EventArgs) Handles
btncalc.Click


        txtTArea.ReadOnly = False

        txtTArea.Text = ""


        txtOArea.ReadOnly = False

        txtOArea.Text = ""


        txtspout.ReadOnly = False

        txtspout.Text = ""


        txtMassFlow.ReadOnly =
False

        txtMassFlow.Text = ""


        txtEVel.ReadOnly = False

        txtEVel.Text = ""


        txtVolFlow.ReadOnly =
False

        txtVolFlow.Text = ""


        Dim Orarea, subt,
finalheight, VolFlowrate, coeff,
gr, EVel, gr1, TCArea, TNCArea,
Theight, mass, time, tankdiam,
ordiam As Double

        ordiam =
Val(txtODiam.Text)

        Orarea = 3.142 * ordiam *
ordiam / 4

        txtOArea.Text = Orarea

        tankdiam =
Val(txtTdiam.Text)

        TCArea = 3.142 * tankdiam
* tankdiam / 4

        TNCArea =
(Val(txtTlength.Text) *
Val(txtTwidth.Text))

        If
cmbTankType.SelectedItem =
"Circular" Then

            txtTArea.Text = TCArea

        Else

            txtTArea.Text =
TNCArea

        End If


        coeff = Val(txtcoeff.Text)

        Theight =
(Val(txtInLevel.Text)) ^ 0.5

        time =
Val(txtTimeTank.Text)

        gr = ((2 / 9.8066) ^ 0.5)

        gr1 = time * Orarea *
coeff / ((Val(txtTArea.Text)) *
gr)

        subt = Theight - gr1

        finalheight = subt * subt

        txtspout.Text =
finalheight


        EVel = coeff * ((2 *
9.8066 * finalheight) ^ 0.5)

        txtEVel.Text = EVel
```

```
        VolFlowrate = Orarea * coeff * ((2 * 9.8066 * finalheight) ^ 0.5)

        txtVolFlow.Text = VolFlowrate


        mass = VolFlowrate * Val(txtfluid.Text)

        txtMassFlow.Text = mass


        txtunitEVel.Text = "m/s"

        txtunitMass.Text = "Kg/s"

        txtunitOArea.Text = "Square Metres"

        txtunitSpout.Text = "Metres"

        txtunitTArea.Text = "square Metres"

        txtunitVolRate.Text = "M^3/s"

    End Sub


    Private Sub ComboBox1_SelectedIndexChanged(ByVal sender As System.Object, ByVal e As System.EventArgs) Handles cmbTankType.SelectedIndexChanged

        If cmbTankType.SelectedItem = "Circular" Then

            txtTlength.ReadOnly = True

            txtTwidth.ReadOnly = True

            txtTwidth.Text = "Not Used"

            txtTlength.Text = "Not Used"

            txtTdiam.ReadOnly = False

            txtTdiam.Text = ""

        Else

            txtTdiam.Text = "Not Used"

            txtTwidth.Text = ""

            txtTlength.Text = ""

            txtTlength.ReadOnly = False

            txtTwidth.ReadOnly = False

            txtTdiam.ReadOnly = True


        End If


    End Sub


    Private Sub btnrefresh_Click(ByVal sender As System.Object, ByVal e As System.EventArgs) Handles btnrefresh.Click

        txtcoeff.Text = ""

        txtEVel.Text = ""

        txtfluid.Text = ""

        txtInLevel.Text = ""

        txtMassFlow.Text = ""

        txtOArea.Text = ""

        txtODiam.Text = ""

        txtspout.Text = ""

        txtTArea.Text = ""

        txtTdiam.Text = ""
```

```
        txtTimeTank.Text = ""

        txtTlength.Text = ""

        txtTwidth.Text = ""

        txtunitEVel.Text = ""

        txtunitMass.Text = ""

        txtunitOArea.Text = ""

        txtunitSpout.Text = ""

        txtunitTArea.Text = ""

        txtunitVolRate.Text = ""

        txtVolFlow.Text = ""

cmbOrificeType.SelectedItem = ""

        cmbTankType.SelectedItem =
""

    End Sub


    Private Sub frmCalc_Load(ByVal
sender As System.Object, ByVal e
As System.EventArgs) Handles
MyBase.Load


    End Sub

End Class
```