Stabilization of Rotary Arm Inverted Pendulum using State Feedback Techniques

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***Abstract*—Inverted pendulum is a pendulum which has its mass above its pivot point and owing to its inherent non-linear nature the control of inverted pendulum is difficult and therefore suitable to test modern control algorithms.The main idea of this paper is to compare and analyse the performance of different control algorithms in order to balance the inverted pendulum in its upright position. Pole placement technique and LQR methods are used to stabilize the pendulum. In this work, the inverted pendulum system was modelled and constructed using Simulink and the performance of the proposed controllers are compared through simulations using Mat lab. Simulation results shows that both LQR Controller and pole placement controller are far more superior in achieving desired response and are more robust when compared to conventional controllers. LQR controller provides better steady state response compared to state feedback controllers and state feedback controller improves transient characteristics.**

Keywords- LQR, Pole Placement, Rotary arm Inverted pendulum, Stabilization

1. INTRODUCTION

By the time of launching a rocket or space craft the main difficulty is to maintain the rocket in the upright position against gravity and keep it on a pre-specified trajectory by overcoming all external disturbances and internal non-linearity present with the system. In the real world examples like balancing a broom stick on hand, Segway, Crane balancing, Robotics, air craft stabilization in the turbulent air flow etc. the stabilization about a desired point is a problem owing to the inherent non-linearity present with these systems. The risk behind these balancing problems can be studied with the help of Inverted pendulum models. Unlike simple pendulum an inverted pendulum has its centre of gravity over its axis of rotation and is highly nonlinear and an open loop unstable system that makes control more challenging and therefore it is well suited for verification and practice of ideas emerging in control theory and robotics. Rotary arm Inverted Pendulum (RIP) is one among the famous inverted pendulum model which is a simple structure, unstable non-minimum phase system subjected to many nonlinear characteristics.

Stabilization of RIP system using classical PID controller is still popular because of its ease of implementation. However in practical RIP system, there may be disturbances which come internally or externally. But for a classical PID controller it is difficult to limit these disturbances rapidly and hence difficult to achieve optimal performance. Only one of the system's parameters can be controlled by PID controller at a time [1]. Therefore for a multivariable system like RIP, more than one PID controller is required. In order to control arm angle and pendulum angle at the same time we need two PID controllers [1] [3], one for controlling the position of arm angle and other for pendulum angle.

Placing poles at the pre-defined locations is another technique so that the characteristics of the system response can be controlled effectively. Dominant pole placement [4] [5] technique and LQR [1][2] methods are very powerful methods which can stabilize the pendulum with minimum control effort and time. The dynamics of the system can be controlled by these methods by changing the location of poles.

Figure.1 represents RIP system which consists of a PMDC servo motor and to the shaft of this motor an L-shaped rotary arm is attached. The position of the PMDC motor is measured by using a quadrature encoder. The pendulum rotates freely in a vertical plane about a shaft mounted on double ball bearings attached to the rotary arm. One end of the shaft (on which the pendulum is locked) drives a servo potentiometer via a shaft coupler. The servo potentiometer measures the position (or angle) of the pendulum in the vertical plane. In its middle, there is a proprietary coupling mechanism which transfers DC power to the servo potentiometer and also transfers a voltage corresponding to pendulum position or angle back to the terminal blocks on the circular base. The stabilizing controller will work only when the pendulum is about a few degrees apart from the vertical position. Once the pendulum is close to vertical position, stabilizing controller will take the control of the pendulum and keep it in upright position. In this paper two different method of stabilization of rotary arm inverted pendulum is discussed viz., Full State Feedback method and LQR.

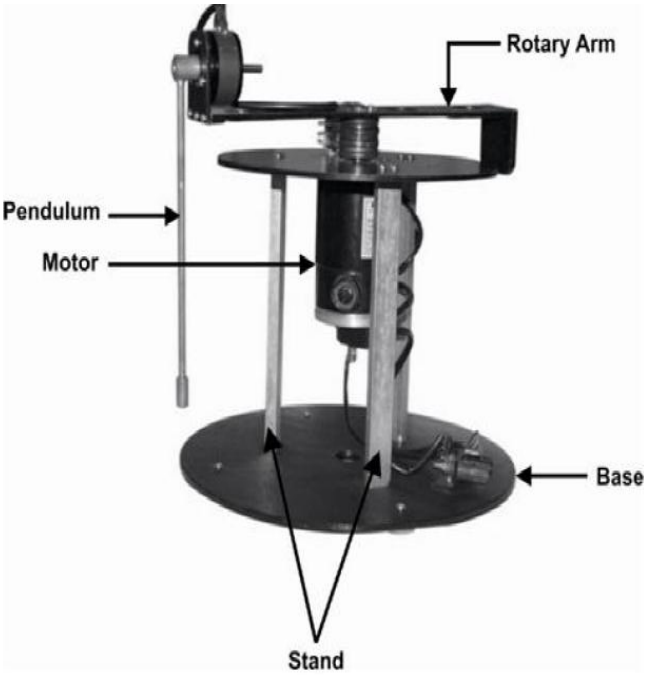

Fig.1. Rotary arm Inverted Pendulum system

The paper is organized as follows: Section 2 deals with the mathematical dynamic model of the system used both for the computer simulations (MATLAB) and for the mathematical design of the controllers. Section 3 goes through the main steps in the design of the control algorithms. Section 4 presents simulation results of the different method discussed, and finally several conclusions are drawn in Section 5.

II. MATHEMATICAL MODELING OF RIP SYSTEM

To analyze the system we need to obtain a mathematical model which describes the dynamics of RIP system. Derivation of mathematical model includes a non-linear model and a linear model of RIP system. The non-linear model describes the whole dynamics of the system. For designing a balancing controller linearized model is useful. The controlling parameters are motor position, pendulum angle and motor velocity. By adjusting these three parameters we control the RIP system. For deriving the mathematical model, pendulum is initially assumed in the direction of gravitational force i.e., in equilibrium state and therefore the initial conditions are taken as zero. Linear model shows close response to the non-linear model only for a particular range of pendulum angle and therefore stabilization controller take its role if the pendulum is of few degrees away from the vertical. Figure.2 shows the arm is rotating about z-axis and reference of arm position θ is taken to the x-axis and the reference of pendulum angle α is taken from the upward vertical. Parameters of RIP system is shown in Table.1

TABLE.I SYMBOLS TO DESCRIBE EQUATION PARAMETERS

Symbol	Description	Unit
r	Length of arm pivot to pendulum pivot	m
θ	Motor shaft position	radians
$\dot{\theta}$	Angular velocity of motor	m/s^2
α	Pendulum angular deflection	radians
$\dot{\alpha}$	Pendulum angular velocity	m/s^2
l_p	Pendulum length	m
m	Mass of pendulum	kg
L	Length of pendulum center of mass from pivot	m
g	Gravitational acceleration	m/s^2
v	Input voltage	volt

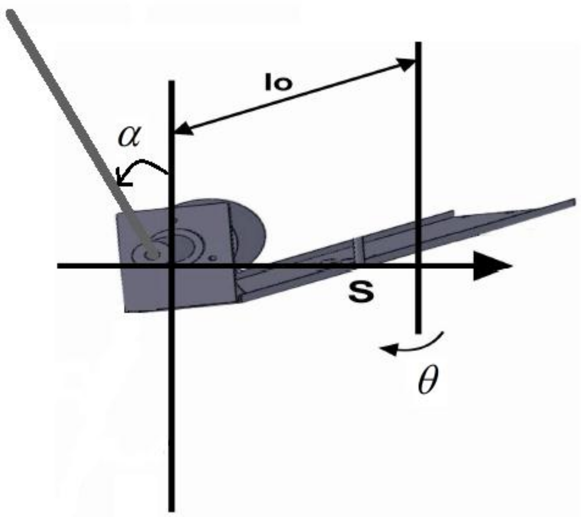

Fig.2. Schematic representation of RIP system

The governing differential equations of the system are as follows:

$$(J_{eq} + mr^2)\ddot{\theta} - \frac{1}{2}mLr\cos\alpha\,\ddot{\alpha} + \frac{1}{2}mLr\sin\alpha\,\dot{\alpha}^2 + B_{eq}\dot{\theta} = T_L \quad (1)$$

$$\frac{1}{2}mL^2\ddot{\alpha} - \frac{1}{2}mLr\cos\alpha\,\ddot{\theta} - \frac{1}{2}mgL\sin\alpha = 0 \quad (2)$$

where,

$$T_L = K_1 v - K_2\dot{\theta} \quad (3)$$

$$K_1 = \frac{K_m}{R_a}$$

$$K_2 = \frac{K_m K_t}{R_a} \quad (4)$$

or,

$$a\ddot{\theta} - b\cos\alpha\,\ddot{\alpha} + b\sin\alpha\,\dot{\alpha}^2 + B_{eq}\dot{\theta} = K_1 v - K_2\omega$$

$$c\ddot{\alpha} - b\cos\alpha\,\ddot{\theta} - d\sin\alpha = 0 \qquad (5)$$

where,

$$a = (J_{eq} + mr^2)$$
$$b = \frac{1}{2}mLr$$
$$c = \frac{1}{3}mL^2 \qquad (6)$$
$$d = \frac{1}{2}mgL$$

Solving for $\ddot{\theta}$ from equation (5), we have

$$\ddot{\theta} = \frac{c}{b\cos\alpha}\ddot{\alpha} - \frac{d}{b\cos\alpha}\sin\alpha \qquad (7)$$

Substituting for $\ddot{\theta}$ in (5) from (7) and solving for $\ddot{\theta}$ and $\ddot{\alpha}$ results in the following nonlinear equations

$$\ddot{\alpha} = \frac{1}{f}[ad\sin\alpha - pb\cos\alpha\,\dot{\theta} - b^2\cos\alpha\sin\alpha\,\dot{\alpha}^2 + bK_1\cos\alpha\, v]$$

$$\ddot{\theta} = \frac{1}{f}[bd\cos\alpha\sin\alpha - pc\dot{\theta} - cb\sin\alpha\,\dot{\alpha}^2 + cK_1 v] \qquad (8)$$

where,

$$p = B_{eq} + K_2$$
$$f = ac - b^2\cos^2\alpha \qquad (9)$$

A. Linearized model

Equation (8) can be linearized by considering the equilibrium state of the system. If we assume α is small (i.e., when the Inverted Pendulum is near its equilibrium point), we can linearize these equations.

For small α, $\sin(\alpha) \approx \alpha$ and $\cos(\alpha) \approx 1$. Also, for small α, square of pendulum angular velocity is negligible, and we get the following linearized equations

$$a\ddot{\theta} - b\ddot{\alpha} + B_{eq}\dot{\theta} = K_1 v - K_2\dot{\theta} \qquad (10)$$

$$c\ddot{\alpha} - b\ddot{\theta} - d\alpha = 0 \qquad (11)$$

$$\ddot{\alpha} = \frac{ad}{e}\alpha - \frac{pb}{e}\dot{\theta} + \frac{cK_1}{e}v \qquad (12)$$

where,

$$e = ac - b^2$$
$$p = B_{eq} + K_2 \qquad (13)$$

The state space model is obtained from the linearized model.

$$\dot{x} = Ax + Bu$$
$$y = Cx \qquad (14)$$

The states of RIP system is chosen as

$$x(t) = [\theta\ \alpha\ \dot{\theta}\ \dot{\alpha}]^T \qquad (15)$$

The linear state space model is given by

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{bd}{e} & \frac{-pc}{e} & 0\\ 0 & \frac{ad}{e} & \frac{-pb}{e} & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{cK_1}{e}\\ \frac{bK_1}{e}\end{bmatrix}v \qquad (16)$$

For obtaining state space model the frictional coefficient of both pendulum section and motor is assumed to be zero. After substituting the parameter values from Table II, the state space model is given by

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 2.1440 & -0.1417 & 0\\ 0 & 82.5677 & -0.19440 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1.0928\\ 1.4991\end{bmatrix}v \qquad (17)$$

$$y = \begin{bmatrix}1\,0\,0\,0\\ 0\,1\,0\,0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}v \qquad (18)$$

TABLE.II. TYPICAL CONFIGURATION OF RIP SYSTEM

Symbol	Description	Value (SI unit)
K_t	Motor torque constant	0.1331
K_m	Back emf	0.1298
J_{eq}	Equivalent moment of inertia at load	0.0200484
m	Mass of pendulum	0.035
L	Length of pendulum center of mass from pivot	0.1848
r	Length of arm pivot to pendulum pivot	0.169
R_a	Armature resistance of motor	6

III. CONTROLLER DESIGN

A. Pole Placement Controller

In this paper, controllers used for balancing pendulum in upright position are linear state feedback controllers. The state feedback controller can be used only if the system is controllable and it tracks the input signal or improve damping of the system. Figure.3 shows the block diagram of a pole placement controller. For a given system, the state feedback gain matrix K is not same but depends on the desired closed loop pole location. This will also determine the speed and damping of the response.

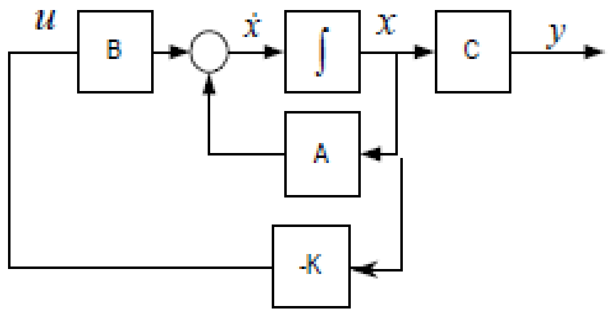

Fig.3. Block diagram of pole placement controller

The linearized state space model (17-18) is used to design balance controller. The dominant pole is chosen so that the damping ratio is 0.7 and natural frequency 4 rad/s. For rest of the two poles, two real poles with very small time constants 0.125 second and 0.025 second are selected. The gain matrix obtained using Ackermann formula is $K = \left[-58.9063\ 501.4796 - 29.5578\ 57.2090\right]$.

B. LQR controller

Linear Quadratic Regulator is an optimal controller used to achieve desired target value with minimum control effort and time. A performance index J is defined in this technique and the control engineer have to find a controller $u = -Kx$ which will minimize the performance index.

$$J = \int (x^T Q x + u^T R u)\, dt$$

where, Q and R are weighing matrices which allow the relative weighting of individual state variables and individual control inputs.

In MATLAB, the command [K] = lqr (A, B, Q, R) calculates the optimal feedback matrix K such that it minimizes the cost function subject to the constraint defined by the state equation. The response of system for different set of state feedback gain matrix is determined by varying Q values, keeping R=1, and chooses the one which give best performance.

The different sets of Q chosen are as follows

Q1=diag($\left[60\ 5\ 1\ 1\right]$), Q2=diag($\left[60\ 0.05\ 60\ 0.05\right]$),

Q3=diag($\left[1000\ 2000\ 600\ 200\right]$)

IV. SIMULATION RESULTS

The pole placement controller and LQR controller can be considered as robust controllers. The simulation results for pendulum position and arm position for pole placement and LQR controllers are shown in the figure 4(a), 4(b), 5(a), 5(b).

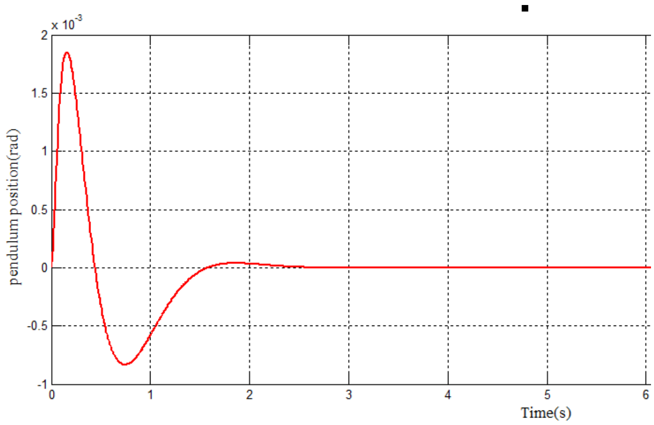

Fig.4(a) Step response of pendulum angle-Pole placement controller

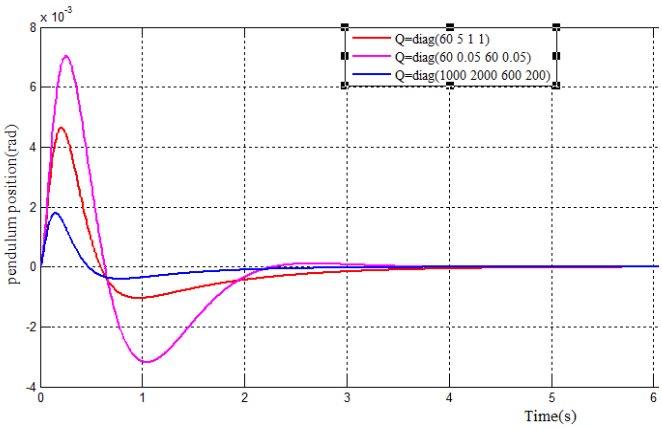

Fig.4(b) Step response of pendulum angle for different set of Q-LQR controller

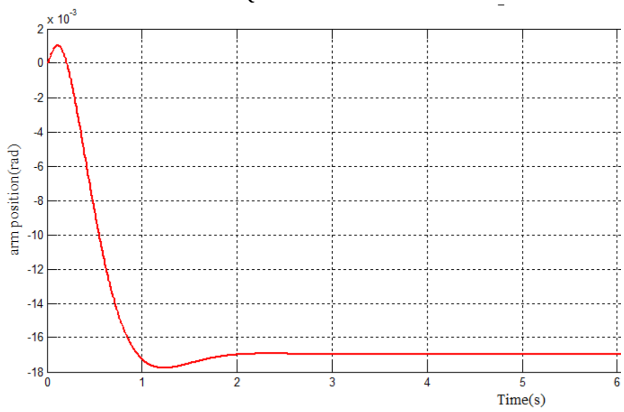

Fig. 5(a) Step response of arm position-Pole placement controller

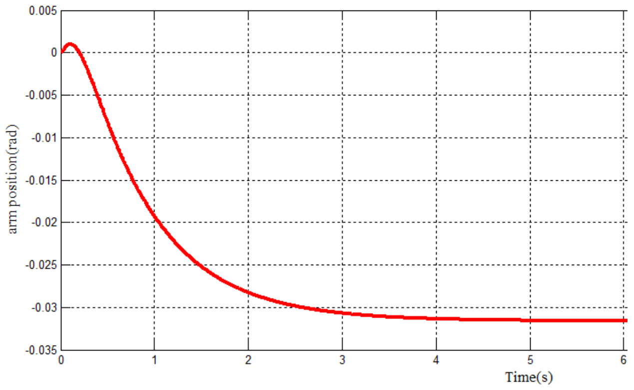

Fig.5(b) Step response of arm position-LQR controller

On observing above figures, LQR controller is better to settle the response with in less time than pole placement controller and pole placement controller is better in reducing the overshoot of the system. However by proper choice of weighing matrices Q and R, LQR response can further improved and is more dominant than state feedback controller. The gain matrix K chosen for the state feedback controller is almost perfect for stabilizing the pendulum. It is always not easy to obtain the gain matrix for state feedback controller easily. But in the case of LQR controller the gain matrix K can be tuned easily to obtain the desired response.

V. CONCLUSION

In this paper state feedback and LQR control methods are discussed for the stabilization of Rotary arm Inverted pendulum system. From the simulation results, it is found that both pole placement technique and LQR method are efficient in satisfying the design requirements and are robust to the parameter variations. The LQR control shows better results in minimizing the steady state value when compared to state feedback control method while pole placement method is better to improve the transient response of the system. Both pole placement and LQR controllers are capable of maintaining the pendulum in its upright position.

ACKNOWLEDGEMENT

This work has been carried out as a part of thesis work done in the Department of Control systems in Mar Baselious College of Engineering. Gratitude is extended to the entire faculty of Electrical Department

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