

Stability of Offshore Barge Subjected to Parametric Rolling in Waves

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Abstract— This paper deals with the stability of an offshore barge when subjected to parametric rolling in longitudinal waves. The main aim of this paper is to understand the roll behaviour of an offshore barge under longitudinal waves, to predict the susceptibility of parametric rolling in longitudinal waves and to identify the region of instability when subjected to parametric rolling. The primary objective is to find the roll response of a geometrically modeled offshore barge in ANSYS AQWA for regular sea conditions followed by the calculation of roll response in a conservative way by the Linear Strip theory to validate with the results obtained from ANSYS AQWA. Two major cases were studied; once when the length of the waves is equal to the length of the barge and the other when the length of the encountering waves is four times the length of the barge. The severity of the parametric rolling for the offshore barge is calculated by the mathematical modeling for solution of Mathieu's Differential equation using Matlab programming, taking in account the stability parameters for the offshore barge for which stability curves are plotted. This Graphical Ince-Strutt diagram was then verified by numerical modeling to illustrate the instability of barge. The boundary approximations for the instability zone for which frequencies prone to parametric roll resonance were identified and their threshold damping coefficient was then calculated.

Keywords— Parametric rolling, rolling, hydrodynamic, roll resonance, barge, longitudinal waves, strip theory.

I. INTRODUCTION

Parametric roll may be defined as the spontaneous rolling motion of the ship moving in head or following seas that come about as a result of the dynamic instability of motion. The development of the parametric roll occurs under the conditions that the encounter angular frequency is approximately twice the roll angular frequency, the wavelength is equal to the ship length and the roll damping is insufficient to dissipate the parametric roll energy. Due to the unexpected nature of the motion as compared with synchronous roll in following or beam seas on smaller and finer ships, parametric roll is quite dangerous and unpredictable in real seas when multiple seas and swells coming from different directions. In head waves roll motion caused by direct wave excitation are not possible. Nevertheless, under certain conditions of encounter period, a rolling can be excited in head seas. The roll motion, once started, may grow to large amplitude limited by roll damping and, in extreme conditions, may result in danger to the ship or its contents. This phenomenon is referred to as "auto

parametrically excited motion" which is usually shortened to "parametric motion".

The objective of the research paper is to understand the roll behavior under longitudinal waves, to predict the susceptibility of parametric rolling of the barge in longitudinal waves and to predict the region of instability Susceptibility of parametric rolling.

A. Mathieu's Equation

The variation of GM with time may result in parametric resonance (Belenky, et al., 2004). To check whether this is possible, we transform the roll equation of motion to the form of Mathieu's equation in order to use the Ince-Strutt diagram to examine the properties of solutions. The Mathieu's equation [France, et al., 2001] can also be written as:

$$\frac{d^2\phi}{d\tau^2} + (\delta + \varepsilon \cos\tau)\phi = 0 \quad (1)$$

Where,

$$\delta = \frac{\omega_n^2}{\omega^2}, \varepsilon = C \frac{\omega_n^2}{\omega^2} \quad (2)$$

C is the fractional change of GM due to waves.

The above equation recognised as Mathieu Equation (France, et al., 2001) is seen to be a linear differential equation with a time varying restoring coefficient. The solutions of this equation has been studied extensively, are found to exhibit unstable behaviour at certain values of the frequency parameter δ . The shaded regions are stable corresponding to the (δ, ε) pairs for which parametric motion cannot exist. In an unstable region, an arbitrarily small disturbance will trigger an oscillatory motion that tends to increase indefinitely with time.

II. RESULTS

A. Validation

Initially an offshore barge was modeled in ANSYS AQWA (Fig. 1) and the response in the six degree of freedom was validated with a Journal Paper (Seung-Chul Lee et al, Analysis of motion response of barges in regular waves, 2012).

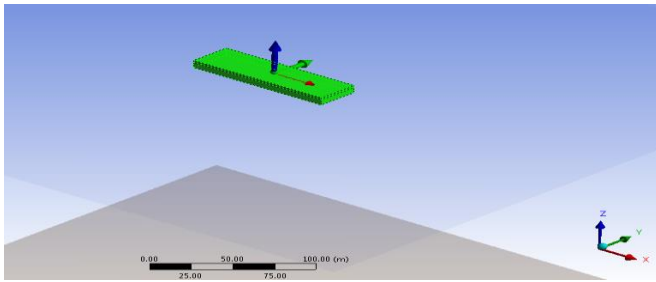


Fig 1. Barge modeled in ANSYS AQWA

TABLE I. PARTICULARS OF THE BARGE

Parameters	Value
Length	100m
Beam	30.75m
Depth	7.5m
Draft	5.5m

TABLE II. CALCULATION CONDITIONS

Parameters	Value
Water depth	300m
Wave frequency	0.25-1.25(0.05) rad/sec [5sec -25 sec]
Xg, yg, zg.	-6.758, 0.0, 2.857
Wave directions	-180 to +180 (degrees)

TABLE III. HYDROSTATIC DETAILS

Parameters	Value
Actual volume displacement	16774.941 m ³
Cut water plane area	3049.988 m ²
BG	5.607 m
GMX	8.487 m
BMX	14.094 m

TABLE IV. DETAILS OF POINT MASS

Parameters	Value
Mass	17194314.94 kg
K _{xx}	10.37 m
K _{yy}	25 m
K _{zz}	26 m
I _{xx}	1849023326.42251 kg.m ²
I _{yy}	10746446838.3789 kg.m ²
I _{zz}	11623356900.3906 kg.m ²

The RAO in different modes of freedom in predominant directions were validated subsequently. The validation is shown in the following figures listed from Fig. 2 to Fig. 7.

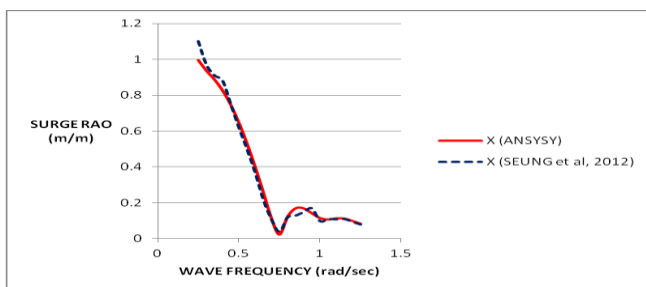


Fig.2 Surge RAO (90 degrees)

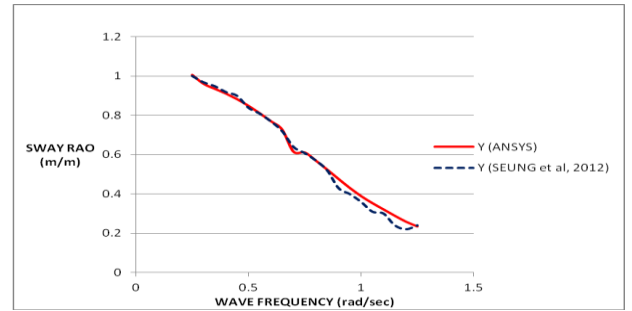


Fig.3 Sway RAO (90 degrees)

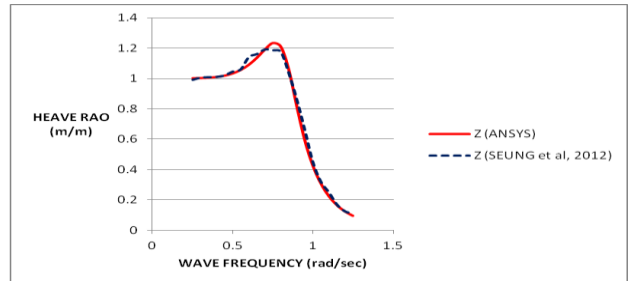


Fig.4 HeaveRAO (90 degrees)

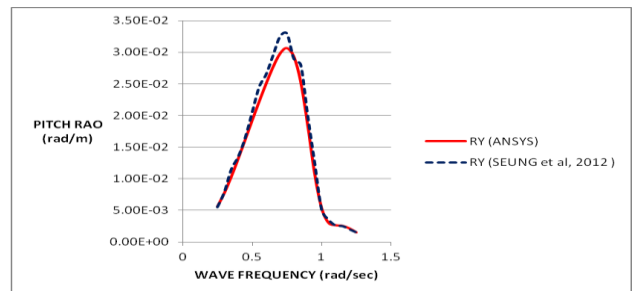


Fig.5 Pitch RAO (150 degrees)

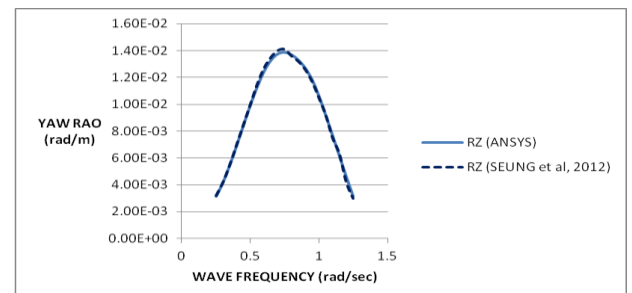


Fig.6 Yaw RAO (120 degrees)

B. Validation of Roll Response

Details for the validation of roll response is given in Table V

TABLE V. ROLL RESPONSE VALIDATION

Parameters when $\omega = 0.7$ rad/sec	Validation AQWA	by	Validation by Strip Theory
Roll Moment of Inertia (kg-m ²)	1.2451e9		12540138280
Stiffness (N-m)	1.43119e9		1429480376
Damping (Nm/s)	3.618e7		3.3544e7
Natural frequency (rad/s)	0.68		0.679

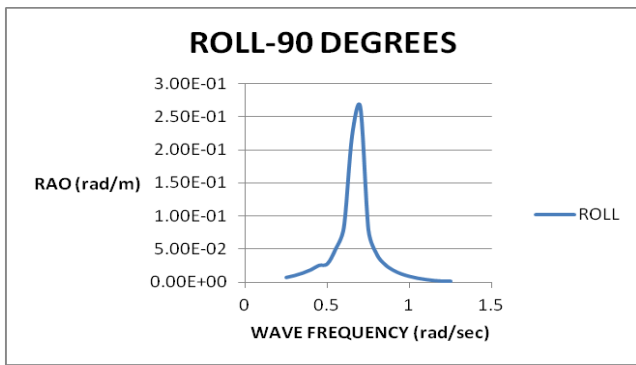


Fig.7 Roll RAO (90 degrees)

C. Static Curve of Stability for Small Angle of Heel

The Barge modeled was then calculated for the Static Stability Curves at calm water. The variation of metacentric radius, vertical centre of buoyancy, metacentric height with respect to the small angle of heel are plotted in Fig. 8 and Fig. 9.

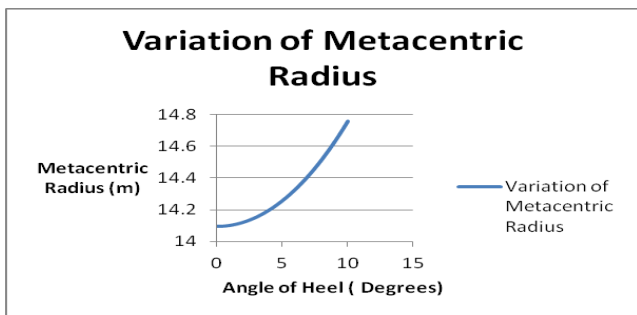


Fig.8 Variation of metacentric radius

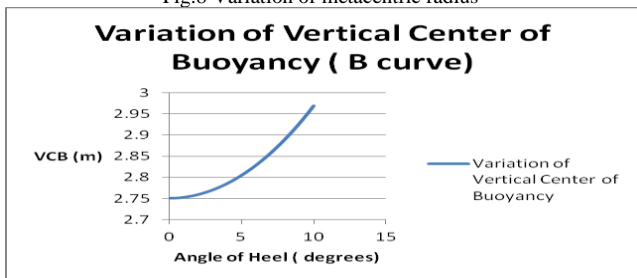


Fig.9 B-Curve

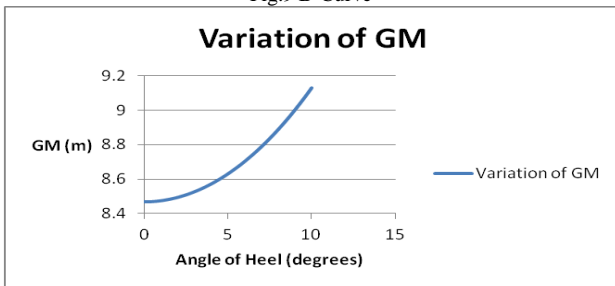


Fig.10 Variation of GM

D. Wave Influence on GM

Two major cases were studied wherein the encountering length of wave was four times the length of the barge and encountering length of the wave was twice that of the length of the barge. For each of the case, the fractional variation of metacentric height was found out using the computational technique for wave influence on GM, which was then incorporated for further two cases viz,

- When the encountering frequency is almost equal to the natural roll frequency.
- When the encountering frequency is half of the natural roll frequency.

Table VI and Table VII show the ship parameters for the different relationship between length of wave and length of ship respectively. Based upon the values in Table VI and Table VII, variation of metacentric height with respect to various crest position was then plotted as shown in Fig. 11 and Fig. 12.

TABLE VI. WHEN LENGTH OF WAVE IS FOUR TIMES LENGTH OF SHIP

Parameters	Value	Units
I_{xx}	1.2451e9	kg-m ²
\bar{I}_{xx}	143956061	kg-m ²
GM_0	8.487	m
GM_{max}	9.629942	m
GM_{min}	7.569313	m
GM_m	8.59133	m
GM_a (C)	1.03036	m
$(\omega_n)^2$	0.46104	rad/sec
\bar{d}	0.9409	-
\bar{v}	0.977	-
L_w	400	m
H_w	1	m
d_m	5.5	m

TABLE VII. WHEN LENGTH OF THE WAVE IS TWICE LENGTH OF THE SHIP

Parameters	Value	Units
I_{xx}	1.2451e9	kg-m ²
\bar{I}_{xx}	143956061	kg-m ²
GM_0	8.487	m
GM_{max}	9.63919	m
GM_{min}	7.551613	m
GM_m	8.59426	m
GM_a (C)	1.04265	m
$(\omega_n)^2$	0.4624	rad/sec
\bar{d}	0.94367	-
\bar{v}	0.9839	-
L_w	400	m
H_w	1	m
d_m	5.5	m

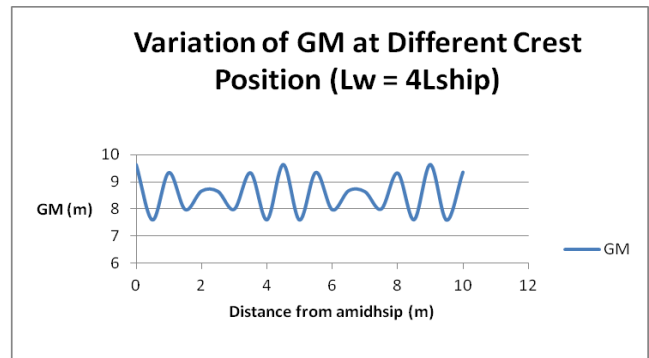


Fig 11. Length of the wave is four times length of ship

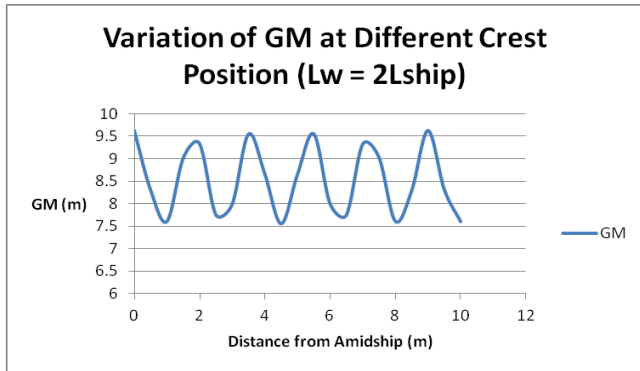


Fig12. Length of the wave is twice the length of the ship

Fig. 13 shows the time varying metacentric height. From the figure.8, it was observed that for any time period, the variation of metacentric height for different location of crest from amidships when the length of the encountering wave was four times as that of the length of the barge, was not very high as compared to the figure.9.

In the figure.9, the length of the encountering wave is twice the length of the barge and comparatively the variation of metacentric height for different location of the wave crest from amidships was found to be higher. Hence one could have a vague idea and inference that parametric roll could be more profound in this case. Thus the more fluctuations in the metacentric variation could be an indication for the parametric rolling.

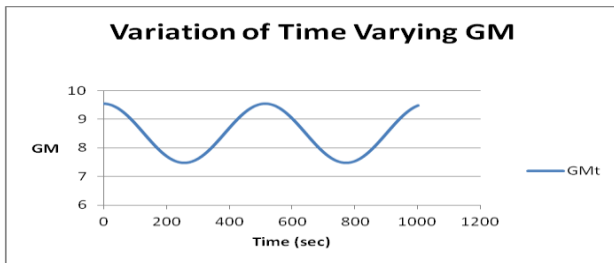


Fig 13. Time Varying metacentric height

E. Solution of Mathieu's Differential Equation

Four cases were studied to find out the stability using the graphical Ince-Strutt diagram. Table VIII and Table IX represents stability of ship for different relationship between length of the wave and length of the ship respectively.

TABLE VIII. STABILITY WHEN LENGTH OF WAVE IS FOUR TIMES LENGTH OF SHIP

	$L_w = 4L_{ship}$ $\omega_n = \omega$	$L_w = 4L_{ship}$ $\omega_n = 2\omega$
C (m)	1.0303	1.0303
d	0.94367	3.94
v	0.97226	4.06
Remarks	Stable	Stable

TABLE IX. STABILITY WHEN LENGTH OF WAVE IS TWICE LENGTH OF SHIP

	$L_w = 2L_{ship}$ $\omega_n = \omega$	$L_w = 2L_{ship}$ $\omega_n = 2\omega$
C (m)	1.0426	1.0426
d	0.94367	4.515
v	0.9839	4.7075
Remarks	Stable	Un-Stable

For (d,v) pairs calculated for four different cases, it was plotted graphically on the Ince-Strutt diagram (Fig. 14.) and the region of instability was indicated. It can be seen that while for different (d,v) considered, they either lie in the shaded region or on the verge of shaded and in-shaded region except for one pair that was seen to lie outside the shaded region.

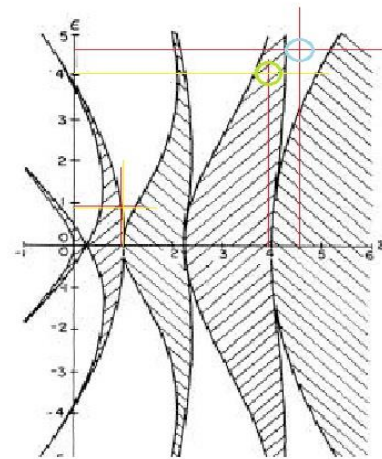


Fig 14. Ince-Strutt diagram

Usually parametric roll is profound when the natural roll frequency is twice the encountering wave frequency but in our analysis we got it when the natural roll frequency was 2.125 times the encountering wave frequency, which was almost near to the condition for parametric roll. This region of instability was further verified mathematically by substituting these values in the Mathieu's differential equation and solving them.

The Mathieu differential equation was finally solved in Matlab using ODE45 inbuilt function and following were the results that were obtained. The Mathieu's equation for the region of stability and instability were obtained from Table IX and was solved subsequently (Fig.15 and Fig. 16)

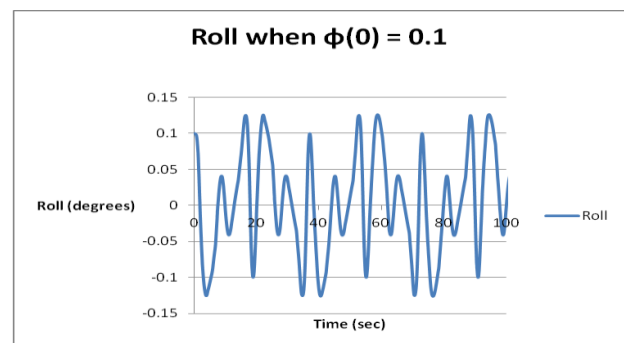


Fig 15. Solution of Mathieu's Equation for region of stability

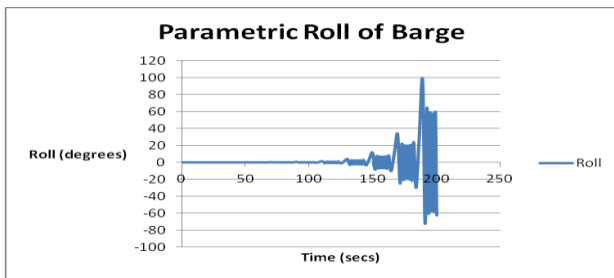


Fig 16. Solution of Mathieu's Equation for region of Instability

III. CONCLUSIONS

In this paper the instability of the barge was analyzed by calculating the fractional change in the metacentric height and incorporating those values at roll resonance to assess how the fractional change can lead to parametric rolling. It should be noted that the Mathieu's equation here was solved just for two cases i.e. when the length of the encountering wave was four times the length of the barge for encountering wave frequency equal to natural roll frequency and when the length of the encountering wave was twice the length of the barge for natural roll frequency is 2.125 times the encountering wave frequency. This was done to signify and illustrate how the fractional variation of metacentric height for the two cases of encountering wave length can cause instability of the ship and how parametric roll occurs.

Moreover it was seen that on the Ince-Strutt stability diagram, the region of instability was mathematically verified by solving the Mathieu's differential equation while the

region of stability was also justified with the mathematical solving of Mathieu differential equation.

Though the fractional variation of the metacentric height for the two cases of encountering wavelength did not vary from each other to a larger extent but it was shown that a small change in the fractional variation of the metacentric height can cause instability of the ship inducing parametric rolling. Both the criteria for the susceptibility of parametric rolling were satisfied wherein the length of the encountering wave should be twice the length of the ship and when the natural roll frequency is twice the encountering wave frequency.

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