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Stability Analysis Of A Closed Loop System With Proportional Controller Having Gain And/or Phase Uncertainties

S. N. Sivanandam , K. Sreekala

Professor Emeritus, Computer Science Engg, Karpagam Institute of Technology, Coimbatore
Associate Professor, Electrical and Electronics Engg, MET'S School of Engg, Mala, Trissur

Abstract

In this paper two algebraic techniques are suggested to compute the stability of a closed loop system, which uses a proportional controller having gain and/or phase uncertainties. In both the schemes, Routh-like table is developed and the elements in the first column are utilized to analyse the stability of the system. In the proposed first scheme; Sign Pair Criterion I(SPC I), the first two rows are formed directly using complex coefficients while in the second scheme; Sign Pair Criterion II (SPC II), the real and imaginary parts of the given complex polynomial are separated to form the first two rows of Routh –like table. These two criteria are very convenient compared to other available algebraic procedures for analysis of stability .

1. Introduction

Any system which is useful to human kind should possess the basic property known as stability. The stability of any system indicates its ability to find a condition of static equilibrium after it has been disturbed. Gain margin of a system can be defined as the maximum gain of the system that can be increased without loosing stability and phase margin is the amount of additional phase lag required to bring the system to the verge of instability. Gain and phase margins are the measures of relative stability analysis and act as a design tool for dynamic systems.

Edward John Routh provided a numerical technique for determining the distribution of roots of a given characteristics equation in the s -plane [1],[2]. Routh test is still used as a basic scheme for analysis of stability in a linear time – invariant continuous system; this algebraic test is simple in application compared to any other procedure and further it can also be used for parameter design. When the characteristic equation has complex coefficients, the given equation C(s) =0 with complex coefficients can be multiplied by its conjugates and the transformed equation T(s) = C(s) x C*(s) =0, [3],[4] and can be handled by Routh's test, but the computational work load will be more in this procedure.

A method for algebraically computing all the stabilizing gains for a given linear system is introduced in [5]. The method is then generalized for PID controllers [6]. Another method is there for computation of stabilizing PI and PID controllers with specified gain and phase margins [7]. The existing methods do not present an efficient algorithm if only proportional controller is used.

In this paper, the procedure utilized in the formulation of direct Routh's test for handling the n^{th} degree characteristic equation with real coefficients is extended suitably for complex coefficients and developed the two stability criteria ;Sign Pair Criterion-I (SPC -I) and Sign Pair Criterion -II (SPC -II) [8],[9]. In the first approach, using the coefficients of characteristic equation , the first – two rows are formed and the Routh multiplication rule is used for computing the remaining elements in the table. In the second scheme , the real and imaginary parts of the given complex polynomial are separated to form the two rows of Routh –like table.

2. Modelling of the System

Consider the single input single output closed loop control system of figure 1 shown below.

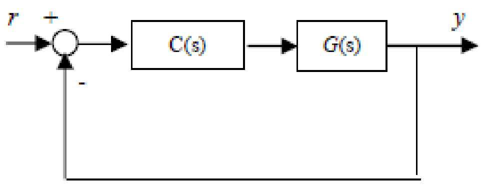

Figure 1. Closed loop system with constant gain

Where 'r' is the input signal and 'y' is the output signal. C(s) is a constant gain proportional controller of the form C(s) = Kp. G(s) is the plant to be controlled which is represented as shown below.

$$G(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1}s^{m-1} + \cdots + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0} \quad (1)$$

The problem is to check the stability of the closed loop system for a specified value of 'Kp' and a particular phase margin. It is possible to represent gain and phase uncertainties in a plant using a complex element $Ke^{-j\theta}$ as illustrated in figure 2 [10].

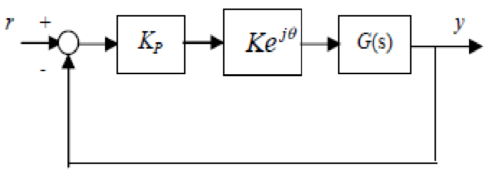

Figure 2. Proportional control of a system with gain and/or phase uncertainties

Gain margin denotes the largest value that K can assume for θ= 0, and phase margin denotes the largest value of θ for K=1 , without affecting closed loop system stability. Consider the phase shifted system $G_{new}(s)$ for K=1;

$$G_{new}(s) = \frac{e^{-j\theta}N(s)}{D(s)} = \frac{(\cos\theta - j\sin\theta)N(s)}{D(s)} \triangleq \frac{N_{new}(s)}{D(s)} \quad (2)$$

In this case the coefficients of the numerator polynomial of equation (2) become complex. The closed loop transfer function for the phase shifted system for K=1 is

$$\frac{Y(s)}{R(s)} = \frac{Kp\ Gnew(s)}{1+Kp\ Gnew(s)} \quad (3)$$

Substituting the value of $G_{new}(s)$ from equation (2) in (3),

$$\frac{Y(s)}{R(s)} = 1 + Kpe^{-j\theta}\ \frac{N(s)}{D(s)} \quad (4)$$

The characteristic equation for the new system is deduced as

$$F(s) = D(s) + Kpe^{-j\theta}\ N(s) = 0 \quad (5)$$

$$= D(s) + Kp\ (\cos\theta - j\sin\theta)\ N(s) = 0 \quad (6)$$

The equation (6) consists of complex coefficients of the form as shown below,

$$f(s) = s^n + (a_1 + jb_1)s^{n-1} + (a_2 + jb_2)s^{n-2} + \ldots + (a_n + jb_n) = 0 \quad (7)$$

The stability analysis can be done for equation (6) and (7) by applying the new proposed criteria SPCI and SPC II [8], [9].

3. Illustrations

3.1 Example 1

For a closed loop system as shown in figure (2), values of N(s) and D(s) are as given below, analyse the stability for Kp= 8 and PM=17.342. [10]

$$N(s) = 0.5s^4 + 2.5s^3 + 5s^2 + 24.375s + 31.22$$
$$D(s) = 1.09s^4 - 13.12s^3 + 64.23s^2 - 151.11s + 70.89$$

$$e^{-j17..3} = 0.9548 - j0.2974$$

Substituting all values in equation (6) ,the characteristic equation is formed as equation (8)

$$F(s) = (4.91 - j1.19)s^4 + (5.976 - j5.952)s^3 + (102.42 - j11.896)s^2 + (35.074 - j57.99)s + (309.362 - j74.28) = 0$$

This equation can be converted to the form of equation (7) and can be written as

$$F(s) = s^4 + (1.43 - j0.866)s^3 + (20.27 + j2.5)s^2 + (9.448 - j9.525)s + (63.017 + j0.17) = 0$$

3.1.1 Stability Analysis using SPCI

The Routh like table for the above equation is formed as shown below.

+1	-j0.866	20.27	-j9.525	63.017
+1.429	j2.5	9.448	j0.17	
-j2.62	13.66	-j9.64	63.02	
-j4.96	4.18	-j34.26		
+11.46	j8.41	63.02		
+0.53	-j6.96			
+j157.63	63.02			
-j6.75				

The fourth pair P4 =[+j157.63, - j6.75] is having the two elements with different signs. So P4 fails to satisfy the stability condition as per SPC I [8]. The system is found to be unstable. The result is in agreement with the result given in [10].

3.1.2 Stability Analysis using SPCII

By substituting s=jω, in the characteristic equation will be as follows,

$$F(j\omega) = R(\omega) + jI(\omega) = 0$$
$$= \omega^4 - 0.866\omega^3 - 20.27\omega^2 + 63.017$$
$$+ j(-1.42\omega^3 - 2.5\omega^2 + 9.45\omega + 0.17) = 0$$

Routh like table is formed as per SPCII [9].

+1	-0.866	-20.27	9.525	63.017
+ 0.01	-1.429	-2.5	9.448	0.17
+142.03	229.73	-935.275	46.017	
-1.4452	-2.4342	9.4448	0.17	
-9.5022	-7.0291	62.7249		
-1.3651	-0.0949	0.17		
-6.3682	61.5415			
-13.2871	0.17			
+61.4601				

Here also the fourth pair P4 =[-13.2871, +61.46] is having the two elements with different signs. So P4 fails to satisfy the stability condition as per SPC II [9]. The system is found to be unstable. The result is in agreement with the result given in [10] and also as per SPC I.

3.2 Example 2

For the system given in example 1, find the stability for Kp = 11 and PM=17.342

Substituting all values in equation (6) ,the characteristic equation is formed as

$$F(s) = s^4 + (2.256 - j0.708)s^3 + (17.893+j2.047)s^2 + (18.565 - j7.782)s + (62.892 + j0.149) = 0$$

3.2.1 Stability Analysis using SPCI

The Routh like table for the above equation is formed as shown below.

+1	- j 0.708	17.893	-j 7.782	62.892
+2.256	j 2.047	18.565	j 0.149	
-j 1.615	9.6638	- j7.848	62.892	
-j11.45	7.6045	-j87.686		
+8.591	j4.5232	62.892		
+1.5763	-j 3.8673			
+j25.6004	62.892			
+j 0.0052				

Here all the pairs obey the SPCI and so the system is stable .This result matches with that of [10]

3.2.2 Stability Analysis using SPCII

By substituting s=jω, in the characteristic equation , the Routh like table is formed as,

+1	-7.08	-17.893	7.782	62.9
+ 0.01	-2.256	-2.047	18.565	0.149
+224.89	186.807	-1848.72	47.992	
-2.264	-1.9648	18.56	0.149	
-0.8.34	-5.045	62.79		
-0.595	1.51	0.149		
-26.22	60.7			
+0.13	0.149			
+ 90				

Here also all the pairs obey the stability condition as per SPCII and so the system is stable .This result matches with that of [10] and also with SPC I.

3.3 Example 3

For the system given in example 1, find the stability for Kp = 11 and PM=20

Substituting all values in equation (6) ,the characteristic equation is formed as

$$F(s) = s^4 + (2.278 - j.82)s^3 + (17.83+j2.36)s^2 + (18.82 - j9)s + (62.9 + j0.13) = 0$$

3.3.1 Stability Analysis using SPCI

The Routh like table for the above equation is formed as shown below.

+1	- j0.82	17.83	- j9	62.9
+2.28	+j 2.36	18.82	j0.13	
- j1.855	9.5756	- j9.06	62.9	
- j9.41	7.6885	-j77.18		

+ 8.06	j6.16	62.9
+0.498	- j3.748	
+j66.83	62.9	
- j3.279		

Here also the fourth pair P4 =[+j66.83,- j3.279] is having the two elements with different signs. So P4 fails to satisfy the stability condition as per SPC I [8]. The system is found to be unstable. The result is in agreement with the result given in [10] and also as per SPC II.

3.3.2 Stability Analysis using SPCII

By substituting s=jω, in the characteristic equation , the Routh like table is formed as,

+ 1	-j0.82	-17.83	j9	62.9
+ 0.01	-2.28	-2.36	18.82	0.13
+227.18	218.17	-1873	49.9	
-2.29	-2.3	18.82	0.13	
-7.8	-5.85	62.8		
-0.563	0.42	0.13		
-11.65	61			
-2.53	0.13			
+ 0.06				

Here also the fourth pair P4 =[-2.53 , + 0.06] is having the two elements with different signs. So P4 fails to satisfy the stability condition as per SPC II [9]. The system is found to be unstable. The result is in agreement with the result given in [10] and also as per SPC I.

4. Conclusion

In this paper, the stability analysis of a closed loop linear time invariant continuous systems with a proportional controller represented in the form of their respective characteristic equations having complex coefficients have been performed with the help of the proposed SPC-I and SPC-II. The proposed algebraic criteria are simple and direct in application compared to other schemes .

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