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Steady State Performance Characteristics of Isoviscous Finite Flexible Oil Journal Bearings Including Fluid Inertia Effect

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Abstract: **The aim of this work is to analyse the steady state on performance characteristics of hydrodynamic journal bearing including combined effect of bearing liner surface deformation and fluid-inertia. The average Reynolds equation that modified to include the fluid inertia effect and surface deformation is used to obtain pressure field in the fluid-film. The solutions of modified average Reynolds equations are obtained using finite difference method and appropriate iterative schemes. The effects of surface deformation factor and modified Reynold's number on circumferential fluid-film pressure distribution, load carrying capacity, attitude angle, and end flow of the bearing are studied for various eccentricity ratio, slenderness ratio, Poisson ratios, and liner thickness to radius ratio. The steady state bearing performance analysis is done through parametric study of the various variables like modified Reynolds number, eccentricity ratio, slenderness ratio, attitude angle, surface deformation factor. The variation of bearing load carrying capacity, attitude angle, end flow, friction parameters has been studied and plotted against various parameters.**

Keyword- Modified Reynolds number, slenderness ratio, attitude angle, sommerfeld number, eccentricity ratio, journal bearings, inertia, and deformation factor.

I INTRODUCTION

The fluid inertia effect cannot be neglected when the viscous and the inertia forces are of the same order of magnitude shown by Pinkus and Sterlincht [1], though the basic assumptions in the classical hydrodynamic theory include negligible fluid inertia forces in comparison to the viscous forces. In recent times synthetic lubricants, low viscosity lubricants, are used in industries and owing to high velocity it is possible to arrive at such a situation where flow is laminar but the fluid inertia effect cannot be neglected. In such cases the classical Reynolds equation is not valid.

Keeping in view of the above, consideration of inertia effect of a lubricant flow may be one of the areas of recent extension of the classical lubrication theory. Among the few studies related to effect fluid inertia effect, Constatinescu and Galetuse [2] evaluated the momentum equations for laminar and turbulent flows by assuming the velocity profiles is not strongly affected by the inertia forces. Banerjee et.al [3] introduced an extended form of Reynolds equation to include the effect of fluid inertia, adopting an iteration scheme. Chen and Chen [5] obtained the steady-state characteristics of finite bearings including inertia effect using the formulation of Banerjee et al.[3]. Kakoty and Majumdar [4, 13] used the method of averaged inertia in which inertia terms are integrated over the film thickness to account for the inertia effect in their studies. The above studies were mainly based on ideally smooth rigid bearing surfaces.

When the fluid-film thickness in a journal bearing system is of the order of few micrometres, the bearing surface is not rigid, rather deformable, then surface deformation due to elastic distortion has a profound effect on bearing performance. In the present study it has been consider that the journal bearing is a cylindrical sleeve bearing made of comparatively soft material than shaft material and a rigid circular shaft rotates inside. The bearing liner is actually a thin tube surrounded by a relatively rigid housing. Since the periphery of the bearing is much larger than its thickness the radial deformation of the latter at a point may be assumed to be proportional to the pressure at that point. Elastic deformation of the journal and the bearing material by hydrodynamic fluid pressure changes the fluid film profile, modifies the pressure distribution and therefore changes the performance characteristics of the journal bearings.

Theoretical research on flexible (soft shell) bearings with a rigid rotor was started with the work of Higginson [6] using a simplified method (the distortion is proportional to the pressure). Since then many workers notably Hooke, Brighton, and O'Donoghue [7-9], Conway and Lee [11], and Oh and Huebner [12] solved the journal bearing problem considering the effect of elastic distortions of the bearing liner.

In the present work, a modified average Reynolds equation and a solution algorithm are developed to include fluid inertia and bearing sleeve surface deformation effects in the analysis of lubrication problems. The developed model is being used to study the influence of fluid inertia and surface deformation effects on the steady state characteristics such as circumferential pressure, load carrying capacity, attitude angle and side leakage of a hydrodynamic oil journal bearing.

II BASIC THEORY

The modified average Reynolds equation for fully lubricated surfaces is derived starting from the Navier-Stokes equations and the continuity equation with few assumptions. The non-dimensional form of the momentum equations and the continuity equation for a journal bearing may be written as (Figure.1)

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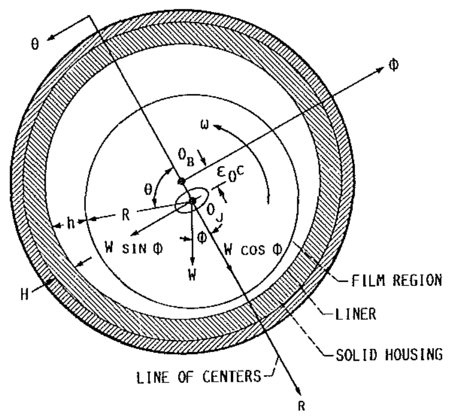

Figure.1 The schematic diagram of oil Journal Bearing

$$R^*_e\left[\Omega\frac{\partial\bar{u}}{\partial\tau}+\bar{u}\frac{\partial\bar{u}}{\partial\theta}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}+\bar{w}\left(\frac{D}{L}\right)\frac{\partial\bar{u}}{\partial\bar{z}}\right]=-\frac{\partial\bar{p}}{\partial\theta}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2} \qquad (1)$$

$$\frac{\partial\bar{p}}{\partial\bar{y}}=0 \qquad (2)$$

$$R^*_e\left[\Omega\frac{\partial\bar{w}}{\partial\tau}+\bar{u}\frac{\partial\bar{w}}{\partial\theta}+\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}}+\bar{w}\left(\frac{D}{L}\right)\frac{\partial\bar{w}}{\partial\bar{z}}\right]=-\left(\frac{D}{L}\right)\frac{\partial\bar{p}}{\partial\bar{z}}+\frac{\partial^2\bar{w}}{\partial\bar{y}^2} \qquad (3)$$

$$\frac{\partial\bar{u}}{\partial\theta}+\frac{\partial\bar{v}}{\partial\bar{y}}+\left(\frac{D}{L}\right)\frac{\partial\bar{w}}{\partial\bar{z}}=0 \qquad (4)$$

Where, $\bar{z}=\frac{z}{L/2}$, $\bar{y}=\frac{y}{c}$, $\theta=\frac{x}{R}$, $\tau=\omega_p.t$, $\Omega=\frac{\omega_p}{\omega}$,

$\bar{u}=\frac{u}{\omega R}$, $\bar{v}=\frac{v}{c\,\omega}$ $\bar{w}=\frac{w}{\omega R}$ $\bar{p}=\frac{pc^2}{\eta\omega R^2}$ and

$$R_e^*=R_e.\frac{c}{R}=\frac{\rho\omega c^2}{\eta}$$

The fluid film thickness in the case of flexible bearing can be written as

$$h=c+e\cos\theta+\delta \qquad (5)$$

$$\bar{h}=1+\varepsilon\cos\theta+\bar{\delta} \qquad (6)$$

where $\bar{\delta}=\frac{\delta}{c}, \varepsilon=\frac{e}{c}, \bar{h}=\frac{h}{c},$

where δ is the elastic deformation of the bearing surface and it is a function of θ and z.

Here the variation in the density with time is considered to be negligible. The momentum equations may be presented in the following form using equation of continuity. However, the second momentum equation is not used any further because there is no variation in pressure across the film. After Constantinescu and Galetuse [2] the velocity components are approximated by the parabolic profiles. The velocity components may be expressed in non-dimensional form as follows:

$$\bar{u}=\left[\frac{\bar{y}}{\bar{h}}+Q_\theta\left(\frac{\bar{y}^2}{\bar{h}^2}-\frac{\bar{y}}{\bar{h}}\right)\right] \qquad (7)$$

$$\bar{w}=\left[Q_z\left(\frac{\bar{y}^2}{\bar{h}^2}-\frac{\bar{y}}{\bar{h}}\right)\right] \qquad (8)$$

Q_θ and Q_z are dimensionless flow parameter in θ and $\bar{z}$ direction respectively.

Substituting these two into momentum equations and integrating give

$$Q_\theta=\frac{\bar{h}^2}{2}\left(\frac{\partial\bar{p}}{\partial\theta}\right)+R_e^*\times I_X \qquad (9)$$

$$Q_z=\frac{\bar{h}^2}{2}\left(\frac{D}{L}\right)\left(\frac{\partial\bar{p}}{\partial\bar{z}}\right)+R_e^*\times I_Z \qquad (10)$$

Where

$$I_X=\frac{\bar{h}}{2}\Bigg[\frac{1}{2}\Omega\left(1-\frac{1}{3}Q_\theta\right)\frac{\partial\bar{h}}{\partial\tau}-\frac{1}{6}\Omega\bar{h}\frac{\partial Q_\theta}{\partial\tau}-\frac{1}{3}\left(1-\frac{1}{2}Q_\theta+\frac{1}{10}Q_\theta^2\right)\frac{\partial\bar{h}}{\partial\theta}+$$
$$\frac{1}{3}\bar{h}\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_\theta}{\partial\theta}+\frac{1}{30}\left(\frac{D}{L}\right)\bar{h}\,Q_z\frac{\partial Q_\theta}{\partial\bar{z}}+$$
$$\frac{1}{6}\left(\frac{D}{L}\right)\bar{h}\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_z}{\partial\bar{z}}+\frac{1}{6}\left(\frac{D}{L}\right)\times Q_z\times\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial\bar{h}}{\partial\bar{z}}\Bigg] \qquad (11)$$

$$I_Z=\frac{\bar{h}}{2}\Bigg[-\frac{1}{6}\Omega\,Q_z\frac{\partial\bar{h}}{\partial\tau}-\frac{1}{6}\Omega\bar{h}\frac{\partial Q_z}{\partial\tau}-\frac{1}{6}Q_z\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial\bar{h}}{\partial\theta}-$$
$$\frac{1}{6}\bar{h}\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_z}{\partial\theta}+\frac{1}{30}\bar{h}Q_z\frac{\partial Q_\theta}{\partial\theta}+$$
$$\frac{1}{15}\left(\frac{D}{L}\right)\bar{h}\,Q_z\frac{\partial Q_z}{\partial\bar{z}}+\frac{1}{30}\times\left(\frac{D}{L}\right)\times Q_z^2\times\frac{\partial\bar{h}}{\partial\bar{z}}\Bigg] \qquad (12)$$

From continuity equation one can obtain the following form of modified Reynold's equation in rotating coordinate system

$$\frac{\partial}{\partial\theta}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}^3\frac{\partial\bar{p}}{\partial\bar{z}}\right)=6\left(1-2.\Omega\frac{\partial\phi}{\partial\tau}\right)\frac{\partial\bar{h}}{\partial\theta}+$$
$$12\,\Omega\frac{\partial\bar{h}}{\partial\tau}-2\times R_e^*\times\left[\frac{\partial}{\partial\theta}\left(\bar{h}\times I_X\right)+\left(\frac{D}{L}\right)\frac{\partial}{\partial\bar{z}}\left(\bar{h}\times I_z\right)\right] \qquad (13)$$

The modified Reynold's equation under steady state condition neglecting all time derivatives can be written as

$$\frac{\partial}{\partial\theta}\left(\bar{h}_0^{\,3}\frac{\partial\bar{p}_0}{\partial\theta}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\bar{z}}\left(\bar{h}_0^{\,3}\frac{\partial\bar{p}_0}{\partial\bar{z}}\right)=6\frac{\partial\bar{h}_0}{\partial\theta}-$$
$$2\times R_e^*\times\left[\frac{\partial}{\partial\theta}\left(\bar{h}_0\times I_X\right)+\left(\frac{D}{L}\right)\frac{\partial}{\partial\bar{z}}\left(\bar{h}_0\times I_z\right)\right] \qquad (14)$$

Where

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$$I_X = \frac{\bar{h}_0}{2}\left[-\frac{1}{3}\left(1-\frac{1}{2}Q_\theta+\frac{1}{10}Q_\theta^{\,2}\right)\frac{\partial \bar{h}_0}{\partial\theta} + \frac{1}{3}\bar{h}_0\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_\theta}{\partial\theta}+\frac{1}{30}\left(\frac{D}{L}\right)\bar{h}_0\,Q_z\frac{\partial Q_\theta}{\partial \bar{z}} + \frac{1}{6}\left(\frac{D}{L}\right)\bar{h}_0\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_z}{\partial \bar{z}}+\frac{1}{6}\left(\frac{D}{L}\right)\times Q_z\times\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial \bar{h}_0}{\partial \bar{z}}\right] \tag{15}$$

$$I_Z = \frac{\bar{h}_0}{2}\left[-\frac{1}{6}Q_z\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial \bar{h}_0}{\partial\theta} - \frac{1}{6}\bar{h}_0\left(\frac{1}{5}Q_\theta-\frac{1}{2}\right)\frac{\partial Q_z}{\partial\theta} + \frac{1}{30}\bar{h}_0\,Q_z\frac{\partial Q_\theta}{\partial\theta}+\frac{1}{15}\left(\frac{D}{L}\right)\bar{h}_0\,Q_z\frac{\partial Q_z}{\partial \bar{z}}+\frac{1}{30}\times\left(\frac{D}{L}\right)\times Q_z^{\,2}\times\frac{\partial \bar{h}_0}{\partial \bar{z}}\right] \tag{16}$$

$$\bar{h}_0 = 1+\varepsilon_0\cos\theta+\bar{\delta}_0 \tag{17}$$

$$Q_\theta = \frac{\bar{h}_0^{\,2}}{2}\left(\frac{\partial \bar{p}_0}{\partial\theta}\right)+R_e^{\,*}\times I_X \tag{18}$$

$$Q_z = \frac{\bar{h}_0^{\,2}}{2}\left(\frac{D}{L}\right)\left(\frac{\partial \bar{p}_0}{\partial \bar{z}}\right)+R_e^{\,*}\times I_Z \tag{19}$$

Where, $\frac{\partial \bar{h}_0}{\partial\theta} = -\varepsilon_0\sin\theta+\frac{\partial\bar{\delta}_0}{\partial\theta}$ and $\frac{\partial \bar{h}_0}{\partial \bar{z}} = \frac{\partial\bar{\delta}_0}{\partial \bar{z}}$

Boundary conditions for equation (14) are as follows

1. The pressure at the ends of the bearing is assumed to be zero (ambient):

$$\bar{p}_0\,(\theta,\pm 1) = 0$$

2. The pressure distribution is symmetrical about the mid-plane of the bearing:

$$\frac{\partial \bar{p}_0}{\partial \bar{z}}\,(\theta,0) = 0$$

3. Cavitation boundary condition is given by:

$$\frac{\partial \bar{p}_0}{\partial\theta}\left(\theta_2,\bar{z}\right) = 0 \;\; and \;\; \bar{p}_0\left(\theta,\bar{z}\right) = 0 \;\; for \;\; \theta_1 \ge \theta \ge \theta_2$$

The equation (14) along with the equations (15) to (19) are first expressed in finite difference form and solved using Gauss-Siedel method in a finite difference scheme.

Before trying to find the solution of equation (14) satisfying the appropriate boundary conditions, the elastic deformation δ_0 is obtained in the following way:

The method is similar to that of Brighton et.al [8,9] and also Majumder et al. [7].In the present calculation the three displacement components *u*, *v*, and *w* are solved simultaneously satisfying the boundary conditions.

The oil film pressure between the shaft and the bearing can be expressed in a double Fourier series of the form as indicated by Brighton et.al [8,9] and Majumder et.al., [7]

$$p_0 = \sum_m^{l}\sum_n^{l} p_{m,n}\cos\frac{2m\pi z}{L}\cos\left(n\theta+\alpha_{m,n}\right) \tag{20}$$

where $\sum^{l}$ indicates that the first term of the series is halved.

$p_{m,n}$ and $\alpha_{m,n}$ are given as follows,

$$p_{m,n} = \frac{4}{\pi L}\left[\left\{\int_0^{2\pi}\int_0^{\frac{L}{2}} p\cos\frac{2m\pi z}{L}\cos n\theta\,dz\,d\theta\right\}^2 + \left\{\int_0^{2\pi}\int_0^{\frac{L}{2}} p\cos\frac{2m\pi z}{L}\sin n\theta\,dz\,d\theta\right\}^2\right]^{\frac{1}{2}} \tag{21}$$

$$\alpha_{m,n} = \tan^{-1}\left[\frac{-\int_0^{2\pi}\int_0^{\frac{L}{2}} p\cos\frac{2m\pi z}{L}\sin n\theta\,dz\,d\theta}{\int_0^{2\pi}\int_0^{\frac{L}{2}} p\cos\frac{2m\pi z}{L}\cos n\theta\,dz\,d\theta}\right] \tag{22}$$

$$p_{0,0} = \frac{2}{\pi L}\int_0^{2\pi}\int_0^{\frac{L}{2}} p\,d\theta\,dz \tag{23}$$

The first term of the right-hand side of equation (20) is $\frac{1}{2}p_{0,0}$.

Using the end condition of the bearing (i,e $p=0$ at $z=\frac{L}{2}$) we can obtain $p_{0,0}$. This term does not contribute any deformation at $z=\frac{L}{2}$. Its effect for the other values of z is included in the total deformation. The boundary conditions of the inner radius are

$$\sigma_r = -p,\;\; \tau_{r\theta} = 0,\;\; \tau_{rz} = 0 \tag{24}$$

After non-dimensionalisation, the equation (21), (22) and (23) becomes

$$\bar{p}_{m,n} = \frac{2}{\pi}\left[\left\{\int_0^{2\pi}\int_0^{1} \bar{p}\cos m\pi\bar{z}\cos n\theta\,d\bar{z}\,d\theta\right\}^2 + \left\{\int_0^{2\pi}\int_0^{1} \bar{p}\cos m\pi\bar{z}\sin n\theta\,d\bar{z}\,d\theta\right\}^2\right]^{\frac{1}{2}} \tag{25}$$

$$\alpha_{m,n} = \tan^{-1}\left[\frac{-\int_0^{2\pi}\int_0^{1} \bar{p}\cos m\pi\bar{z}\sin n\theta\,d\bar{z}\,d\theta}{\int_0^{2\pi}\int_0^{1} \bar{p}\cos m\pi\bar{z}\cos n\theta\,d\bar{z}\,d\theta}\right] \tag{26}$$

and $$\bar{p}_{0,0} = \frac{1}{\pi}\int_0^{2\pi}\int_0^{1} \bar{p}\,d\theta\,d\bar{z} \tag{27}$$

where $\bar{p}_{m,n} = \frac{p_{m,n}c^2}{\eta_0\omega R^2}$, $\bar{p} = \frac{p\,c^2}{\eta_0\omega R^2}$, $\bar{z} = \frac{z}{L/2}$

The outer surface of the bearing is rigidly enclosed by the housing, preventing any displacement of the outer surface. The ends of the bearing are prevented from expanding axially, but are free to move circumferentially or radially.

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The displacement components in r, θ and z directions are found from the pressure distribution, which has been expressed in a Fourier series. It is apparent that the displacements will also be harmonic functions.

These displacements were substituted in the stress-strain relationships using Lame's constants. The six components of stresses were then used in the equations of equilibrium to obtain the following three displacement equations.

$$C^* \frac{d^2 u^*}{d\bar{y}^2} + \frac{C^*}{\bar{y}}\frac{d u^*}{d\bar{y}} - \left(C^* + n^2\right)\frac{u^*}{\bar{y}^2} + \left(C^* - 1\right)\frac{n}{\bar{y}}\frac{d v^*}{d\bar{y}} - \left(C^* + 1\right)\frac{n}{\bar{y}^2} v^* - k^2 u^* + \left(C^* - 1\right) k \frac{d w^*}{d\bar{y}} = 0 \quad (28)$$

$$\frac{d^2 v^*}{d\bar{y}^2} + \frac{1}{\bar{y}}\frac{d v^*}{d\bar{y}} - \left(1 + C^* n^2\right)\frac{v^*}{\bar{y}^2} - k^2 v^* - \left(C^* - 1\right)\frac{n}{\bar{y}}\frac{d u^*}{d\bar{y}} - \left(C^* + 1\right)\frac{n}{\bar{y}^2} u^* - n k \left(C^* - 1\right)\frac{w^*}{\bar{y}} = 0 \quad (29)$$

$$\frac{d^2 w^*}{d\bar{y}^2} + \frac{1}{\bar{y}}\frac{d w^*}{d\bar{y}} - \frac{n^2}{\bar{y}^2}.w^* - C^* k^2 w^{*/} - k\left(C^* - 1\right)\frac{d u^*}{d\bar{y}} - k\left(C^* - 1\right)\frac{u^*}{\bar{y}} - n\left(C^* - 1\right).k\frac{v^*}{\bar{y}} = 0 \quad (30)$$

Where, $C^* = 2 + \frac{\lambda}{\mu}$ & $k = \frac{2 m \pi r_i}{L}$

The boundary conditions are, at $\bar{y} = 1$,

$$C^* \frac{d u^*}{d\bar{y}} = -\frac{1}{\mu} p_{m,n} - \left(C^* - 2\right)\left(\frac{n v^*}{\bar{y}} + \frac{u^*}{\bar{y}} + k w^*\right) \quad (31)$$

$$\frac{d v^*}{d\bar{y}} = \frac{n u^*}{\bar{y}} + \frac{v^*}{\bar{y}} \quad (32)$$

$$\frac{d w^*}{d\bar{y}} = u^* k \quad (33)$$

and at $\bar{y} = \frac{b}{a}$,

$$u^* = v^* = w^* = 0 \quad (34)$$

The equations (28), (29) and (30) expressed first in finite difference form solving the displacement equations with the boundary conditions (31 to 34) the values of the distortion coefficient $d_{m,n}$ were obtained and expressed as,

$$d_{m,n} = \frac{\mu u^*}{R p} \quad (35)$$

The radial deformation δ_0 of the bearing surface will be $\delta_0 = u^*$

or, $\delta_0 = d_{m,n} \frac{R p_0}{\mu} r_i \cos\left(n\theta + \alpha_{m,n}\right) \cos\frac{2 m \pi z}{L}$

Considering the bearing clearance is very small in compare to the diameter of the journal, the total radial deformation will be

$$\delta_0 = \frac{R p_{0,0} d_{0,0}}{2\mu} + \sum_{\substack{m=0 \\ (m,n)\neq(0,0)}}^{\infty} \sum_{n=0}^{\infty} d_{m,n} \frac{R}{\mu} p_{m,n} \cos\left(n\theta + \alpha_{m,n}\right) \cos\frac{2 m \pi z}{L} \quad (36)$$

Using $\bar{p}_{m,n} = \frac{p_{m,n} c^2}{\eta_0 \omega R^2}$, $\bar{z} = L/2$ and μ is replaced by $\frac{E}{2(1+\nu)}$,

the radial deformation in the inner surface will be in the form,

$$\bar{\delta} = 2(1+\nu) F\left[\bar{p}_{0,0} d_{0,0} + \sum_{\substack{m=0 \\ (m,n)\neq(0,0)}}^{\infty} \sum_{n=0}^{\infty} \bar{p}_{m,n} d_{m,n} \cos\left(n\theta + \alpha_{m,n}\right) \cos m\pi\bar{z}\right] \quad (37)$$

In steady condition radial deformation in the inner surface may be written as,

$$\bar{\delta}_0 = 2(1+\nu) F\left[\bar{p}_{0,0} d_{0,0} + \sum_{\substack{m=0 \\ (m,n)\neq(0,0)}}^{\infty} \sum_{n=0}^{\infty} \bar{p}_{m,n} d_{m,n} \cos\left(n\theta + \alpha_{m,n}\right) \cos m\pi\bar{z}\right] \quad (38)$$

Where $\bar{\delta}_0 = \frac{\delta_0}{c}$ & $F = \frac{\eta_0 \omega R^3}{E c^3}$

III METHOD OF SOLUTION

A. Steady-State Analysis.

To find out steady-state pressure all the time derivatives are set equal to zero and the non-dimensionalised all equations (14) to (19) and also equations (25) to (30) and equation (37) are written in finite difference form along with all required boundary conditions to proceed for calculation. For $\varepsilon_0 \leq 0.2$ the pressure distribution and flow parameters Q_θ and Q_z are evaluated from inertia less ($\mathrm{Re}^* = 0$) solution, i.e., solving classical Reynold's equation. These values are then used as initial value of flow parameters to solve Eqs.(18) and (19) simultaneously for Q_θ and Q_z Using Gauss-Siedel method in a finite difference scheme. Then updated I_x and I_z and then calculate Q_θ and Q_z for use to solve Eq.(14) with initial zero surface deformation for new pressure $\bar{p}$ with inertia effect by using a successive over relaxation scheme. The latest values of Q_θ and Q_z and $\bar{p}$ are used iteratively to solve the set of equations until all variables converges using a finite-difference method (Gauss-Seidel) with successive over relaxation scheme. The convergence criterion adopted for pressure is $\left|\left(1 - \left(\sum \bar{p}_{new} \Big/ \sum \bar{p}_{old}\right)\right) \leq 10^{-5}\right|$ and also same criterion for Q_θ and Q_z .. The distribution was expressed as a double Fourier series as given by equation (20). The deformation equation (37) was then calculated for a given F using distortion coefficients from equation (35) after calculating displacement components solving equations (28), (29), and (30) using boundary conditions (31) to (35). The film thickness equation was then modified using equation (17). The fluid film pressure was again obtained from equation (14) simultaneously with equation (15), (16) and equation (18) and (19) and then get modified film shape. The process was repeated until a compatible film shape and pressure distribution was determined.

For higher eccentricity ratios ($\varepsilon_0 > 0.2$) the initial values for the variables are taken from the results corresponding to the previous eccentricity ratios. Very small increment in ε is to be provided as Re^* increases. The procedure converges up to a value of $\mathrm{Re}^* = 1.5$ which should be good enough for the present study.
Since the bearing is symmetrical about its central plane ($\bar{z}$ =0),only one half of the bearing needs to be considered for the analysis, Once the pressure distribution is evaluated fluid film forces and the load bearing capacity $\bar{W}_o$ and attitude angle (φ) are calculated

B. Steady State Fluid Film Forces.
The non-dimensional fluid film forces along line of centres and perpendicular to the line of centres are given by

Re*	ε	$\bar{W}_0$	$\bar{W}_0$	$\bar{W}_0$	ϕ_0	ϕ_0	ϕ_0
		Present	Kakoty	Chen-Chen	Present	Kakoty	Chen-Chen
0	0.2	0.499	0.504	0.501	77.377	73.71	73.90
	0.5	1.728	1.790	1.779	58.847	56.64	56.80
	0.8	7.046	7.459	7.1460	36.641	34.66	36.20
	0.9	16.91	17.714	16.982	26.370	23.90	26.40
0.28	0.2	0.494	0.5055	0.504	77.427	73.75	74.20
	0.5	1.734	1.7980	1.7850	58.460	56.72	57.00
	0.8	7.101	7.4837	7.1510	36.543	34.72	36.30
	0.9	17.05	17.761	16.993	26.365	23.93	26.40
0.56	0.2	0.495	0.5070	0.505	77.538	73.79	74.50
	0.5	1.740	1.8058	1.790	58.674	56.79	57.20
	0.8	7.156	7.5081	7.159	36.446	34.78	36.40
	0.9	17.094	17.809	17.00	26.36.1	23.97	26.40
1.4	0.2	0.508	0.5112	0.508	77.622	73.95	75.30
	0.5	1.7562	1.830	1.587	56.893	57.05	58.00
	0.8	7.3172	7.585	7.187	36.149	35.02	36.70
	0.9	17.359	------	17.03	26.339	----	26.60

$$\bar{F}_{r_0} = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_0 \cos\theta \, d\theta \, d\bar{z} \qquad (39)$$

$$\bar{F}_{\phi_0} = \int_0^1 \int_{\theta_1}^{\theta_2} \bar{p}_0 \sin d\theta \, d\bar{z} \qquad (40)$$

Where, $\bar{F}_{r_0} = \dfrac{F_{r_0} C^2}{\eta\omega R^3 L}$, $\bar{F}_{\phi_0} = \dfrac{F_{\phi_0} C^2}{\eta\omega R^3 L}$

where θ_1 and θ_2 are angular coordinates at which the fluid film commences and cavitates respectively.

C. Steady-State Load and Attitude Angle.

The steady state non-dimensional load and attitude angle are given by

$$\bar{W}_0 = \sqrt{\left(\bar{F}_{r_0}{}^2 + \bar{F}_{\phi_0}{}^2\right)} \qquad (41)$$

$$\phi_o = \tan^{-1}\left(\frac{-\bar{F}_{\phi_0}}{\bar{F}_{r_0}}\right) \qquad (42)$$

Since the steady state film pressure distribution has been obtained at all the mesh points, integration of equations (39) and (40) can be easily performed numerically by using Simpson's 1/ 3 rd. rule to get $\bar{F}_r$ and $\bar{F}_\phi$. The teady state load $\bar{W}_0$ and the attitude angle (ϕ_0) are calculated using equations (41) and (42).

The present theoretical study has been done considering combine effect of fluid Inertia effects and bearing surface deformation The results have been compared with available data of researchers.

IV RESULTS AND DISCUSSION

The present steady state results (considering only fluid inertia effect) are compared to the results of Kakoty et.al., [4] and Chen & Chen [5] (for $L/D = 1.0$) as given in Table 3. These three results are in good agreement

Table 1: Comparison of Steady -state characteristics of a oil journal bearings for $L/D = 1.0, F = 0.0$ i.e, with considering Inertia Effect only

The steady-state results are compared to the results of Kakoty & Majumder [4] Chen and Chen [5] for $L/D = 1.0$ and $F = 0.0$ as given in Table 1. These two results are in good agreement. A slight increase in load capacity with modified Reynolds number R_e^* is observed in the present study. In the present study it is observed that the attitude angle increases slightly for eccentricity ratio 0.2, whereas the attitude angle reduces slightly for eccentricity ratio 0.8 & 0.9.

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Table 2. Comparison of maximum steady state pressure obtained by the present method to that of Brighton et.al [9] and Majumder, Brewe et.al [7]
$[L/D=1.0, \varepsilon_0=0.85, H/R=0.3, \nu=0.4]$

Deformation factor, F	P max (Present)	P max (Brighton)	P max (Majumder)
0.0	18.2231	16.8	17.25
0.05	14.9515	14.10	13.5
0.1	12.4542	11.40	11.5
0.2	8.8761	8.7	9,0
0.4	5.8980	6.3	6.25

It may be seen that the peak pressure decreases with increase in the elasticity parameter / deformation factor.

In Table 2 the comparison of maximum centreline pressures in the circumferential direction of the present solution for a finite bearing with $L/D=1.0, \varepsilon_0=0.85$, and for values of F varying from 0 to 0.4 with those of reference [7, 9] are shown.

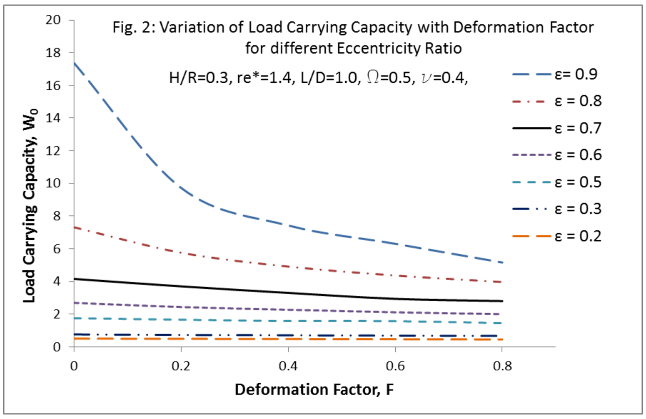

Fig. 2: Variation of Load Carrying Capacity with Deformation Factor for different Eccentricity Ratio

Figure 2 shows the steady-state load capacity variation with elasticity parameter for seven eccentricity ratios (i.e., $\varepsilon_0=0.2$, 0.3, 0.5, 0.6, 0.7, 0.8, 0.9). Although there is little variation of load with F at low eccentricity ratios, the load drops very sharply with F at $\varepsilon_0=0.9$. The increase in F increases the minimum film thickness as shown in fig. 3. This in effect reduces the true eccentricity ratio, therefore pressures and load capacity drop. It may be mentioned that a similar observation has been made by Conway and Lee [11] while analyzing a flexible bearing using the short bearing approximation

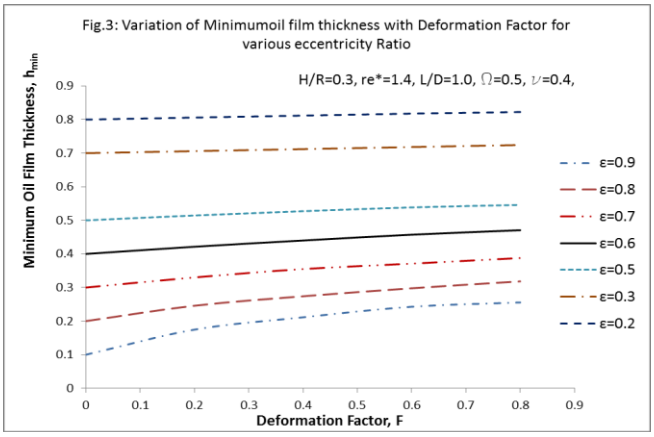

Fig.3: Variation of Minimumoil film thickness with Deformation Factor for various eccentricity Ratio

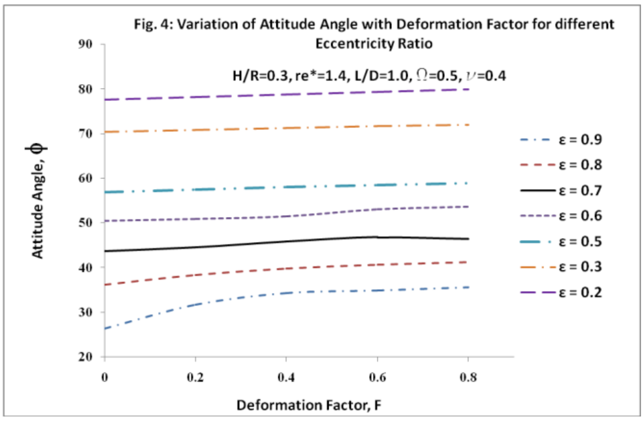

Fig. 4: Variation of Attitude Angle with Deformation Factor for different Eccentricity Ratio

Figure 4 shows the steady-state attitude angle variation with elasticity parameter for seven eccentricity ratios (i.e., $\varepsilon_0=0.2$, 0.3, 0.5, 0.6, 0.7, 0.8, 0.9). The increasing trend of the variation of attitude angle with F is observed

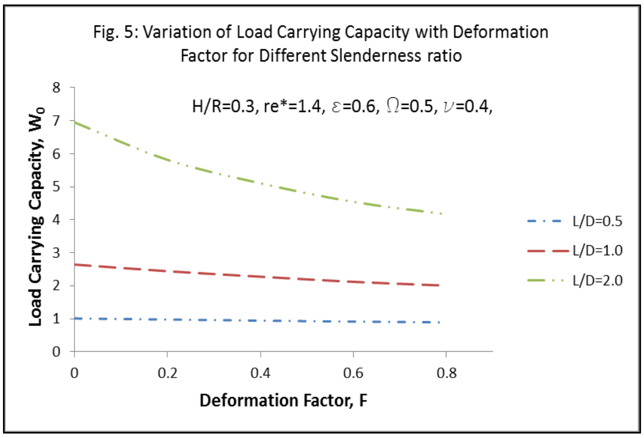

Fig. 5: Variation of Load Carrying Capacity with Deformation Factor for Different Slenderness ratio

Figure 5 shows the steady-state Load carrying capacity variation with elasticity parameter for different slenderness ratio (i.e., $L/D=0.5, 1.0, 2.0$). Although there is little variation of load with F at low slenderness ratios, the load drops very sharply with F at $L/D=2.0$

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Fig. 6: Variation of Load Carrying Capacity with Deformation for different Modified Reynold's Number

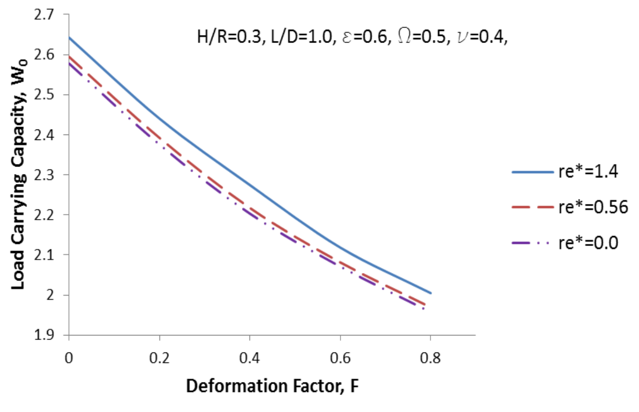

Figure 6 shows the steady-state Load carrying capacity variation with elasticity parameter for different modified Reynold's number (i.e., $R_e^* = 0.0,\ 0.56,\ 1.4$). The load drops very sharply with F . The load carrying capacity increases as modified Reynold's number increases.

Fig. 7: Variation of Load Carrying Capacity with Deformation Factor for different H/R ratio.

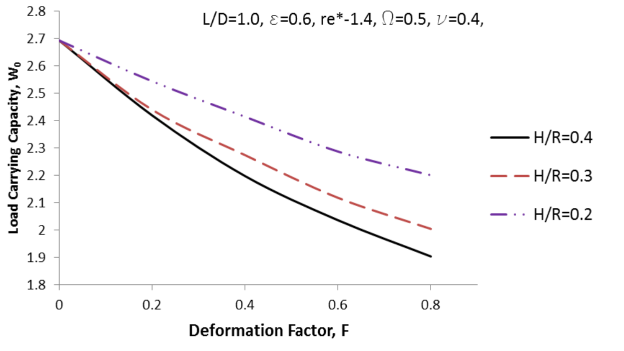

Figure 7 shows the steady-state Load carrying capacity variation with elasticity parameter for different H/R ratio. The load drops very sharply with F . The load carrying capacity increases as H/R ratio decreases.

Fig. 8: Variation of Load Carrying Capacity with Deformation Factor for different Poisson Ratio

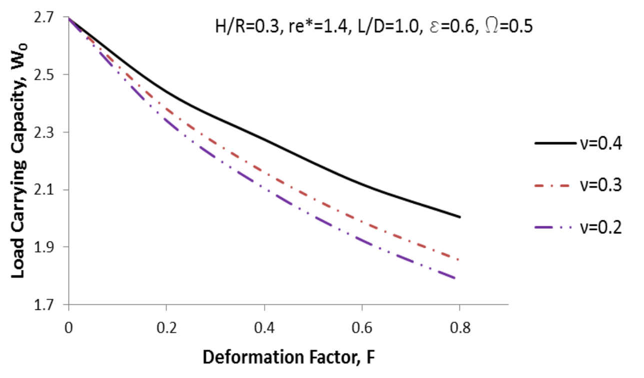

Figure 8 shows the steady-state Load carrying capacity variation with elasticity parameter for different poisson ratio. The load drops very sharply with F . The load carrying capacity increases as Poisson ratio increases.

Fig. 9: Variation of End Flow with Deformation Factor for different Eccentricity Ratio

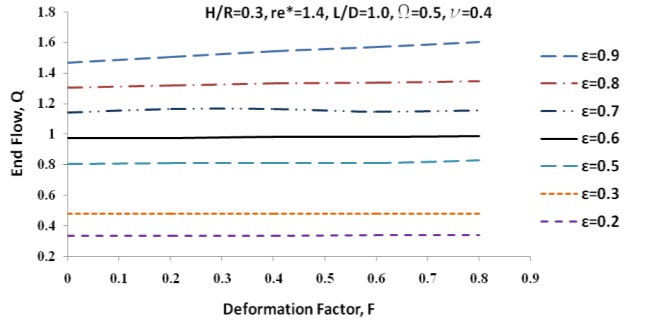

Figure 9 shows the steady-state End flow variation with elasticity parameter for different Eccentricity ratio. The End flow increases marginally at higher eccentricity ratio $\varepsilon_0 \geq 0.8$ but at low eccentricity ratio the variation is almost constant with F .

V CONCLUSIONS

1. The region of load carrying capacity decreases as the bearing liner is made more flexible for high eccentricity ratios
(i.e., $\varepsilon_0 > 0.7$). For $\varepsilon_0 < 0.7$, the flexibility of the bearing liner had little or no effect on stability.
2. As L/D is increased, distortion effects are more prominent. This leads to a decrease in load carrying capacity.
3. The hydrodynamic pressure and hence the load capacity is reduced as the bearing liner becomes more flexible, especially at eccentricities greater than 0.8.
4. As the Reynolds number increases the load carrying capacity increases but drop when bearing liner is made more flexible
5. As the Poisson ratio increases the load carrying capacity increases but drop sharply when bearing liner is made more flexible.
6. As the liner thickness to radius ratio increases the load carrying capacity decreases but drop when bearing liner is made more flexible

NOMENCLATURE

$a = r_i$	Inner radius of the bearing liner [m]
$b = r_0$	Outer radius of the bearing liner [m]
c	Radial clearance [m]
R	Journal radius [m]
D	Journal diameter [m]
$d_{m,n}$	Distortion coefficient of $_{m,n}$ harmonic
m,n	Axial and circumferential harmonics

e	Eccentricity [m]
e_0	Steady state eccentricity [m]
E	Young's modulus [N/m^2]
F	Elasticity parameter or deformation factor,
$\bar{F}_r$	Nondimensional fluid film force along the line of centers
$\bar{F}_\phi$	Nondimensional fluid film force perpendicular the line of centres
$\bar{F}_{r_0}$, $\bar{F}_{\phi_0}$	Non-dimensional steady state fluid film forces
h	Oil film thickness [m]
h_0	Steady state oil film thickness [m]
$\bar{h}_0$	Non-dimensional steady state oil film thickness
H	Thickness of bearing liner [m]
J	Mechanical equivalent of heat
L	Length of bearing [m]
p	Oil film pressure [Pa]
p_0	Steady state film pressure [Pa]
$\bar{p}_0$	Dimensionless oil pressure
Q	End flow of oil [m^3/s]
$\bar{Q}$	Nondimensional End flow
$\bar{u}, \bar{v}, \bar{w}$	Components of fluid velocity in the x, y, and z direction, respectively. [m/s]
U	Shaft peripheral speed [m/s]
W_0	Steady state load [N]
$\bar{W}_0$	Dimensionless steady state load
x, y, z	Circumferential, radial and axial coordinates
$\theta, \bar{y}, \bar{z}$	Dimensionless coordinates in circumferential, radial and axial directions
η_0	Viscosity at inlet condition [Pa s]
ρ	Density [kg/m^3]
ν	Poisson's ratio
ε	Eccentricity ratio
ε_0	Steady state eccentricity ratio
ϕ	Attitude angle [rad]
ϕ_0	Steady state attitude angle [rad]
θ_1	Angular coordinates at which the fluid film commences [rad]
θ_2	Angular coordinates at which the fluid film cavitates [rad]
ω	Angular velocity of journal [rad / s]
Ω	Whirl ratio. [$\frac{\omega_p}{\omega}$]
$\tau = \omega_p t$	Non dimensional time.
δ	Deformation of bearing surface. [m]
δ_0	Steady state deformation of bearing surface. [m]
$\bar{\delta}_0$	Non-dimensional deformation of bearing surface
λ, μ	Lame's constants
R_e =	Reynolds number, $\frac{\rho c R \omega}{\eta}$
R^*_e =	Modified Reynolds number, $\left(\frac{c}{R}\right) R_e$
Q_θ =	Dimensionless flow parameter in θ direction
Q_z =	Dimensionless flow parameter in $\bar{z}$ direction

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