Static Analysis of Laterized Concrete Cylindrical Shells under Food Grain Pressures by Initial Value Method

T. U. Nwakonobi [1]; N. N. Osadebe[2] and C. C. Mbajiorgu[3]

[1]PhD, Senior Lecturer, Department of Agricultural and Environmental Engineering, University of Agriculture, P.M.B.2373, Makurdi, Benue State, Nigeria.

[2]Professor of Civil Engineering, Department of Civil Engineering, Faculty of Engineering, University of Nigeria, Nsukka, Enugu State, 410001, Nigeria.

[3] Professor of Agricultural and Bioresources Engineering, Department of Agricultural and Bioresources Engineering, Faculty of Engineering,University of Nigeria, Nsukka, Enugu State, 410001, Nigeria.

Abstract: In this study, the performance of a laterized concrete cylindrical shell for storage of food grains was investigated. The structural characteristics values of laterized concrete at optimum mix proportion were applied in performing the static analysis of the cylindrical shell structure under the action of food grain pressures using Pasternack's equations formulated on the basis of theory of shell of revolution. The reduced equation of static equilibrium was solved by the initial- value method. A generalized solution of cylindrical shell made of laterized concrete was obtained and used to evaluate the performance of the structure through the determination of deflection and stresses. The results of the analysis indicate that the maximum stress developed due to soybean grain loading is greater than the material strength in silo thickness of 100mm, 20m in diameter and 8m in height. The wall thickness of 150mm was found adequate for silo of the same size.

Keywords: static analysis; Food grain; storage; laterized concrete; cylindrical shell; Initial-value method

1. Introduction

The post harvest handling of food grains and seeds has constituted a matter of great concern to agricultural engineers and other agricultural experts. In developed countries, efficient storage and processing of food have been recognized as major factors in the solution of food problems (Aboaba, 1996).

In Nigeria, about 70% of farming population stores their farm produce using traditional storage structures as the use of modern structures has become unaffordable. The use of these structures are associated with numerous problems which, according to several writers (Olumeko 1996; Adejum and Raji 2007; Nwakonobi and Mbajiougu, 2009); they are not durable, not watertight, not rodent proof, susceptible to insert attack, low viability of grains, change in grain texture, appearance and flavour and of low capacity. These defects and shortcoming of the traditional structures usually result in tremendous losses estimated to be up to 30 – 40% (Igbeka and Olumeko, 1991). To overcome these problems efforts are now being directed towards evolving low-cost, effective storage structures made of locally available materials for use by low to medium scale farmers. Laterized concrete has been identified by (Lasisi and Osunade, 1990) as a possible low-cost construction material.

Laterized concrete is concrete in which some or the entire fine aggregate is from lateritic soil. Sand (fine aggregate) which is the second largest of the concrete volume is not always available. According to Osunade (1997), sand is the second most costly item per unit volume of concrete produced. Thus, replacing sand with lateritic soil can reduce construction cost since the later is abundant and readily available at construction sites. Several investigators have studied the applications of lateritic soils as a component in concrete (Osunade et al 1990; Osunade, 1993; [Osunade, 1994 and Osadebe and Nwakonobi, 2007, Nwakonobi, et al, 2012) and some of the research findings were very encouraging. However a lot still remain to be done in order to establish the behaviour of laterized concrete cylindrical shells under different types of loadings.

The primary objective of this study was to establish the behaviour of laterized concrete cylindrical shells under food grain loadings. The specific objectives are to:

- Analyze the cylindrical shell structure made of laterized concrete using pasternack's semi-moment equation which was formulated on the basis of cylindrical theory of shell of revolution.
- Develop a general solution for the design of a shell cylindrical silo built of laterized concrete.
- Evaluate the performance of the structure through the determination of deflection and stresses

1.1 Background

A known problem in the study of shells is how to determine whether analysis based on a particular theorem is adequate, satisfactory or successful. Because the analysis of a shell structure is very tedious and cumbersome, theories have been developed to simplify the design of a shell structures. Some of the shell theories includes; Membrane theory, variational theory, Bending or moment theory and Half (or semi) moment theory.

According to Zingoni et al, (2000) membrane theory is applicable to shell with finite bending rigidity but in which the moments that are developed are so small as to be negligible. The theory assumes that silo walls are subjected to tensile forces only. However, there are some limitations associated with the application of membrane theory in the analysis of the shell structure. This theory according to Timoshenko and Woinowsky-Krieger (1959) fails to represent the true stress in those portions of the shell close to the edges, since the edge conditions usually cannot be completely satisfied by considering only membrane stresses. Zingoni et al, (2000) also noted similar conditions under which membrane theory is not valid which include conditions at certain locations in the interior of the shell. There exist at these locations transverse shearing force and moments to satisfy the condition of continuity of the shell. Any component of the forces reaction at the support in the direction normal to the shell's middle surface would induce transverse shear in the shell, which the, membrane theory does not admit.

The variational theory is deficient as 'bending term' are not adequately accounted for due to approximation in the strain energy function Muhammad (2000). For the bending or moment theory, it can be used in obtaining reduced (finite) set of equations which involve no more than stress resultants and stress couples for the description of stress distribution, together with the approximate boundary conditions. Bending theory is the most accurate of all the theories but the analysis is very cumbersome as the analyst are involved in the generally very difficult task of searching for the actual particular solution corresponding to a given surface loading. But in this theory, transverse shearing force and bending terms are accounted for in the stress analysis of shell structure. However, half or semi moment theory which is also known as pasternack's theory tried to simplify the moment theory and this theory was adopted in this study based on its strength.

Several researchers using various methods solve reduced equation of static equilibrium of circular cylindrical shell structure and Ren and Kuan-Chan (2001) tried to develop series solutions to the differential equations for a complete cylindrical shell under various point loads. Instead of expanding the solution series in both axial and circumferential coordinates and each terms in the series represent an exact solution to the set of basic equations. An exact three- dimensional elasticity solution is obtained from an infinite long, thick transversely isotropic circular cylindrical shell panel, simply supported along the longitudinal edges and subjected to a radial patch load (Chandrashekhera and Rao, 1996). The boundary value problem is reduced to Bessel's differential equation using a set of three displacement functions. In another study, Chandrashekhera and Rao,(1996) presented an approximate three dimensional elastic solution for an infinitely cross-ply laminated circular cylindrical shell panel with simply supported boundary conditions, subjected to an arbitrary discontinuous transverse loading. The solution is based on the principal assumption that the ratio of the thickness of the lamina to its middle surface radius is negligible compared to unity.

Mohammad (2000) carried out analysis of the cylindrical shell subject to local and continuous axi-symmetric loads using pasternack's equations. The reduced differential equation was solved using classical and initial value methods. The results obtained in each of the methods are the same indicating the applicability of either of the two methods in solving equation of static equilibrium of cylindrical shell. Initial value method has been applied in the analysis of circular cylindrical shell under hydrostatic pressure and ring force (Amodou, 2002). Reinforced concrete dome structures designed with reinforcement ratios were analyzed to check the applicability of the finite element analysis techniques (Song et al, 2002). Their results showed that the technique could be applied effectively to failure analysis of various types of reinforced concrete shell structures.

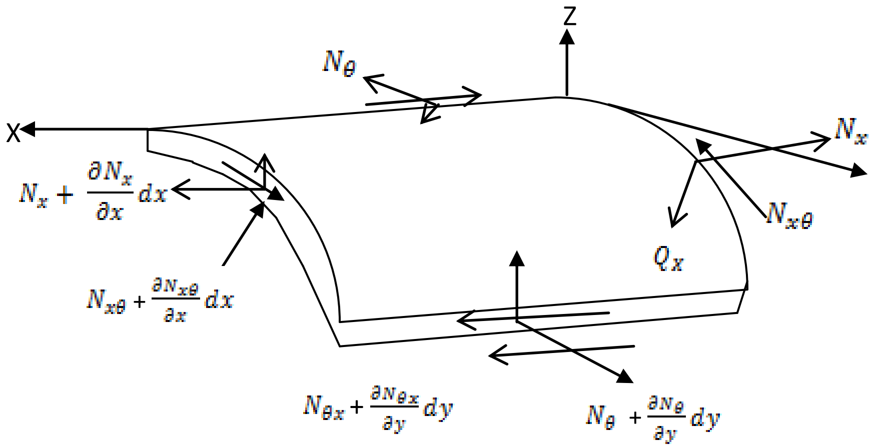


Fig. 1: Direct and shear forces in an element of a shell

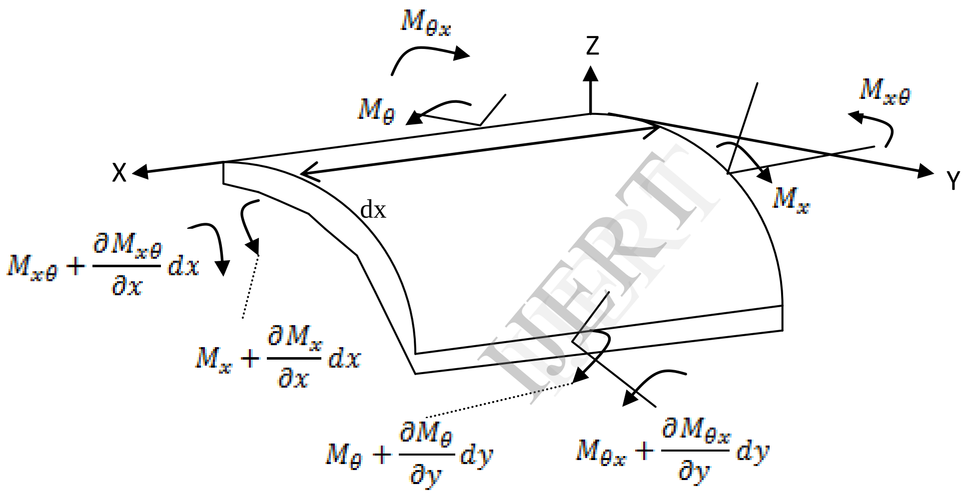


Fig. 2: Bending and twisting moments in an element of a shell

3. Stress Equilibrium Equations

The x, y and $z - axes$ are taken at a given point on the middle surface to be the direction to the axis of the cylinder, the tangent to the circumference, and the normal to the middle surface of the shell respectively. Assuming a transverse distributed load whose components in x, y and z directions are X, Y and Z respectively, R is the radius of the cylinder, d_x and d_y are the sizes of the element in x and y directions. The direct and shear forces as well as bending and twisting moments in an element of a shell are shown in Figs.1 and 2, respectively.

The equilibrium of forces in x, y and z-direction are given as follows:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{\theta x}}{\partial y} + X = 0 \tag{1}$$

$$\frac{\partial N_\theta}{\partial y} + \frac{Q_\theta}{R} + \frac{\partial N_{x\theta}}{\partial x} + y = 0 \tag{2}$$

$$Z + \frac{\partial Q_x}{\partial x} + \frac{\partial Q_\theta}{\partial y} - \frac{N_\theta}{R} = 0 \tag{3}$$

The moment equilibrium with respect to x and y- axes result in:

$$\frac{\partial M_\theta}{\partial y} + \frac{\partial M_{x\theta}}{\partial x} - Q_\theta = 0 \tag{4}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{\theta x}}{\partial y} - Q_x = 0 \tag{5}$$

In semi- moment (or pasternack's) theory as applied to cylindrical shell, twisting moments are usually neglected. This result in:

$$M_{\theta x} = M_{x\theta} = 0 \tag{6}$$

Based on the law of reciprocity of shearing stresses, we have:

$$N_{\theta x} = N_{x\theta} = S \tag{7}$$

Substitution of equation (6) into equations (4) and (5) gives:

$$\frac{\partial M_\theta}{\partial y} = Q_\theta \tag{8}$$

$$\frac{\partial M_x}{\partial x} = Q_x \tag{9}$$

Substitution of equations (7), (8) and (9) into equations (1), (2) and (3) to obtain:

$$\frac{\partial N_x}{\partial x} + \frac{\partial S}{\partial y} + X = 0 \tag{10}$$

$$\frac{\partial M_\theta}{R\partial y} + \frac{\partial N_\theta}{\partial y} + \frac{\partial S}{\partial x} + Y = 0 \tag{11}$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_\theta}{\partial y^2} + \frac{N_\theta}{R} + Z = 0 \tag{12}$$

Equations (10) through (12) represent the equation of the static equilibrium of a cylindrical shell under distributed forces of components X, Y and Z in x, y and z directions only. The systems are statically indeterminate since there are five unknowns for only three equations. In order to solve the systems, the following stress strain relations were needed to facilitate the determination of the unknowns namely $N_{x,}$ $N_{\theta,}$ $S_{,}$ $M_{\theta,}$ and M_x:

$$N_x = \frac{Eh}{1-\mu^2}\ (\varepsilon_x + \mu\varepsilon_\theta) \tag{13}$$

$$N_\theta = \frac{Eh}{1-\mu^2}\ (\varepsilon_\theta + \mu\varepsilon_x) \tag{14}$$

$$S = Gh\gamma_{x\theta} \tag{15}$$

$$M_x = -D\left(X_{x-}\mu X_\theta\right) \tag{16}$$

$$M_\theta = D\left(X_{\theta-}\mu X_x\right) \tag{17}$$

Where:

$$\varepsilon_x = \frac{\partial u}{\partial x} \tag{18}$$

$$\varepsilon_\theta = \frac{\partial v}{\partial y} + \frac{w}{R} \tag{19}$$

$$X_x = \frac{\partial^2 w}{\partial x^2} \tag{20}$$

$$X_\theta = \frac{\partial}{\partial y}\left(\frac{v}{R} - \frac{\partial w}{\partial y}\right) \tag{21}$$

$$\gamma_{x\,\theta} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{22}$$

$$G = \frac{E}{2(1-\mu)} \tag{23}$$

Substituting equations (18) through (23) into equations (13) through (17) gives the stress resultants and stress couples in terms of the displacements μ, v and w.

$$N_x = \frac{Eh}{1-\mu^2}\left[\frac{\partial u}{\partial x} + \mu\left(\frac{\partial v}{\partial y} + \frac{w}{R}\right)\right] \tag{24}$$

$$N_\theta = \frac{Eh}{1-\mu^2}\left[\frac{\partial v}{\partial y} + \frac{w}{R} + \mu\frac{\partial u}{\partial x}\right] \tag{25}$$

$$S = \frac{Eh}{2(1-\mu)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \tag{26}$$

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} - \mu\frac{\partial}{\partial y}\left(\frac{v}{R} - \frac{\partial w}{\partial y}\right)\right] \tag{27}$$

$$M_\theta = D\left[\frac{\partial}{\partial y}\left(\frac{v}{R} - \frac{\partial w}{\partial y}\right) - \mu\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{28}$$

Substitution of above expressions into equations (10), (11) and (12) gives:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1-\mu}{2}.\frac{\partial^2 u}{\partial y^2} + \frac{1+\mu}{2}.\frac{\partial^2 v}{\partial x \partial y} + \frac{u}{R}.\frac{\partial w}{\partial x} + \frac{1-\mu^2}{Eh}.X = \tag{29}$$

$$\frac{1+\mu}{2}.\frac{\partial^2 u}{\partial x \partial y} + \frac{1-\mu}{2}.\frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial y}\left(\frac{w}{R}\right) + \frac{h^2}{12R}.\left[\frac{\partial^2}{\partial y^2}\left(\frac{v}{R}\right) - \frac{\partial^3 w}{\partial y^3} - \mu\frac{\partial^3 w}{\partial y^2 \partial y}\right] + \frac{1-\mu^2}{Eh}.\ Y = 0 \tag{30}$$

$$\frac{\partial^4 W}{\partial X^4} + \frac{\partial}{\partial y}\left[\frac{u}{R}.\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2}{\partial y^2}\left(\frac{v}{R}\right) - \frac{\partial^4 W}{\partial y^{4)}}\right] - 2\mu.\frac{\partial^4 w}{\partial x^2 \partial y^2} - \frac{12}{Rh^2}\left[\frac{\partial v}{\partial y} + \frac{w}{R} + u\frac{\partial u}{\partial x}\right] + \frac{12(1-\mu^2)}{Eh^3}.Z = 0$$

(31)

From the Pasternak's theory on cylindrical shell, the following assumptions were made:

(1) The middle plane of the shell is inextensible in y direction such that $v = 0$.

(2) It is assumed that the stress generated by imposed load from stored grain do not depend on the y but only on the length x.

(3) The normal force N_x acting on the transverse section of the shell is neglected.

Consequently, all derivatives with the respect to y are set to be zero.

In case of grain pressure, the external forces acting on the cylindrical shell (see Fig.3) are such that y and $z = k\gamma x$, where γ stands for the weight per unit volume of stored grain material (soybean).

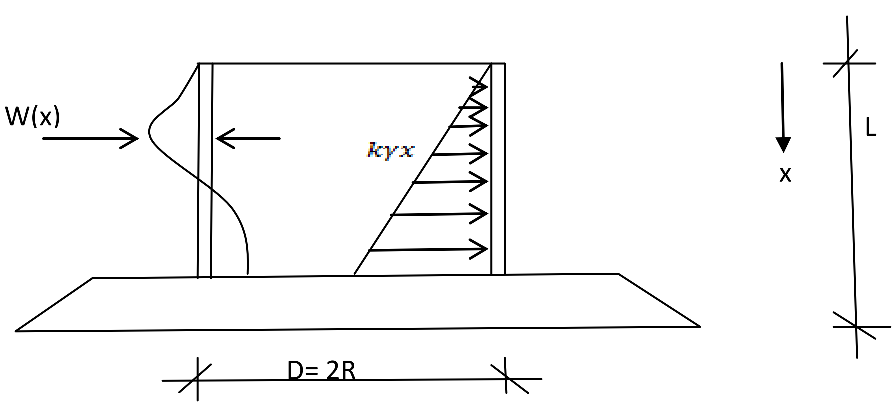


Fig.3: A section of circular cylindrical shell

Applying the above assumptions to equations (24) through (28) to get.

$$N_x = \frac{Eh}{1-\mu^2}\left[\frac{du}{dx} + \mu\frac{w}{R}\right] = 0 \Rightarrow \frac{du}{dx} = \mu' = -\mu\frac{w}{R} \tag{32}$$

$$N_\theta = \frac{Eh}{1-\mu^2}\left[\frac{w}{R} + \mu\frac{du}{dx}\right] = \frac{Eh}{1-\mu^2}\left[\frac{w}{R} + \mu^2\frac{w}{R}\right] \Rightarrow N_\theta = \frac{Ehw}{R} \tag{33}$$

$$S = \frac{Eh}{2(1+\mu)}[0+0] \Rightarrow S = 0 \tag{34}$$

$$M_x = \mathrm{M} = -DW'' \tag{35}$$

$$M_\theta = -\mu DW'' \Rightarrow M_\theta = \mu M \tag{36}$$

Where; $W'' = \frac{\partial^2 w}{\partial x^2}$

3.1 Differential Equation of Equilibrium

Making use of the previous results, it can be seen that equations (29) and (30) are satisfied.

Substituting these same results into equation (31) to get:

$$\frac{d^4 w}{dx^4} + 4\alpha^4 w = \frac{k\gamma x}{D}$$

Which can be written as $w^{iv} + 4\alpha^4 w = \frac{k\gamma}{D}\,x$ (37)

Where: $w^{iv} = \frac{d^4 w}{dx^4}$

$\alpha^4 = \frac{3(1-\mu^2}{R^2 h^2}$

$D = \frac{Eh^3}{12(1-\mu^2)}$

Equation (37) is due to pasternack and is differential equation for cylindrical shell.

3.2 General Solution of Static Equilibrium by Initial Value Method

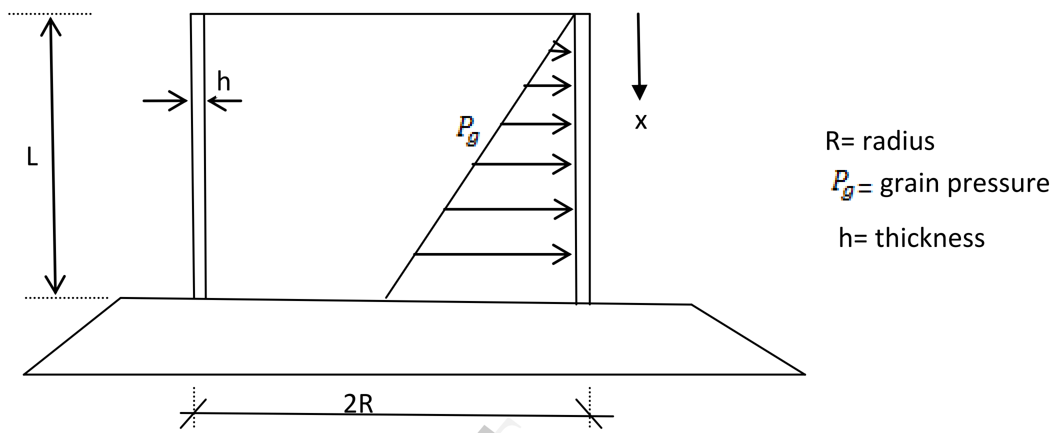
In case of food grain pressure, the distribution of forces is triangular (see Fig. 4)

The static differential equation is given by:

$$w^{iv} + 4\alpha^4 w = \frac{p_g}{D} \quad (38)$$

Where; w, α, and D are as defined in section 3.1 and

$p_g = k\gamma x$



Fg.4: Grain pressure distribution.

Where; k is the lateral- to- vertical pressure ratio given by Rankine as:

$K = tan^2\,[45^0 - \frac{\emptyset}{2}]$, $\emptyset =$ angle of internal friction of grain

γ = unit weight of grain

Equation (38) can be written as;

$$w^{iv} + 4\alpha^4 w = \frac{k\gamma x}{D} \quad (39)$$

From the initial value method;

Let Z $=\alpha x$

Differentiating above with respect to x gives:

$$\frac{dz}{dx} = \alpha \quad (40)$$

Generally,

$$\frac{d^n w}{dx^n} = \frac{d^n}{dz^n}\left(\frac{dz}{dx}\right)^n = \frac{\alpha d^n w}{dz^n} \tag{41}$$

Substitution of equations (40) and (41) into equation (39) gives:

$$= \frac{\alpha^4 d^4 w}{dz^4} + 4\alpha w(z) = \frac{k\gamma x}{D\alpha}$$

Dividing all through by α^4 gives:

$$= \frac{d^4 w}{dz^4} + 4\, w(z) = \frac{k\gamma z}{D\alpha^4}\ \frac{z}{\propto} \tag{42}$$

Substitute for D and α^4 as defined in equation (37), to get:

$$\frac{d^4 w}{dz^4} + 4\, w(z) = \frac{4k\gamma R^2}{Eh\alpha}\, \mathrm{z} \tag{43}$$

3.3 The Initial Value Homogenous Solution

The Homogenous equation is given by:

$$\frac{d^4 w}{dd^4} + 4\, w(z) = 0 \tag{44}$$

This solution is found by using the result of the classical homogenous solution obtained as:

$$wh(x) = c_1 e^{\alpha x} + e^{i\alpha x} + c_2 e^{\alpha x} + e^{-i\alpha x} + c_3 e^{-\alpha x} + e^{-i\alpha x} + c_4 e^{-\alpha x} + e^{i\alpha x} \tag{44}$$

By taking $\propto = 1$ and $x = 2$; equation (44) is put as:

$$wh(z) = c_1 e^{z} * e^{iz} + c_2 e^{z} * e^{-iz} + c_3 e^{-z} * e^{-iz} + c_4 e^{-z} * e^{iz} \tag{45}$$

Where: $c_1, c_{2,} c_{3,} c_4$ are arbitrary constants

$ez = \cos h\, z + \sin hz$

$e - z = \cos h\, z - \ \sin hz$

$e\ iz =$ cosz + $\ isinz$

$e - iz = \cos z - \ isinz$

Equation (45) can now be put as:

$$wh(z) = \sum_{k=1}^{4} \mathrm{C_k}\, \mathrm{Y_k}(\mathrm{z}) \tag{46}$$

Where: $\mathrm{y_k}\ (z) = a_k coshzcosz + b_k coshzsinz + c_k\, sinhzcosz + d_k\, sinhzsinz$ (47)

The constants $a_k, b_{k,} c_k$, and d_k $(k = 1, 2, 3, 4)$, are 16 in number and are obtained by differentiating equation (47) with respect to z till the fourth order and making use of the supplementary initial conditions which are given as:

$Y_1(0) = 1$	$Y_1^i(0) = 0$	$Y_1^{ii}(0) = 0$	$Y_1^{iii}(0) = 0$
$Y_2(0) = 0$	$Y_2^i(0) = 1$	$Y_2^{ii}(0) = 0$	$Y_2^{iii}(0) = 0$
$Y_3(0) = 0$	$Y_3^i(0) = 0$	$Y_3^{ii}(0) = 1$	$Y_3^{iii}(0) = 0$
$Y_4(0) = 0$	$Y_4^i(0) = 0$	$Y_4^{ii}(0) = 0$	$Y_4^{iii}(0) = 1$

The constants are obtained as:

$a_1 = 1$	$a_2 = 0$	$a_3 = 0$	$a_4 = 0$
$b_1 = 0$	$b_2 = \frac{1}{2}$	$b_3 = 0$	$b_4 = \frac{1}{4}$
$c_1 = 0$	$c_2 = \frac{1}{2}$	$c_3 = 0$	$a_4 = -\frac{1}{2}$
$d_1 = 0$	$d_2 = 0$	$d_3 = \frac{1}{2}$	$d_4 = 0$

These constants are substituted into equation (47) to get:

$$Y(z) = \cos h\, z \cos z \quad (48)$$

$$Y_2(z) = ½(\cos h\, z \sin z + \sin hz \cos z) \quad (49)$$

$$Y_3(z) = ½\ (\sin h\ z \sin z) \quad (50)$$

$$Y_4(z) = ¼(\cos h\, z \sin z - \sin hz \cos z) \ldots \quad (51)$$

$Y_k,\ k = 1, 2, 3, 4$ are four independent expressions. Therefore, the homogenous solution is written as:

$$W_h(z) = C_1 Y_1(z) + C_2\ Y_2(z) + C_3\ Y_3(z) + C_4\ Y(z) \ldots \quad (52)$$

The arbitrary constants $C_{1,}$ $C_2\, C_{3,}$ and C_4 are determined using the initial conditions.

The relationships between the deflection $W(z)$ and the rotation $\theta(z)$, moment M(z), shear force Q(z) and hoop tension N(z) are derived as follows:

Recalling equation (46)

$$wh(z) = \sum_{k=1}^{4}(\mathrm{C_k}\,\mathrm{Y_k}\,(z)$$

Using it together in the following expression

$\frac{d^n w}{dx^n} = \frac{\alpha^n d^n w}{dz^n}$ to obtain:

$$\theta_h(z) = \alpha C_1 Y_4(z) + \alpha C_2 Y_1(z) + \alpha C_3 Y_2(z) + \alpha C_4 Y_3(z) \quad (53)$$

$$M_h(z) = \frac{Eh}{4\alpha^2 R^2}[4C_1 Y_3(z) + 4C_2 Y_4(z) - C_3 Y_1(z) - C_4 Y_2(z)] \quad (54)$$

$$Q_h(z) = \frac{Eh}{4\alpha R^2}[4C_1 Y_2(z) + 4C_2 Y_3(z) + C_3 Y_4(z) - C_4 Y_1(z)] \quad (55)$$

$$N_h(z) = \frac{Eh}{R}[C_1 Y_1(z) + C_2 Y_2(z) + 4C_3 Y_3(z) + C_4 Y_4(z)] \quad (56)$$

Taking W_0 , θ_0, M_0 and Q_0 as initial value of $Wh(z), \theta_h(z), M_h(z)$ and $Q_h(z)$ respectively, the arbitrary constants are C_k are found by making use of the initial supplementary conditions given above. The constants are obtained as follows:

$$\theta_0 = \alpha[-4C_1 Y_4(0) + C_2 Y_1(0) + C_3 Y_2(0) + C_4 Y_3(0)]$$

$$\Rightarrow C_2 = \frac{\theta_0}{\alpha} \quad (57)$$

$$M_0 = \frac{Eh}{4\alpha^2 R^2}[4C_1 Y_3(0) + 4C_2 Y_4(0) - C_3 Y_1(0) - C_4 Y_2(0)]$$

$$\Rightarrow C_3 = -\frac{4\alpha^2 R^2}{Eh} M_0 \quad (58)$$

$$Q_0 = \frac{Eh}{4\alpha R^2}[4C_1 Y_2(0) + 4C_2 Y_3(0) + 4C_3 Y_4(0) - C_4 Y_1(0)]$$

$$\Rightarrow C_4 = -\frac{4\alpha R^2}{Eh} Q_0 \quad (59)$$

$$W_0 = [C_1 Y_1(0) + C_2 Y_2(0) + C_3 Y_3(0) + C_4 Y_4(0)]$$

$$\Rightarrow C_1 = W_0 \quad (60)$$

Substituting for $C_1,$ $C_2,$ $C_3,$ and $C_4,$ into equations (46) and (53) through (56) to obtain

$$W_h(z) = Y_1(z)W_0 + \frac{Y_2(z)}{\alpha}\theta_0 - \frac{4\alpha^2 R^2}{Eh}Y_3(z)M_0 - \frac{4\alpha R^2}{Eh}Y_4(z)Q_0 \tag{61}$$

$$\theta_h(z) = -4\alpha Y_4(z)W_0 + Y_1(z)\theta_0 - \frac{4\alpha^3 R^2}{Eh}Y_2(z)M_0 - \frac{4\alpha^2 R^2}{Eh}Y_3(z)Q_0 \tag{62}$$

$$M_h(z) = \frac{Eh}{\alpha^2 R^2}Y_3(z)W_0 + \frac{Eh}{\alpha^3 R^2}Y_4(z)\theta_0 + Y_1(z)M_0 + \frac{Y_2}{\alpha}Q_0 \tag{63}$$

$$Q_h(z) = \frac{Eh}{\alpha R^2}Y_2(z)W_0 + \frac{Eh}{\alpha^2 R^2}Y_3(z)\theta_0 - 4\alpha Y_4(z)M_0 + Y_1(z)Q_0 \tag{64}$$

$$N_h(z) = \frac{Eh}{R}Y_1(z)W_0 + \frac{Eh}{\alpha R}Y_2(z)\theta_0 - 4\alpha^2 R Y_3(z)M_0 - 4\alpha R Y_4(z).Q_0 \tag{65}$$

3.4 The Initial Value Particular Solution

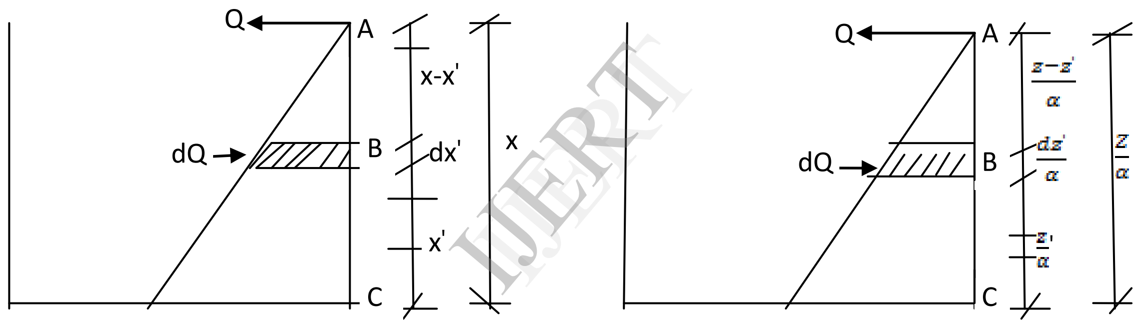


Fig. 5: Elemental Force Acting on side of Grain Silo

Considering the above Figure 5, the particular solution is found by assuming an origin transformation from A to B;

Let $Z' = \alpha x'$

$$dx' = \frac{dz'}{\alpha}$$

The distributed load at new origin B is given by

$$q(z') = \frac{z-z'}{\alpha}\, k\gamma$$

Therefore the elemental force is expressed as:

$$dQ = q(z')\frac{dz'}{\alpha} = k\gamma\, \frac{(z-z')}{\alpha^2}\, dz' \tag{66}$$

To get the particular solution, use is made of the homogeneous solution in which only parameters similar to dQ are considered. Therefore, the coefficients of Q_0 in the homogeneous solution are used with the appropriate sign to formulate the particular solutions:

$$dW_p = \frac{4\alpha R^2}{Eh} Y_4(z').\frac{k\gamma(z-z')}{\alpha^2} dz' \Rightarrow dW_p = \frac{4k\gamma R^2}{Eh\alpha}(z-z')Y_4(z')dz' \tag{67}$$

$$d\theta_p = \frac{4\alpha^2 R^2}{Eh} Y_3(z').\frac{k\gamma(z-z')}{\alpha^2} dz' \Rightarrow d\theta_p = \frac{4k\gamma R^2}{Eh}(z-z')Y_3(z')dz' \tag{68}$$

$$dM_p = \frac{Y_2(z')}{\alpha}.\frac{k\gamma(z-z')}{\alpha^2} dz' \Rightarrow dM_p = -\frac{k\gamma}{\alpha^3}(z-z')Y_2(z')dz' \tag{69}$$

$$dQ_p = -Y_1(z').\frac{k\gamma(z-z')}{\alpha^2} dz' \Rightarrow dQ_p = -\frac{k\gamma}{\alpha^2}(z-z')Y_1(z')dz' \tag{70}$$

$$dN_p = 4\alpha R Y_4(z').\frac{k\gamma(z-z')}{\alpha^2} dz' \Rightarrow dN_p = \frac{4Rk\gamma}{\alpha}(z-z')Y_4(z')dz' \tag{71}$$

Integrating the above expressions gives the actual particular solutions;

$$W_p = \frac{k\gamma R^2}{Eh\alpha}[Z - Y_2(z)] \tag{72}$$

$$\theta p = \frac{k\gamma R^2}{Eh}[1 - Y_1(z)] \tag{73}$$

$$M_p = \frac{-k\gamma}{\alpha^3} Y_4(z) \tag{74}$$

$$Q_p = \frac{-k\gamma}{\alpha^2} Y_3(z) \tag{75}$$

$$N_p = \frac{k\gamma R}{\alpha}[z - Y_2(z)] \tag{76}$$

3.5 The Initial Value General Solution

This is obtained by adding homogeneous and particular solutions:

$$W(z) = Y_1(z)\, W_0 + [\frac{Y_2(z)}{\alpha}]\theta_0 - [\frac{4\alpha^2 R^2}{Eh} Y_3(z)]M_0 -$$

$$[\frac{4\alpha R^2}{Eh} Y_4(z)]Q_0 + \frac{k\gamma R^2}{Eh\alpha}[z - Y_2(z)] \tag{77}$$

$$\theta(z) = -4\alpha Y_4(z)\, W_0 + Y_1(z)\,\theta_0 - [\frac{4\alpha^3 R^2}{Eh} Y_2(z)]M_0 -$$

$$[\frac{4\alpha^2 R^2}{Eh} Y_3(z)]Q_0 + \frac{k\gamma R^2}{Eh}[1 - Y_1(z)] \quad (78)$$

$$M(z) = [\frac{Eh}{\alpha^2 R^2} Y_3(z)]W_0 + [\frac{Eh}{\alpha^3 R^2} Y_4(z)]\theta_0 +$$

$$Y_1(z)\, M_0 + \frac{Y_2(z)}{\alpha} Q_0 - \frac{k\gamma}{\alpha^3} Y_4(z) \quad (79)$$

$$Q(z) = [\frac{Eh}{\alpha R^2} Y_2(z)]W_0 + [\frac{Eh}{\alpha^2 R^2} Y_3(z)]\theta_0 -$$

$$4\alpha Y_4(z) M_0 + Y_1(z)\, Q_0 - \frac{k\gamma}{\alpha^2} Y_4(z) \quad (80)$$

$$N(z) = [\frac{Eh}{R} Y_1(z)]W_0 + [\frac{Eh}{\alpha R} Y_2(z)]\theta_0 - [4\alpha^2 R Y_3(z)]M_0 -$$

$$[4\alpha R Y_4(z)]Q_0 + \frac{k\gamma R}{\alpha}[z - Y_2(z)] \quad (81)$$

Boundary conditions are applied in equations (76) through (80)

At $Z = 0, [M(z) = W(z)] = 0 \Rightarrow W_0 = M_0 = 0$

At $Z = L, [W(z) = \theta(z)] = 0$ (82)

Substitution of $Z = \alpha L$ and equation (81) into equations (76) and (77) which gives:

$$\frac{Y(\alpha L)}{\alpha}\theta_0 - \frac{4\alpha R^2}{Eh} Y_4(\alpha L) Q_0 + \frac{4\alpha R^2}{Eh\alpha}[\alpha L - Y_2(\alpha L)] = 0 \quad (83)$$

$$Y_1(\alpha L)\theta_0 - \frac{4\alpha^2 R^2}{Eh} Y_3(\alpha L) Q_0 + \frac{k\gamma R^2}{Eh}[1 - Y_1(\alpha L)] = 0 \quad (84)$$

Solving equations (82) and (83), to obtain

$$\theta_0 = \frac{k\gamma R^2 [Y_4(\alpha L) + Y_2(\alpha L) Y_3(\alpha L) - Y_1(\alpha L) Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{Eh\, [Y_2(\alpha L) Y_3(\alpha L) - Y_1(\alpha L) Y_4(\alpha L)} \quad (85)$$

$$Q_0 = \frac{k\gamma [Y_2(\alpha L) - Y_3(\alpha L) - \alpha L Y_1(\alpha L)]}{4\alpha^2 [Y_2(\alpha L) Y_3(\alpha L) - Y_1(\alpha L) Y_4(\alpha L)} \quad (85)$$

Substituting for W_0, M_0, θ_0, and Q_0 in equations (76) through (80) to get:

$$W(z) = \frac{k\gamma R^2 [Y_4(\alpha L) + Y_2(\alpha L) Y_3(\alpha L) - Y_1(\alpha L) Y_4(\alpha L) - \alpha L Y_3(\alpha L)]}{\alpha Eh [Y_2(\alpha L) Y_3(\alpha L) - Y_1(\alpha L) Y_4(\alpha L)]} Y_2(z)$$

$$- \frac{k\gamma R^2 [Y_2(\alpha L) - \alpha L Y_1(\alpha L)]}{\alpha Eh [Y_2(\alpha L) Y_3(\alpha L) - Y_1(\alpha L) Y_4(\alpha L)]} Y_4(z) + \frac{k\gamma R^2}{\alpha Eh}[z - Y_2(z)] \quad (86)$$

$$\theta(z) = \frac{k\gamma R^2\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_1(z)$$

$$- \frac{k\gamma R^2\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{Eh\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_3(z) \quad + \frac{k\gamma R^2}{Eh}\left[1 - Y_1(z)\right] \tag{87}$$

$$M(z) = \frac{k\gamma\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha^3\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_4(z)$$

$$+ \frac{k\gamma\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{4\alpha^3\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_2(z) \quad - \frac{k\gamma}{\alpha^3} Y_4(z) \tag{88}$$

$$Q(z) = \frac{k\gamma\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_3(z)$$

$$+ \frac{k\gamma\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{4\alpha^2\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_1(z) \quad - \frac{k\gamma}{\alpha^2} Y_3(z) \tag{89}$$

$$N(z) = \frac{k\gamma R\left[Y_4(\alpha L) + Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L) - \alpha L Y_3(\alpha L)\right]}{\alpha\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_2(z)$$

$$- \frac{k\gamma R\left[Y_2(\alpha L) - \alpha L Y_1(\alpha L)\right]}{\alpha\left[Y_2(\alpha L)Y_3(\alpha L) - Y_1(\alpha L)Y_4(\alpha L)\right]} Y_4(z) \quad + \frac{k\gamma R}{\alpha}\left[z - Y_2(z)\right] \tag{90}$$

The initial value general solutions given in equations (86) through (90) are used to evaluate the performance of the cylindrical shell under food grain pressures.

4. Materials and Method

The values of laterized concrete properties at optimum mix proportion as determined by Osadebe and Nwakonobi (2007) were adopted in performing the static analysis of the circular cylindrical shell under the action of food grain pressure using pasternack,s equations formulated on the basis of cylindrical theory of shell of revolution

4.1 Characteristics of the Silo Shell

The silo size 20m in diameter (D), 8m in height (L) and wall thickness (H) of 0.1m and 0.150m were selected. The value of laterized concrete properties such as young modules, E_L and poisson’s ratio, μ are

18,888.9kN/m^2 and 0.26 respectively (Osadebe and Nwakonobi, 2007). The characteristics of the soybean grain adopted from Nwakonobi and Idike (2002) are given as follows:

Unit weight of soybean, γ_g = 11.7kN/m^3.

Angle of internal friction for soybean grain, θ = 29^0.

4.2 Static Analysis of Laterized Cylindrical Shell The general solutions of the initial value method given in equations (86) through (90) were used to evaluate the performance of a cylindrical shell under soybean grain pressures. The values for different parameters given in section 4.1 were substituted into equations (86) through (90) and the values of deflection $W(z)$, bending moment $M(z)$, slope $\theta(z)$, shear force $Q(z)$and hoop tension $N(z)$ computed for each of the ten segment of the silo height for the silo wall thicknesses of 100mm and 150mm as presented in Tables 1 and 2, respectively.

Table 1: Results of analysis by Initial Value method for a shell thickness of 100mm

X m	W(z) m	θ(z) radian	M(z) kN-m	Q(z) kN	N(z) kN
0	0	0.000215	0	0.000866	0
0.8	0.000172	0.000215	0.001027	0.002044	32.5036
1.6	0.000344	0.000214	0.001375	0.002679	64.96722
2.4	0.000514	0.000212	0.002081	-0.009135	97.13281
3.2	0.000685	0.000215	-0.019099	-0.048181	129.30923
4.0	0.000863	0.000236	-0.071395	-0.066367	163.06735
4.8	0.00107	0.000273	-0.054079	0.190728	201.61726
5.6	0.00128	0.000217	0.409837	1.073931	241.42261
6.4	0.00131	-0.000242	1.606573	1.577469	246.89532
7.2	0.000761	-0.00112	1.373896	-3.968366	143.71501
8.0	-6.58E-09	-8.107E-09	-8.77132	-23.90073	-8.69E-05

Table 2: Results of analysis by Initial Value method for a shell thickness of 150mm

X	W(z)	θ(z)	M(z)	Q(z)	N(z)
0	0	0.0001425	0	0.00758826	0
0.8	0.000114	0.0001422	0.0040973	-0.0000459	32.27428
1.6	0.000227	0.00014199	-0.0055399	-0.028110	64.45364
2.4	0.000342	0.00014519	-0.046359	-0.074745	96.893805
3.2	0.000462	0.0001564	-0.113917	-0.074376	130.893123
4.0	0.000594	0.0001737	-0.100014	0.175309	168.33233
4.8	0.000734	0.0001663	0.308625	0.9444009	207.92997
5.6	0.000831	0.0000495	1.495723	1.9673535	235.5476
6.4	0.000757	-0.0002711	2.950649	0.966383	214.4430
7.2	0.000379	-0.0006294	1.0167647	-7.663666	107.65697
8.0	-1.856E-10	-5.59E-09	-12.84057	-28.93818	-4.749E-05

The maximum stress developed due to the bending moment and hoop tension for both wall thicknesses are as presented in Table 3.

Table 3: Maximum stress developed by two different wall thicknesses

Wall thickness mm	Bending stress N/mm^2	Hoop tension stress N/mm^2
100	5.26	2.47
150	3.42	1.57

5.0 Results and Discussion

Tables 1 and 2 show the results of stresses and deflection analysis for a grain silo shell of thickness 100 mm and 150, respectively.

- **Direct deflection**

  From Table 2, it is observed that the deflection is zero at both end of silo. The maximum value of deflection is 0.831mm and occur at a height of 5.6m from the top of silo which is 8m high and 150 mm thick.

- **Rotation**

  From Table 2 the rotation is 0.0001425 radians at the top and is zero at the bottom end. The maximum rotation is negative of 0.0006294 radians and occurs at a height 7.2m from the top.

- **Bending moment**

  The bending moment is zero at the top of the grain silo (see Table 2). The maximum sagging moment is 2.9506 kN-m and occurs at height 6.4 m from the top while the maximum hogging moment is 12.8405 kN-m and occurs at the bottom.

- **Shear force**

  Table 2 showed the shear force to be 0.007588 kN at the top and is -28.93818 kN at the bottom end of the silo. The maximum positive shear is 1.96735 kN and occurs at height 5.6 m from the top of the silo.

- **Hoop tension**

  The maximum hoop tension is 235.5476KN and occurs at height 5.6 m from the top of the silo (see Table 2).

- **Silo Shell thickness**

It is seen from Table 3 that the maximum stress developed due to soybean grain loading is 5.26N/mm^2 in wall thickness of 100mm, 20m in diameter and 8m in height. While the maximum stress developed within wall thickness of 150mm was 3.42N/mm^2. These stresses developed were compared with the allowable stress of the laterized concrete material which is 4.12 N/mm^2 (Nwakonobi and Osadebe, 2007). The wall thickness of 150 mm was within safe limit while that of 100mm was not.

5. Conclusion

From the cylindrical shell analysis using the initial-value method, a wall thickness of 100mm was found inadequate to retain soybean grain to the silo height of 8m and 20m in diameter without reinforcement. A wall thickness of 150mm was found adequate. It is however recommended that minimum reinforcement may be provided in a laterized concrete silo to prevent crack due to thermal effect or temperature variations. Cement plaster should be applied on both inner and outer surface for more environmental protection and longevity. But in order to validate this analytical study it is recommended that the laterized concrete shell be constructed to measure the deflection that will develop during the storage of food grains. This study has however demonstrated that the theory of shell structure can therefore be applied in the analysis and design of structure built of laterized concrete.

References

Aboaba, F.O., (1996). Storage and processing of major crops in Nigeria, AFP symposium proceeding, 131-135.

Amodou, A., (2002). Application of the initial value method in the analysis of circular cylindrical shell under hydrostatic pressure and ring forces. M.Eng thesis submitted to the Department of civil engineering, university of Nigeria, Nsukka.

Adejumo, B.A.; Raji, A.O., (2007). Technical appraisal of storage systems in the nigerian sudan savanna.Agricultural Engineering International: the CIGR Ejournal, IX (11).

Chandrashekhera, K. Rao, K.S.N., (1996). Analysis of a long thick orthotropic circular cylindrical shell panel. J. Eng. Mech., 122(6) 575 – 579.

Igbeka, J.C.; Olumeko, D.O.,(1991). An appraisal of village – level Grain Practices in Nigeria. Agricultural Mechanization in Asia, Africa and Latin America. 27(2) 29-33.

Lasisi, F.; Osunade, J.A., (1990). Laterized concrete masonry as an alternative in building and rural infrastructures. in proc. of international conference on Structural Engineering, seam 2, Kumasi, Ghana.

Muhammad, A.K., (2000). Analysis of thin cylindrical shell subjected to local and continuous axi-symmetric loads. unpublished project report submitted to the Department of Civil Engineering, KNUST Kumasi-Ghana.

Nwakonobi, T.U; Idike, F.I., (2002). Physical properties of soybean. Nigeria J. Tech, 21(1) 90 –91.

Nwakonobi, T.U.; Mbajiorgu, C.C., (2009). Review of agricultural storage structures for rice in Nigeria. Paper presented at south-east regional conference of the Nigerian society of Agricultural Engineers held at Abakiliki, Ebonyi State, 16 – 18 of September.

Nwakonobi, T.U.; Osadebe, N.N.; Mbajiorgu, C.C., (2012). Application of initial value method in the static analysis of laterised concrete cylindrical shells under hydrostatic loading. Int. J. of Structural Engineering, 3(4) 229-249).

Olumeko, D.O., (1996). Assessment of On-farm storage structures in Nigeria. In a Paper presented at NCAM national training workshops, Ilorin, Kwara State, Nigeria.Osunade, J.A.,(1997). Laterized concrete masonry as an alternative in building and rural infrastructures. International Conference Proc. on Structural Engineering, 151 – 161.

Osunade, J.A.,(1990). Adeyefa PO, F. Lasisi F. Shear and Tensile Properties of Unreinforced Laterized Concrete. Ife J. of Tech, 2 (1) 8-17.

Osunade, J.A., (1993). The compressive strength of laterized concrete. the effect of types and sizes of coarse aggregates. Masonry International J. Bulletin, (1) 104.

Osunade, J.A., (1994). Effect of grain size ranges of laterite fine aggregate on the shear and tensile strengths of laterized concrete. International J. for Housing Sci. and its Application, USA, 18 (2) 91 – 104.

Osadebe, N.N.; Nwakonobi, T.U., (2007). Structural characteristics of laterized concrete at optimum mix proportion. Nigeria J. of Tech, 26(1)12 – 17.

Ren, D.; Kuan-Chen, F., (2001). Solution of complete circular cylindrical shell under concentrated loads. J. Eng. Mech., 127(3) 248 – 253.

Song, H.; Shim.; S.; Byun, K., (2002). Failure analysis of reinforced concrete shell structures using layered shell element with pressure mode. J. Struct. Eng. 2002; 128(5) 655 – 664.

Timoshenko, S.; Woinosky-krieyer, S., (1959). Theory of plate and shell. McGraw Hill Book. Inc. New York.

Zingoni, A.J.; Nwakal, I.A; Salahuddin, A., (2000). Theory and analysis of structures: trusses, beams, frames, plates, and shells. UNESCO Publ. Paris, France, Chap 5, 165 –214.

NOMENCLATURE

b = Breadth of section

d = Depth of section

D = Cylindrical rigidity

E_c = Static modulus of elasticity of laterized concrete

F_c = Compressive strength of Laterized- concrete

F_{tb}, = Tensile strength of laterized concrete

h = Thickness of shell

k = Lateral to vertical pressure ratio

L = Height of cylindrical silo

M = Bending moment

N = Hoop tension

P_w = Hydrostatic pressure

P_g = Grain pressure

Q = Shear force

R = Radius of cylindrical shell

W =,Deflection

α = A constant

γ = Unit weight of water

γ_g = Unit weight of grain

θ = Rotation

$\emptyset$= Angle of internal friction of grain

σ = Stress

μ = Poisson's ratio
u = displacement component along the x- axis

v = displacement component along the y- axis

w = displacement component along the z – axi

h = thickness of the shell

$D = \frac{Eh^3}{12(1-\mu^{2)}}$ is the flexural rigidity of the shel

μ = Poisson's ratio

E = Young modulus of the shell material