Stabilization of Inverted Pendulum (IP) Using Two Fuzzylogic Controllers (FLC's) Having Nine Linguistic Variables"

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ABATRACT- This paper presents a Modelling and Simulation study of a control strategy for an Inverted Pendulum system. Inverted Pendulum is well known as a testing bed for various controllers. For independent control of Cart and Pendulum two FLC's have been defined separately using nine linguistic variables. Modelling and Simulation has been done in Matlab and Simulink respectively. In addition to Step-responses and Pulse-responses, an Open-loop and Closed-loop response using PID controllers are also shown. The proposed fuzzy control scheme successfully fulfils the control objectives and also has an excellent stabilizing ability to overcome the external impact acting on the pendulum system.

KEYWORDS- Inverted Pendulum, Fuzzy logic Controller, Unit-Step response, Pulse-response.

I. INTRODUCTION

The inverted pendulum is a classical problem in control system [1]. It is a system that is inherently unstable. The pendulum is mounted onto a non-stationary cart in the vertical position or at equilibrium position. The pendulum is unstable and free to fall over if there are any disturbances. On the other hand, if the pendulum is too thin and the cart is move by a force, it can flex and cause vibrations. These vibrations are one of the

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the major concerns in industry machineries. In conventional control theory, most of control problems are usually solved by mathematical tools based on system models. But in true sense, there are many complex systems whose accurate mathematical models are not available or difficult to formulate. As an alternative to conventional control approach, the fuzzy control techniques can provide a good solution for these problems by introducing linguistic information [2]. The control of inverted pendulum is fundamentally same as those involved in rocket or missile propulsion, walking robot, flying objects in space etc [3]. Figure 1.0 shows a view of inverted pendulum [3].

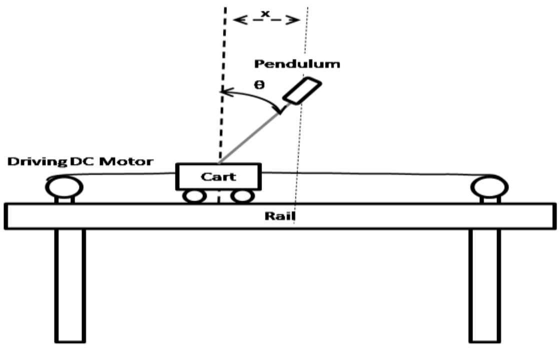

Figure 1.0 view of Inverted Pendulum

An Inverted Pendulum has two equilibrium points: vertical upright equilibrium point and downward equilibrium point [4]. When the pendulum is standing at vertical upright position and the resultant of forces acting from all sides is zero it is said to be in vertical upright equilibrium point. The vertical upright equilibrium point is inherently unstable, as any small disturbance may cause the pendulum to fall on the either side when the cart is at rest. When there is no external force acting on pendulum it will come into rest in downward position. This equilibrium position is stable.

II. MATHEMATICAL MODELLING OF INVERTED PENDULUM

The system consists of a pole with mass, m , hinged by an angle θ from vertical axis on a cart with mass, M, which is free to move in the x direction as shown in figure 1.1[2] . A force, F is required to push the cart horizontally, the friction coefficient of cart b, the length between axle centre and the centre of pendulum L, the inertia of pendulum I [6].

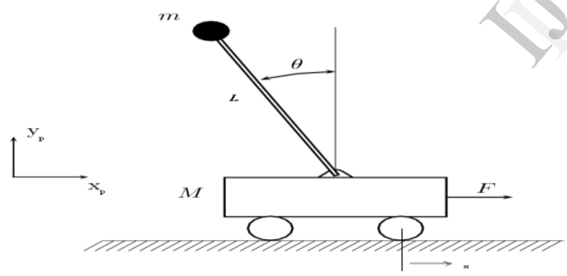

Figure 1.1 Diagram of Inverted Pendulum

For simulation the dynamic equations of the actual inverted pendulum system are derived as follows consider figure 1.2[2,7].

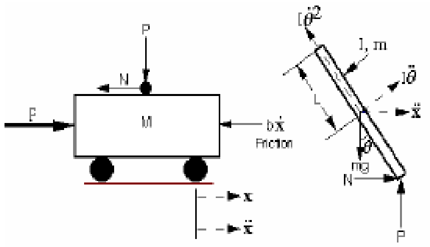

Figure 1.2 FBD of Cart & Pendulum

By Newton's Equation:

For Cart:

$$M\frac{d^2x}{dt^2} = \sum_{cart} F \qquad \cdots 1$$

$$F = M\ddot{x} + b\dot{x} + N \qquad \cdots 2$$

Where, N and P are the interaction forces between the cart and pendulum.

$$\frac{d^2x}{dt^2} = \frac{1}{M}\left(F - N - b\frac{dx}{dt}\right) \qquad \cdots 3$$

For Pendulum:

$$I\frac{d^2\theta}{dt^2} = \sum_{pend} \tau \qquad \cdots 4$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{I}\left(Nlcos\theta + Plsin\theta\right) \qquad \cdots 5$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics. The inclusion of these forces requires modelling the x and y dynamics of the pendulum in addition to its theta dynamics. Therefore, the additional x and y equations for the pendulum are modelled as given by the equations below:

$$m\frac{d^2x_p}{dt^2} = \sum_{pend} F_x = N \qquad \cdots 6$$

$$N = m\frac{d^2x_p}{dt^2} \qquad \cdots 7$$

$$m\frac{d^2y_p}{dt^2} = \sum_{pend} F_y = P - mg \qquad \cdots 8$$

$$P = m\left(\frac{d^2y_p}{dt^2} + g\right) \qquad \cdots 9$$

However Xp and Yp are exact functions of theta. Therefore, their derivatives are represented in terms of the derivatives of theta

$$x_p = x - l\sin\theta \qquad \cdots 10$$

$$\frac{d\,x_p}{dt} = \frac{dx}{dt} - l\cos\theta\frac{d\theta}{dt} \qquad \cdots 11$$

(2)

$$\frac{d^2x_p}{dt^2} = \frac{d^2x}{dt^2} + l\sin\theta\left(\frac{d\theta}{dt}\right)^2 - l\cos\theta\frac{d^2\theta}{dt^2} \qquad \cdots 12$$

$$y_p = l\cos\theta \qquad \cdots 13$$

$$\frac{dy_p}{dt} = -l\sin\theta\frac{d\theta}{dt} \qquad \cdots 14$$

$$\frac{d^2y_p}{dt^2} = -l\cos\theta\left(\frac{d\theta}{dt}\right)^2 - l\sin\theta\frac{d_\theta^2}{dt^2} \qquad \cdots 15$$

After substituting these equations into eq. 7 and 9, we get:

$$N = m\frac{d^2x_p}{dt^2} = m\left(\frac{d^2x}{dt^2} + l\sin\theta\left(\frac{d\theta}{dt}\right)^2 - l\cos\theta\frac{d^2\theta}{dt^2}\right) \cdots 16$$

$$P = m\frac{d^2y_p}{dt^2} = m\left(-l\cos\theta\left(\frac{d\theta}{dt}\right)^2 - l\sin\theta\frac{d^2\theta}{dt^2}\right) \qquad \cdots 17$$

Using the above nonlinear equations, a Matlab Simulink model has been developed [8]. This Simulink model is used in this work for stabilizing the upright position of the pendulum and to move the cart at desired position.

III. DESIGN OF FUZZY LOGIC CONTROLLER FOR INVERTED PENDULUM SYSTEM

In order to implement four inputs to the controller, the FLC were divided into two parts as can be seen in the figure 1.3[2,4]. The 'FLC 1' is used for controlling the cart's position, where as the 'FLC 2' controls the pendulum's angle. The 'FLC 1' receives Position (x) and Del Position (x-dot) as the inputs while the 'FLC 2' receives Angle (θ) and Del Angle (θ-dot) as the inputs. The output variable of both the FLC's is force.

(3)

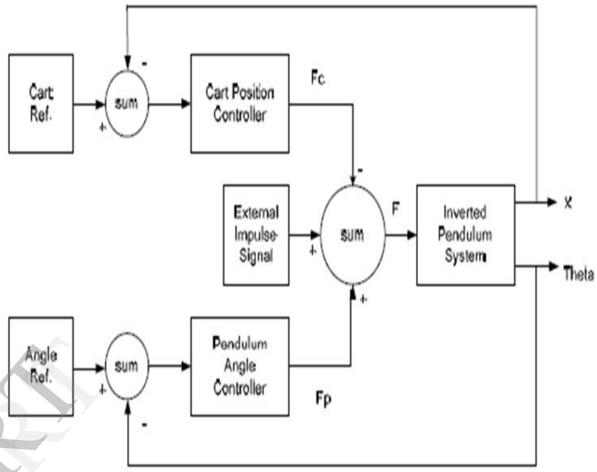

Figure1.3 FLC Overview

IV. MEMBERSHIP FUNCTIONS:FLC's

According to the complexity of this inverted pendulum system, we have taken nine fuzzy subsets to quantize each fuzzy variable for both FLC as shown in table.

LINGUISTIC TERM	LABEL
Negative Extreme	NE
Negative Big	NB
Negative Medium	NM
Negative Small	NS
Zero	ZE
Positive Small	PS
Positive Medium	PM
Positive Big	PB
Positive Extreme	PE

Table 1.3 Standard labels of quantization.

Figure 1.4 till figure 1.6 show the membership functions of FLC's.

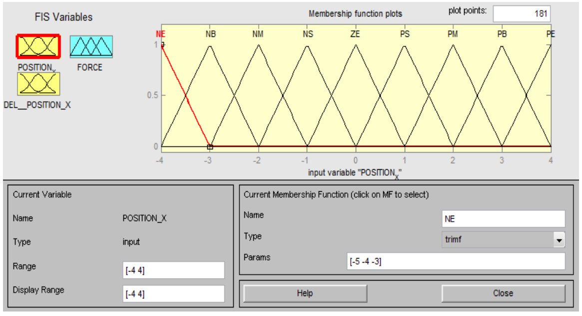

Figure 1.4 input 1 for ‘FLC 1’

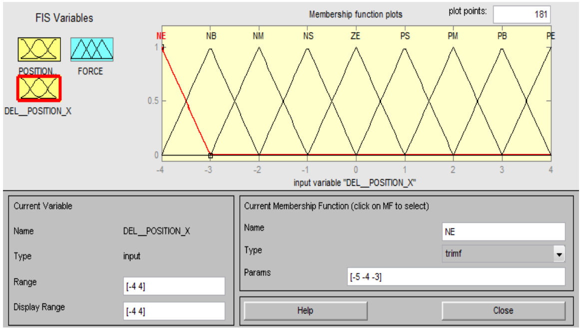

Figure 1.5 input 2 of ‘FLC 1’

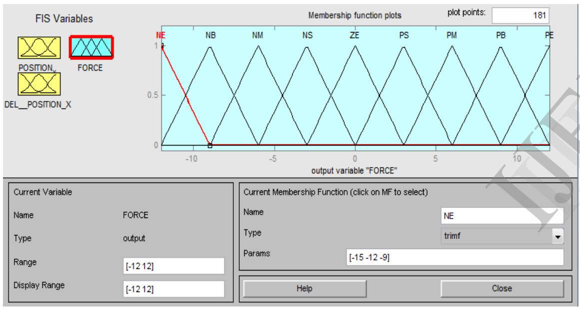

Figure 1.6 output of ‘FLC 1’

V. RULE BASE:

The following rule bases are applied for simulation study to control the Inverted Pendulum.

POSITION X

DEL POSITION X	NE	NB	NM	NS	ZE	PS	PM	PB	PE
NE	NE	NE	NE	NE	NB	NM	NM	NS	ZE
NB	NE	NB	NB	NB	NM	NS	NS	ZE	PS
NM	NE	NB	NM	NM	NM	NS	ZE	PS	PM
NS	NE	NB	NM	NS	NS	ZE	PS	PM	PB
ZE	NE	NB	NM	NS	ZE	PS	PM	PB	PE
PS	NB	NM	NS	ZE	PS	PS	PM	PB	PE
PM	NM	NS	ZE	PS	PM	PM	PM	PB	PE
PB	NS	ZE	PS	PS	PM	PB	PB	PB	PE
PE	ZE	PS	PM	PM	PB	PE	PE	PE	PE

Table1.4 Fuzzy Rule Matrix (9x9) :‘FLC 1’

ANGLE

DEL ANGLE	NE	NB	NM	NS	ZE	PS	PM	PB	PE
NE	NE	NE	NE	NE	NB	NM	NM	NS	ZE
NB	NE	NB	NB	NB	NM	NS	NS	ZE	PS
NM	NE	NB	NM	NM	NM	NS	ZE	PS	PM
NS	NE	NB	NM	NS	NS	ZE	PS	PM	PB
ZE	NE	NB	NM	NS	ZE	PS	PM	PB	PE
PS	NB	NM	NS	ZE	PS	PS	PM	PB	PE
PM	NM	NS	ZE	PS	PM	PM	PM	PB	PE
PB	NS	ZE	PS	PS	PM	PB	PB	PB	PE
PE	ZE	PS	PM	PM	PB	PE	PE	PE	PE

Table 1.5 Fuzzy Rule Matrix (9x9): ‘FLC 2’

(4)

VI. SURFACE VIEWERS

Figure 1.7 and Figure 1.8 shows surface viewer for FLC 1 and FLC 2 respectively.

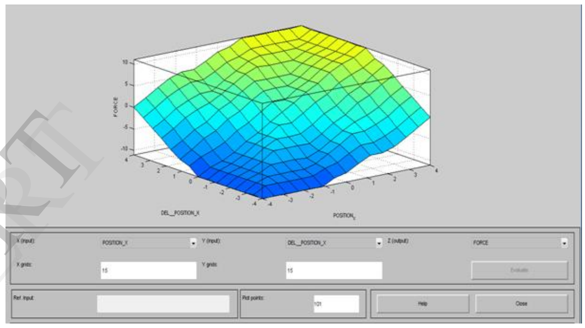

Figure 1.7 Surface Viewer for ‘FLC 1’

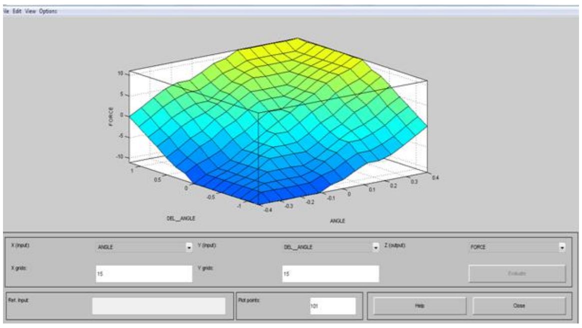

Figure 1.8 Surface Viewer for ‘FLC 2’

VII. SIMULINK MODEL:INVERTED PENDULUM

(a) Simulink model for Step-response of Inverted Pendulum using two FLC' s

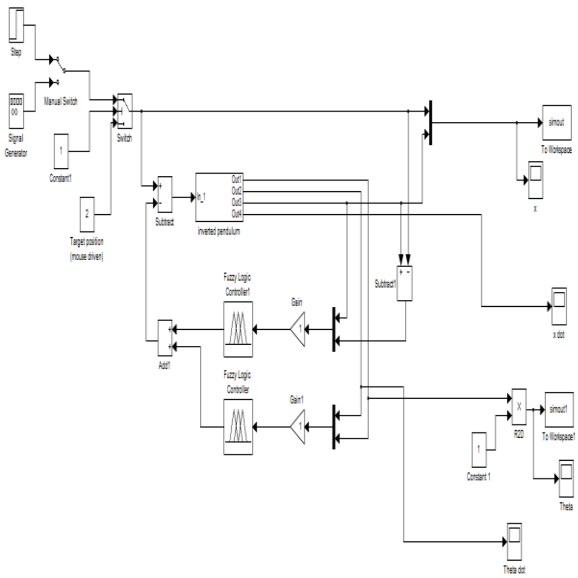

Figure 1.9 Simulink model using two FLC' s

(b) Simulink Sub-System model for Inverted Pendulum

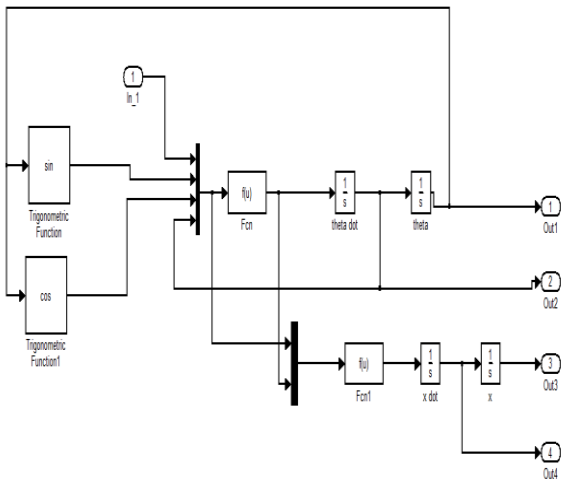

Figure 2.0 IP: sub-system model

(c) Open-loop response for Inverted Pendulum

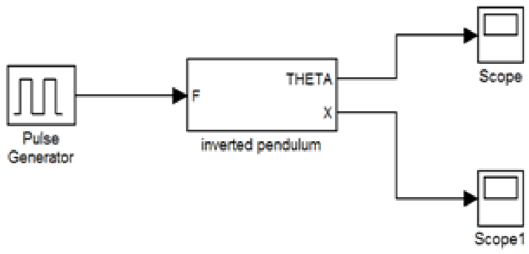

Figure 2.1 Open-loop pulse response

(d) Closed-loop response : using PID controller

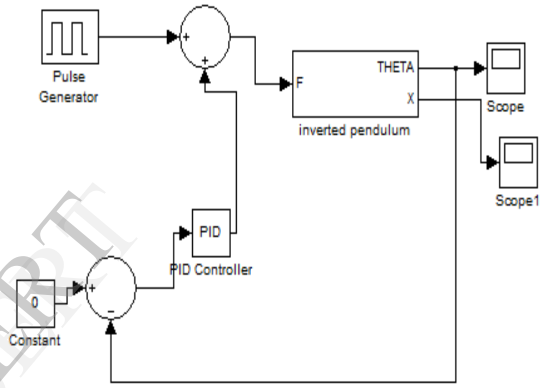

Figure 2.2 Closed-loop pulse response

(5)

VIII. RESULTS:SIMULATION (t:100 sec)

(a) Step-Responses

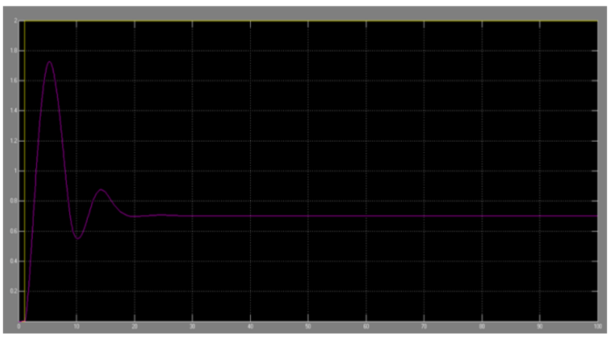

Figure 2.3 output step response :'Position x'

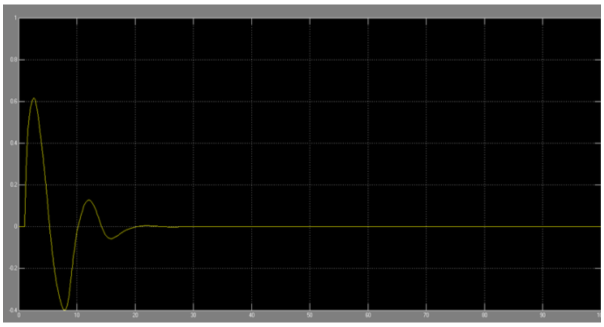

Fig 2.4 output step response 'Del Position x'

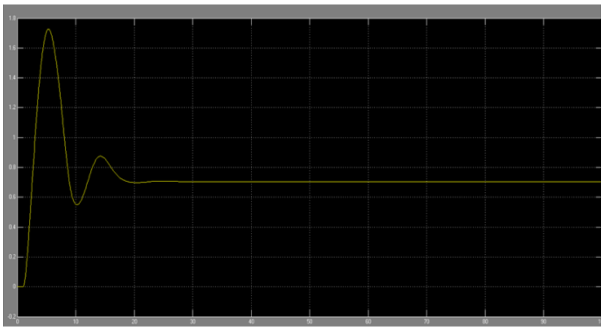

Figure 2.5 output step response for 'Angle'

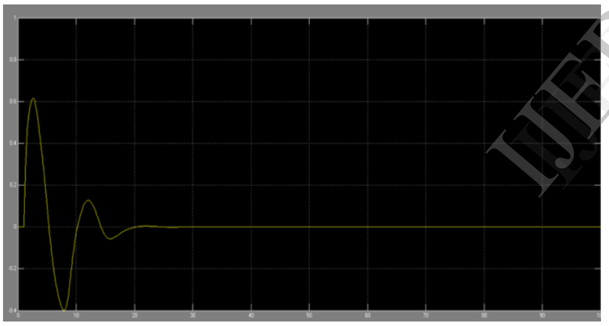

Fig 2.6 output step response for 'del_ angle'

(b) Pulse-responses

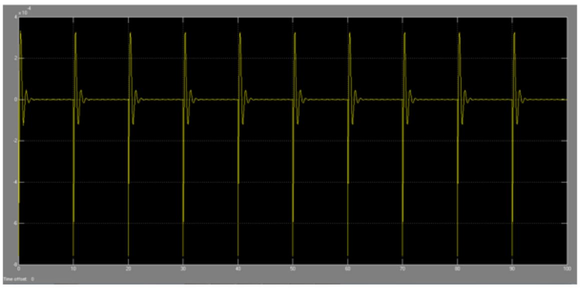

Fig2.8 Oputut pulse-response 'Del position'

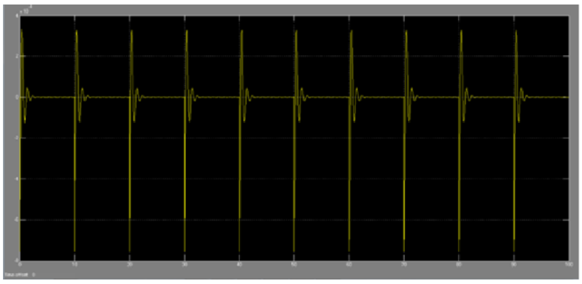

Fig 3.0 Output pulse-response 'Del_angle'

(c)Open-loop responses

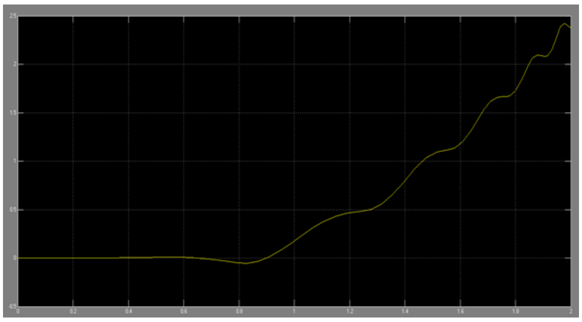

Fig3.1 open-loop impulse response 'Position x'

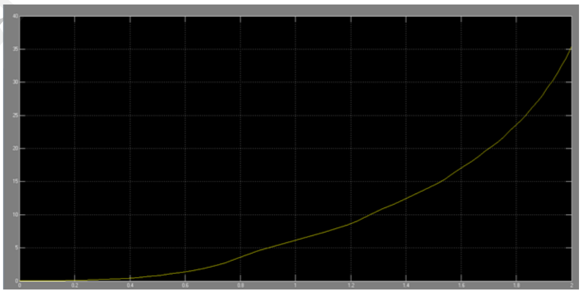

Figure 3.2 open loop impulse response for 'angle'

(6)

(d) Closed-loop response

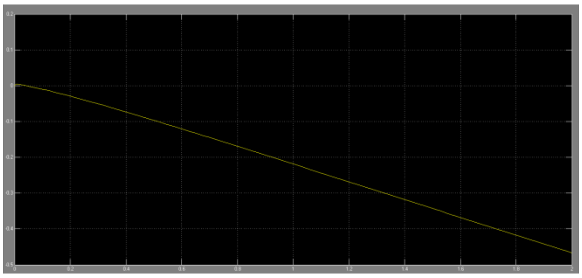

Figure 3.3 closed-loop response for 'Position' implementing PID control

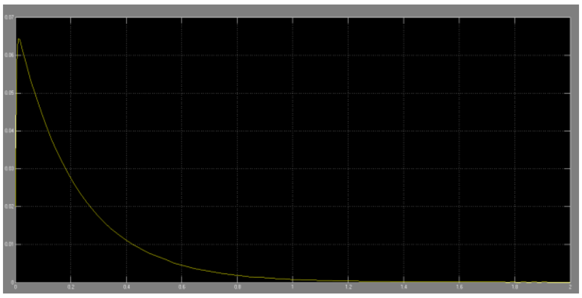

Figure 3.4 closed-loop response for 'Angle' implementing PID control

CONCLUSION

As a conclusion, the objective in stabilizing the inverted pendulum has been achieved by using two block of Fuzzy Logic Controllers[5]. This can be verified through output responses of the system which satisfy the design criteria. The Output Step-response for both the FLC's shows that after 20sec all the transient behaviour stops and the Pendulum is stabilized. The Output Pulse-response for both the FLC's shows that there is no carry over of the previous pulses and the system need not to be re-initialized. As can be seen from the Open-loop response that the Pendulum swings all the way around due to impact, and the cart travels along with a jerky motion due to pendulum. We can infer from the Closed-Loop response that the PID Controller handles the nonlinear system very well because the angle is very small (.04 radians).

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