

# Speed and Current Observer for Induction Motor using Extended Kalman Filter

Anwin John

MTech Research Scholar  
Electrical and Electronics Engineering  
Govt.Engineering College,Idukki  
Idukki, India

Dr.Jayanand B

Professor  
Electrical and Electronics Engineering  
Govt.Engineering College,Thrissur

**Abstract-** This paper deals with the estimation of rotor currents and speed of an induction motor using Extended Kalman Filter Algorithm(EKF).Kalman filter in its basic form its an state estimator which can be used to estimate the rotor currents.By using extended kalman filter we can estimate other parameters along with rotor currents.In this paper we are estimating the rotor currents as well as speed using EKF.Simulation results using MATLAB software is analysed in this paper.

**Index Terms—**Induction motor,Kalman Filter,Extended Kalman filter(EKF)

## NOMENCLATURE

$V_{ds}, V_{qs}$	Stator voltages in a fixed reference frame [V].
$i_{ds}, i_{qs}$	Stator currents in fixed reference frame [A].
$V_{dr}, V_{qr}$	Rotor voltages in a fixed reference frame [V].
$i_{dr}, i_{qr}$	Rotor currents in fixed reference frame [A].
$L_s(L_r)$	Stator (rotor) inductance [H].
$L_m$	Mutual inductance [H].
$R_s(R_r)$	Stator (rotor) resistance [ $\Omega$ ].
$\omega$	Electrical angular rotor speed [el.rad/s].
$\omega_r$	Mechanical angular rotor speed [rad/s].
$t_s$	Sampling t .

## I.INTRODUCTION

In the recent years speed estimation have got great interest in induction motor control researches.Elimination of speed sensors and associated cables reduces the cost and it has got good reliability.Kalman filter is a linear filter,compared to other nonlinear filters[1] it has got good dynamic behaviour.Kalman filter provided optimal filtering of noises in the measurement and inside the system.The EKF is based on the nonlinear extended induction motor model that includes the rotor speed as a state variable

In this paper we are modeling the induction motor as a discrete time varying system and analyses are carried out.

## II. INDUCTION MOTOR MODEL

Estimation is based on a discrete time varying linear model of induction motor.

$$x(k+1) = F(k)x(k) + G(k)u(k) \quad (1)$$

$$y(k) = Hx(k) \quad (2)$$

where

$$F(k) = \begin{bmatrix} a_{11} & a_{12}\omega_r & a_{13} & a_{14}\omega_r \\ a_{21}\omega_r & a_{22} & a_{23}\omega_r & a_{24} \\ a_{31} & a_{32}\omega_r & a_{33} & a_{34}\omega_r \\ a_{41}\omega_r & a_{42} & a_{43}\omega_r & a_{44} \end{bmatrix} \quad (3)$$

$$G(k) = \begin{bmatrix} a_{15} & 0 \\ 0 & a_{25} \\ a_{35} & 0 \\ 0 & a_{45} \end{bmatrix} \quad (4)$$

$$x(k) = [i_{ds}(k) \ i_{qs}(k) \ i_{dr}(k) \ i_{qr}(k)]^T \quad (5)$$

$$u(k) = [v_{ds}(k) \ v_{qs}(k)]^T \quad (6)$$

where the coefficients

$$a_{10} = L_1L_2 - L_m^2 \quad a_{11} = 1 - (R_1L_2t_s/a_0)$$

$$a_{12} = L_m^2t_s/a_0 \quad a_{13} = L_mR_2t_s/a_0$$

$$a_{14} = L_mL_2t_s/a_0 \quad a_{15} = L_2t_s/a_0 \quad a_{21} = -a_{12}$$

$$a_{22} = a_{11} \quad a_{23} = -a_{14} \quad a_{24} = a_{13}$$

$$a_{25} = a_{15} \quad a_{31} = L_mR_1t_s/a_0 \quad a_{32} = -L_mL_1t_s/a_0$$

$$a_{33} = 1 - (R_2L_1t_s/a_0) \quad a_{34} = -L_1L_2t_s/a_0 \quad a_{35} = -L_mt_s/a_0$$

$$a_{41} = L_m L_1 t_s / a_0 \quad a_{42} = L_m R_1 t_s / a_0 \quad a_{43} = L_1 L_2 t_s / a_0$$

$$a_{44} = 1 - (R_2 L_1 t_s / a_0) \quad a_{45} = a_{35}$$

$t_s$  = discrete sampling interval

Output is required and stator currents are taken as output in practical case.

$$y(k) = [i_{ds} \ i_{qs}]^T \tag{7}$$

which gives output matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{8}$$

In the above representation the plant is assumed to be perfect but in actual case it is not. So for handling plant uncertainties a stochastic model is used which is given by

$$x(k+1) = F(k)x(k) + G(k)u(k) + w(k) \tag{9}$$

$$y(k) = Hx(k) + v(k) \tag{10}$$

where  $w(k)$  and  $v(k)$  are characterized by

$$E\{w(k)\} = 0, \quad E\{v(k)\} = 0$$

$$E\{w(k)w(j)^T\} = Q\delta_{kj} \quad Q \geq 0$$

$$E\{v(k)v(j)^T\} = R\delta_{kj} \quad R \geq 0$$

Initial state is characterized by

$$E\{x(0)\} = \hat{x}_0$$

$$E\{(x(0) - \hat{x}_0)(x(0) - \hat{x}_0)^T\} = P_0$$

### III. KALMAN FILTER

The main advantage of Kalman filter is its ability to measure the states which are not measurable. In cage induction motor the rotor parameters are very difficult to measure. So we can estimate the rotor parameters using Kalman filter. The Kalman filter algorithm is as follows.

Prediction :

$$\hat{x}(k+1/k) = F(k)\hat{x}(k/k) + G(k)u(k) \tag{11}$$

$$P(k+1/k) = P(k)P(k/k)F(k)^T + Q \tag{12}$$

Kalman gain:

$$K(k+1) = P(k+1/k)H(k+1)^T [H(k)P(k+1/k)H(k)^T + R]^{-1} \tag{13}$$

Time Update :

$$\hat{x}(k+1/k+1) = \hat{x}(k+1/k) + K(k+1)[y(k+1) - H(k+1)\hat{x}(k+1/k)] \tag{14}$$

$$P(k+1/k+1) = P(k+1/k) - K(k+1)H(k+1)P(k+1/k) \tag{15}$$

Where  $K(\cdot)$  is the kalman gain matrix

$\hat{x}(\cdot)$  is the state estimate

$k/k$  denotes the prediction at time  $k$  based on data upto and including time  $k$ .  $(k+1)/k$  denotes the prediction at time  $k+1$  based on data upto and including time  $k$ .

### IV. ROTOR CURRENT ESTIMATION

In this section the application of Kalman filter for the estimation of rotor current is described. The KF requires the measurement of plant input, plant output and rotor speed. The output. The output of KF are the estimates of stator and rotor currents.

### A. Simulation Results

The application of KF for current estimation is illustrated by a computer simulation in MATLAB software as shown in fig.1

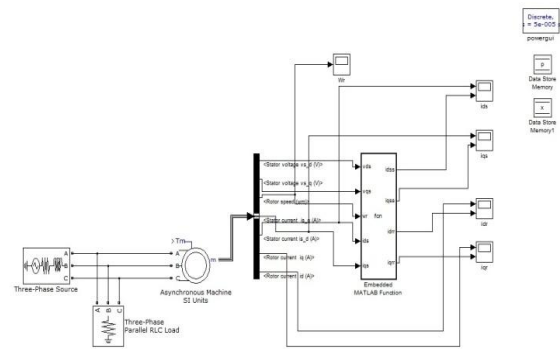


Fig.1 Matlab simulation set up for KF

The measurement noise covariance is

$$R = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix} \tag{16}$$

The process noise covariance is

$$Q = \begin{bmatrix} 0.0001 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix} \tag{17}$$

This covariance is chosen primarily to force the KF to use the stator current measurements by specifying that the model is not perfect. Selection of  $Q$  matrix is based on trial and error.

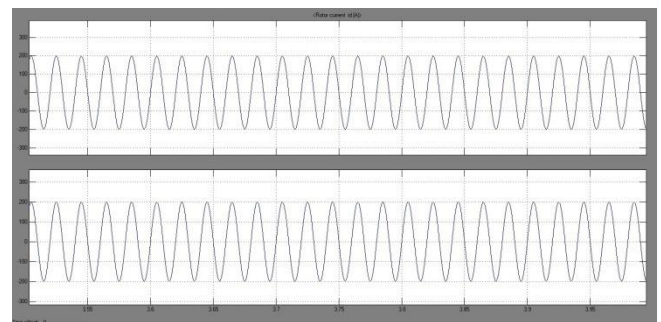


Fig 2.(a)Measured value of rotor d-axis current (b)Estimated value of rotor d-axis current.

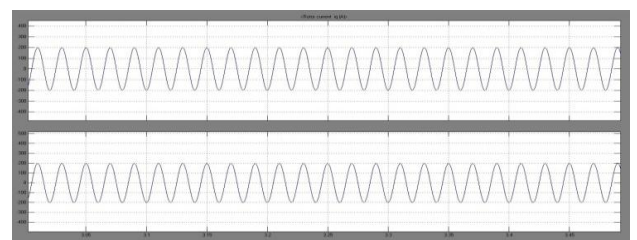


Fig 3.(a)Measured value of rotor q-axis current (b)Estimated value of rotor q-axis current.

From fig 2-3 it is clear that the estimated value follows the actual/measured value. Thus using KF we are getting an optimal estimate of rotor currents.

V. THE EXTENDED KALMAN FILTER(EKF)

The EKF is a non-linear version of Kalman filter which linearizes about an estimate of current mean and covariance. The EKF can be used for state estimation and parameter estimation by treating the parameter to be estimated as an extra state and thus forming augmented model. This augmented model is nonlinear because there is multiplication of states. This can be directly used in EKF.

To use the KF with nonlinear model we have to first linearize the model about a nominal or auxiliary state to produce a linear perturbation model.

Consider the nonlinear state space model[2]

$$x(k+1) = f(x(k),u(k)) + w(k) \tag{18}$$

$$y(k) = Hx(k)+v(k) \tag{19}$$

the associated EKF algorithm is

$$\hat{x}(K+1) = f(\hat{x}(k),u(k))+K(k)[y(k)-H\hat{x}(k)] \tag{20}$$

$$K(k) = F(k)P(k)H^T[HP(k)H^T+R]^{-1} \tag{21}$$

$$P(k+1) = F(k)P(k)F(k)^T + Q - K(k)[HP(k)H^T + R]K(k)^T \tag{22}$$

$$F(k) = \frac{\partial f(\cdot)}{\partial x(k)} \Big|_{\hat{x}(k),u(k)} \tag{23}$$

A. Rotor Speed Estimation

Here EKF theory is applied for the simultaneous estimation of stator and rotor currents together with rotor speed.

$$x(k) = [x_1(k) \ x_2(k) \ x_3(k) \ x_4(k) \ x_5(k)]^T$$

$$[i_{ds}(k) \ i_{qs}(k) \ i_{dr}(k) \ i_{qr}(k) \ W_r(k)]^T$$

$$x_1(k+1) = a_{11}x_1(k) + a_{12}x_2(k) + a_{13}x_3(k) + a_{14}x_4(k)x_5(k) + a_{15}u_1(k) + w_1(k) \tag{20}$$

$$x_2(k+1) = a_{21}x_1(k)x_5(k) + a_{22}x_2(k) + a_{23}x_3(k)x_5(k) + a_{24}x_4(k) + a_{25}u_2(k) + w_2(k) \tag{21}$$

$$x_3(k+1) = a_{31}x_1(k) + a_{32}x_2(k)x_5(k) + a_{33}x_3(k) + a_{34}x_4(k)x_5(k) + a_{35}u_1(k) + w_3(k) \tag{22}$$

$$x_4(k+1) = a_{41}x_1(k)x_5(k) + a_{42}x_2(k) + a_{43}x_3(k)x_5(k) + a_{44}x_4(k) + a_{45}u_2(k) + w_4(k) \tag{23}$$

$$x_5(k+1) = x_5(k) + n(k) \tag{24}$$

Terms  $w_1(k)$ - $w_4(k)$  and  $n(k)$  are the zero mean process noise sequences.

The partial or jacobian derivative matrix for the induction motor is

$F(k) =$

$$\begin{bmatrix} a_{11} & a_{12}x_5(k) & a_{13} & a_{14}x_5(k) & a_{12}x_2(k) + a_{14}x_4(k) \\ a_{21}x_5(k) & a_{22} & a_{23}x_5(k) & a_{24} & a_{21}x_1(k) + a_{23}x_3(k) \\ a_{31} & a_{32}x_5(k) & a_{33} & a_{34}x_5(k) & a_{32}x_2(k) + a_{34}x_4(k) \\ a_{41}x_5(k) & a_{42} & a_{43}x_5(k) & a_{44} & a_{41}x_1(k) + a_{43}x_3(k) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

B. Simulation Results

Fig 4 shows the simulation set up for estimation of rotor speed along with rotor currents.

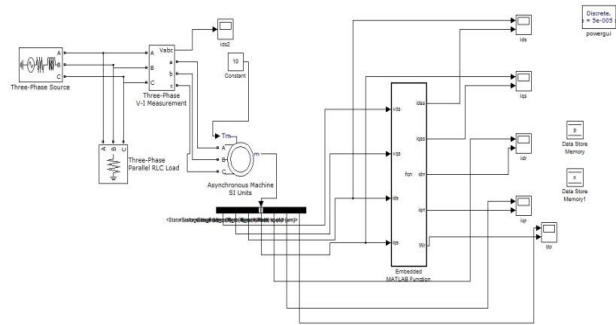


Fig.4 Matlab simulation set up for estimation of rotor speed

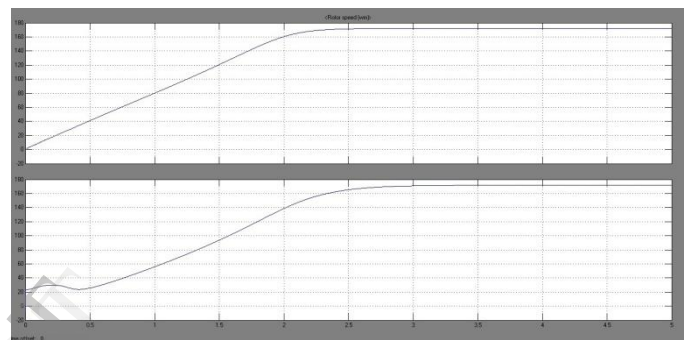


Fig 5.(a)Measured value of rotor speed (b)Estimated value of rotor speed

From the fig.5 it is clear that the estimated value follows the actual value. By using EKF we are getting an optimal filtering. In fig 6 -7 the rotor currents are plotted. From fig 6-7 it is clear that the estimated value follows the actual value. During the initial stage(transient state) the estimation of currents is optimal.

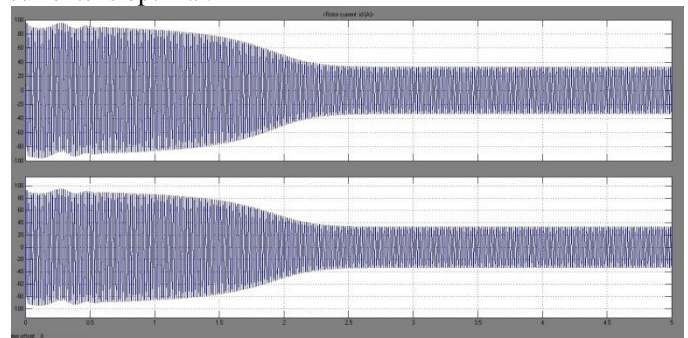


Fig 6.(a)Measured value of rotor d-axis current (b)Estimated value of rotor d-axis current.

## REFERENCES

- [1] C. Manes, F. Parasiliti, and M. Tursini, "A comparative study of rotor flux estimation in induction motors with a nonlinear observer and the extended Kalman filter," in *Proc. IEEE IECON'94*, 1994, pp. 2149–2154
- [2] D. J. Atkinson, P. P. Acarnley, and J. W. Finch, "Parameter identification techniques for induction motor drives," presented at the *European Power Electron. Conf.*, Aachen, Germany, 1989, pp 307 -312.

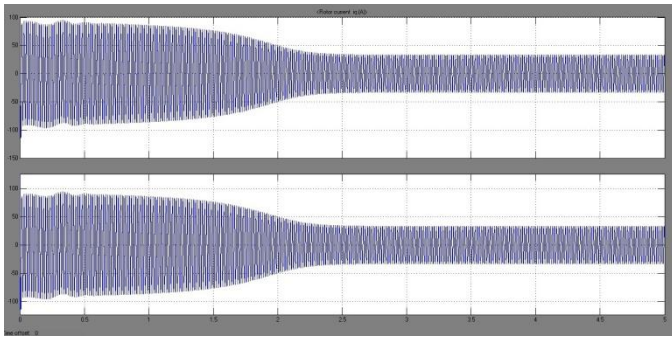


Fig 7.(a)Measured value of rotor q-axis current (b)Estimated value of rotor q-axis current.

## VI.CONCLUSION

This paper shows how Kalman filter can be used to estimate the rotor currents by combining informations from the plant model along with output measurements to produce an optimal state of the motor. Extended Kalman filter is used to estimate the rotor speed along with rotor currents.

IJERT