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Stochastic Modelling and Reliability Analysis of a RO Membrane System Used in Water Purification System with Patience –Time for Repair

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Abstract - **This study presents stochastic model for analysis of real industrial system model of a RO Membrane (ROM) used in water purification system. The system consists of a single unit of RO membrane which plays important role in water purification system. In this study, two repairman's regular and expert repairman is available. If regular repairman is able to repair the failed unit within fixed amount of time known as "patience-Time" then it is ok otherwise the expert repairman will be called. The failure time distributions fallows negative exponential while repair time distributions are general. By using semi-Markov processes and regenerative point technique we have obtained measures of system effectiveness. The relevant data's/information gathered from industry is used for study, Graph have been plotted for economical analysis of system.**

1. INTRODUCTION

Reliability models for system have widely been analyzed by a number of authors under various assumptions, including Singh. S. K.et al (1991) analyzed cost benefit analysis of a 2-unit priority standby system with patience –time for repair, Tuteja, R. K. et al (1994) discussed A system with pre inspection and two type of repairman, Chander, S. (2005) discussed Reliability model with priority for inspection and repair with arrival time of server, M. N. Gopalan et al (1985) analyzed cost benefit analysis of one-server two-unit cold standby system with repair and preventive maintenance.

Most of these studies are not based on the real data's. However very few researchers considered the real existing system models like Singh, J.and Mahajan, P. (1999) analyzed Reliability of Utensils manufacturing plant- A case study. Guines, M. and Devici,I.(2002) discussed Reliability of service system and application in student office.Gupta,R.et al(2007) discussed benefit analysis of distillery plant system Studied some reliability models in real data of failure and repair rates in such systems. In present paper we studied performances of RO Membrane system used in water purification system model. The RO Membrane is a very important part of water purification system.

"A ROM is thin film Composite (TFC) membrane with 0.0001 micron pore size, in which the water is passed under high pressure through TFC. The output of membrane is purified water, with reduced Total Dissolved Solids TDS free from micro- organisms like bacteria, virus protozoa and cysts, hardness, pesticides, and heavy metals like arsenic lead and mercury".

In view of above advantages of RO membrane in water purification industry its failure in case is not tolerable. These RO membranes may fail in following some main reasons:

(a) Failure of electrical motor pump.
(a) Failure due to deposition of salt, dissolved solids etc
(b) Failure due to non supply of water in the membrane.

Keeping the above view, introducing concept of reliability modelling, a one unit RO membrane system has been analyzed in the present paper in which the policy of preventive maintenance is applied after continuous working for a random amount of time to make the system more reliable. In the system two repair facilities are considered known as regular and expert. If regular repairman is able to repair the failed unit within the fixed amount of time known as "patience –time" then it is O.K. otherwise expert repairman will be called. System is analyzed using semi-Markov process and regenerative techniques we obtain many system measures such as MTSF, availability, mean down time expected profit etc. Graphs have been plotted for economic analysis of system.

2. MODEL

(1) The system consists of a single unit R O Membrane (ROM), which is operable initially.
(2) RO membrane can fail due to following three reasons.
(a) Failure of electric motor pump.
(b) Failure due to deposition of salt, heavy metals etc.
(c) Failure due to non supply of water in the membrane

(3)The probabilities that a membrane will fail due to reasons (a), (b) and, (c) are fixed.

(4)The policy of PM is applied after continue working for random amount of time to make the system more reliable .In this the system becomes down (not Failed) and the complete unit inspected, flushing, servicing etc, are applied.

(5)There are two repair facilities are considered known as regular and expert. Whenever a RO membrane fails with any of the reasons, the failed unit is sent for repair by regular repair facility. If regular repairman is able to repair the failed unit within the fixed amount of time known as "Patience –time" then it is O.K. otherwise expert repairman will be called. Once the expert repairman enters, it will complete all the jobs related to the system. The PM will be completed by the regular repair facility only.

(6)The distribution of time to failure of a working ROM is negative exponential while completing PM and repair of failed ROM are in general.

2.1 Notations

E	Set of regenerative state
α	constant failure rate of operative ROM
β	constant rate of applying PM policy
g(.)/G(.)	pdf and cdf of time to complete PM
h1(.)/h2(.)/h3(.)	pdf of time to complete repair of ROM failed due to reason of (a), (b), (c) respectively.
H1(.)/H2(.)/H3(.)	cdf of time to complete repair of ROM failed due to reason of (a), (b), (c) respectively
f(.)/F(.)	pdf and cdf of time to completing patience-time for regular repairman
k(.)/K(.)	pdf and cdf of time to complete repair of a ROM by expert repairman
p/ q/ r	probability that operative ROM will fail due to reason of (a), (b), (c) respectively
m_1	mean patience-time $=\int_0^\infty t\, g(t)\, dt$
m2	mean repair time of ROM by expert repairman = $\int_0^\infty t\, k(t)\, dt$
$/©	Symbol for Stieltjes convolution/ Laplace convolution
~/*	Symbol for Laplace Stieltjes Transform (LST)/ Laplace Transform
$S_i (i = 1\ to 5)$	The possible transition

2.2 Symbol for States of the system

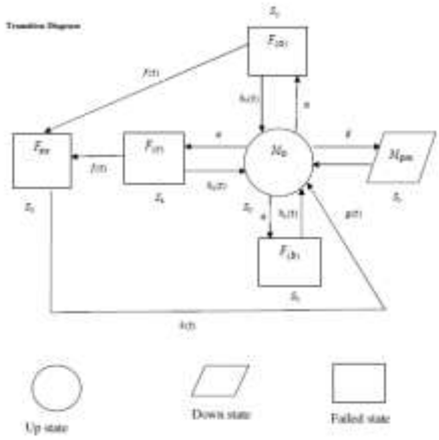
M_0	Normal unit of ROM under operative
M_{pm}	Normal unit of ROM under preventive maintenance
$F_{(a)}$	ROM under regular repairman failed due to reason (a)
$F_{(b)}$	ROM under regular repairman failed due to reason (b)
F(c)	ROM under regular repairman failed due to reason (c)
F_{ex}	Failed ROM under expert repairman

Up state $S_0 = (M_0)$, Down state $S_1 = (M_{pm})$

Failed state $S_2 = (F_{(a)}\ or)$, $S3 = (F_{(b)}\ or)$,

$S_4 = (F_{(c)}\ or)$, $S_5 = (F_{ex})$,

Fig-1. State Transition Diagram



3. TRANSITION PROBABILITIES AND MEAN SOJOURN TIME

Simple probabilistic consideration yield the following equations for the non zero elements

$$p_{ij} = Q(\infty) = \lim_{t\to\infty} Q_{ij}(t) \tag{1}$$

$$p_{01} = \frac{\alpha}{\alpha+\beta}, \quad p_{02} = \frac{p\alpha}{\alpha+\beta}$$

$$p_{03} = \frac{q\alpha}{\alpha+\beta}, \quad p_{04} = \frac{r\alpha}{\alpha+\beta},$$

$$p_{10} = 1, \quad p_{20} = \int_0^\infty \overline{F}(t) h_1(t) dt$$

$$p_{25} = \int_0^\infty f(t) \overline{H}_1(t) dt, \; p_{35} = \int_0^\infty f(t) \overline{H}_2(t) dt$$

$$p_{30} = \int_0^\infty \overline{F}(t) h_2(t) dt, \; p_{40} = \int_0^\infty \overline{F}(t) h_3(t) dt$$

$$p_{45} = \int_0^\infty f(t) \overline{H}_3(t) dt, \quad p_{50} = 1 \tag{2}$$

From the above probabilities the following relation can be easily verified as

$$p_{01} + p_{02} + p_{03} + p_{04} = 1$$

$$p_{20} + p_{25} = 1, \quad p_{30} + p_{35} = 1$$

$$p_{40} + p_{45} = 1 \tag{3}$$

If μ_i is the mean sojourn time in state S_i, then mean sojourn time in state, in given by

$$\mu_i = \int_0^\infty P[T > t] dt \tag{4}$$

Where

T is the time of stay in state S_i by the system.

Therefore, the mean sojourn time for various states is as follows

$$\mu_0 = \int_0^\infty e^{-(\alpha+\beta)} dt = \frac{1}{\alpha+\beta},$$

$$\mu_1 = \int_0^\infty \overline{G}(t) dt = \int_0^\infty t\, g(t)\, dt = m_1$$

$$\mu_2 = \int_0^\infty \overline{F}(t) H_{1(t)} dt,$$

$$\mu_3 = \int_0^\infty \overline{F}(t) H_{2(t)} dt$$

$$\mu_4 = \int_0^\infty \overline{F}(t) H_{3(t)} dt,$$

$$\mu_5 = \int_0^\infty \overline{K}(t) dt = \int_0^\infty t\, k(t) dt = m_2 \tag{5}$$

The unconditional mean time taken by the system to transit to any regenerative state S_j when it is counted from epoch of entrance into S_i is

$$m_{ij} = \int_0^{\infty} t dQ_{ij}(t) = -q'^{*}_{ij}(0) \qquad (6)$$

Thus

$m_{01} + m_{02} + m_{03} + m_{04} = \mu_0$

$m_{10} = m_1 = \mu_1$, $m_{20} + m_{25} = \mu_2$,

$m_{30} + m_{35} = \mu_3$, $m_{40} + m_{45} = \mu_4$

$m_{50} = m_2 = \mu_5$ (7)

4. MEAN TIME TO SYSTEM FAILURE (MTSF)

To investigate the distributions function $\pi_i(t)$ of the time to system failure with starting state S_0, the failed states are taken to be absorbing. Using the arguments for a regenerative process, we obtain the following relation for $\pi_i(t)$

$\pi_0(t) = Q_{01}$ \$ $\pi_1(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t)$

$\pi_1(t) = Q_{10}$ \$ $\pi_0(t)$ (8)

Taking LST of relation (8) and solving for $\tilde{\pi}_0(s)$ by omitting the argument 's' for brevity, we get

$$\tilde{\pi}_0(s) = \frac{N_1'(s)}{D_1(s)} \qquad (9)$$

Where

$N_1(s) = \tilde{Q}_{02} + \tilde{Q}_{03} + \tilde{Q}_{04}$ (10)

$D_1(s) = 1 - \tilde{Q}_{01}\tilde{Q}_{02}$ (11)

Taking the limit s→0 in equation (9), one gets $\tilde{\pi}_0(s) = 1$, which implies that $\tilde{\pi}_0(s)$ is proper distribution function. Therefore, mean time to system failure when the initial state is S_0, is

$$E(T) = -\left[\frac{d}{ds}\tilde{\pi}_0(s)\right]_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)} = \frac{N_1}{D_1} \qquad (12)$$

Where

$N_1 = \mu_0 + m_1 p_{01}$ (13)

$D_1 = 1 - p_{01}$ (14)

5. AVAILABILITY ANALYSIS

Using the probabilistic arguments, we have the following recursive relations

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{02}(t)©A_2(t) + q_{03}(t)©A_3(t) + q_{04}(t)©A_4(t)$

$A_1(t) = q_{10}(t)©A_0(t)$

$A_2(t) = q_{20}(t)©A_0(t) + q_{25}(t)©A_5(t)$

$A_3(t) = q_{30}(t)©A_0(t) + q_{35}(t)©A_5(t)$

$A_4(t) = q_{40}(t)©A_0(t) + q_{45}(t)©A_5(t)$

$A_5(t) = q_{50}(t)©A_0(t)$ (15)

Where

$M_0(t) = e^{-(\alpha+\beta)t}$ (16) Taking Laplace-Transform of above equation (15) and solving for $A^*_0(s)$ and using this, we get steady-state availability of system as:

$$A_0 = \lim_{s\to 0} s A^*_0(s) = \frac{N_2}{D_2} \qquad (17)$$

Where

$N_2 = \mu_0$ (18)

$D_2 = \mu_0 + m_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} + \mu_4 p_{04}$ (19)

6. BUSY PERIOD ANALYSIS

6.1 Busy Period For Regular Repair Facility

Using the probabilistic argument, we have the following recursive relation for

$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t) + q_{03}(t)©B_3(t) + q_{04}(t)©B_4(t)$

$B_1(t) = W_1(t) + q_{10}(t)©B_0(t)$

$B_2(t) = W_2(t) + q_{20}(t)©B_0(t) + q_{25}(t)©B_5(t)$

$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{35}(t)©B_5(t)$

$B_4(t) = W_4(t) + q_{40}(t)©B_0(t) + q_{45}(t)©B_5(t)$

$B_5(t) = q_{50}(t)©B_0(t)$ (20)

Where

$W_1 = \bar{G}(t)$, $W_2 = \bar{H}_1(t)$, $W_3 = \bar{H}_2(t)$, $W_4 = \bar{H}_3(t)$ (21)

Taking Laplace-Transform of above equation (20) and solving for $B^*_0(s)$) and using this, we get steady-state, the function of time for which the regular repair facility is busy in repair is given by

$$B_0 = \lim_{s\to 0} s B^*_0(s) = \frac{N_3}{D_2} \qquad (22)$$

We have

$N_3 = p_{01} w_1 + p_{02} w_2 + p_{03} w_3 + p_{04} w_4$ (23)

6.2 Busy period for Expert Repair Facility

Using the probabilistic argument, we have the following recursive relation for $E_i(t)$

$E_0(t) = q_{01}(t)©E_1(t) + q_{02}(t)©E_2(t) + q_{03}(t)©E_3(t) + q_{04}(t)©E_4(t)$

$E_1(t) = q_{10}(t)©E_0(t)$

$E_2(t) = q_{20}(t)©E_0(t) + q_{25}(t)©E_5(t)$

$E_3(t) = q_{30}(t)©E_0(t) + q_{35}(t)©E_5(t)$

$E_4(t) = q_{40}(t)©E_0(t) + q_{45}(t)©E_5(t)$

$E_5(t) = W_5(t) + q_{50}(t)©E_0(t)$ (24)

Taking Laplace-Transform in above equations (24) and solving for $E^*_0(s)$ and using this, we get steady-state, the function of time for which the regular repair facility is busy in repair is given by

$$E_0 = \lim_{s\to 0} s E^*_0(s) = \frac{N_4}{D_2} \qquad (25)$$

Where

$N_4 = (p_{02} p_{25} + p_{03} p_{35} + p_{04} p_{45}) w_5$ (26)

Where

$$w_5 = \int_0^{\infty} t\, k(t)\, dt \qquad (27)$$

7. EXPECTED DOWN TIME OF THE SYSTEM

Let $DT_{i(t)}$ be the probability that the system is down under PM by regular repair facility at time t. Thus following recursive relation can be obtained as:

$DT_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t) + q_{03}(t)©B_3(t) + q_{04}(t)©B_4(t)$

$DT_1(t) = W_1(t) + q_{10}(t)©DT_0(t)$

$DT_2(t) = q_{20}(t) © DT_0(t) + q_{25}(t) © DT_5(t)$
$DT_3(t) = q_{30}(t) © DT_0(t) + q_{35}(t) © DT_5(t)$
$DT_4(t) = q_{40}(t) © DT_0(t) + q_{45}(t) © DT_5$
$DT_5(t) = q_{50}(t) © DT_0(t)$ (28)

Taking Laplace-Transform in above equations (28) and solving for $DT^*_0(s)$ and using this, we get steady-state, the function of time for which the system is down is given by

$DT_0 = \lim_{s \to 0} s\, DT^*_0(s) = \frac{N_5}{D_2}$ (29)

Where

$N_5 = w_1 p_{01}$ with $w_1 = e^{-\beta t}$ (30)

8. RESULTS AND DISCUSSION

The expected profit per unit time incurred to the system is given by:

$P = C_0(A_0) - C_1(B_0) - C_2(E_0) - C_3(DT_0) - C_4,$ (31)

Where

C_0 - Revenue per unit up time when system works in full capacity

C_1- Cost per unit time for which regular repairman is busy for repairing

C_2- Cost per unit time for which expert repairman is busy for repairing

C_3- Cost per unit up time when system is down

C_4- Payment per unit time made to repairman's

Particular case

Consider $\bar{G}(t) = \gamma e^{-\gamma t}$ $\bar{H}_1(t) = e^{-r_1 t}$ $\bar{H}_1(t) = r_1 e^{-r_1 t}$
$\bar{H}_3(t) = e^{-r_3 t}$, $f(t) = \lambda e^{-\lambda t}$
$\bar{F}(t) = e^{-\lambda t}, \bar{K}(t) = e^{-\delta t}$ (32)

Using the values estimated from the data collected i.e. $p = .2, q = .7, r = .1, \alpha = (.2, .4, .6, .8, 1.0), \beta = .002, \gamma = .2, \delta = 0.3, r_1 = .25, r_2 = ..57, r_1 = 15, \lambda = .1, C_0 = 5000, C_1 = 1500, C_2 = 1000, C_3 = 700, C_4 = 500$ (33)

The mean time to system failure (MTSF) of the system, which decreases with the increase of failure rates of the unit shown in Fig.2. The behaviour of availability shown in Fig.3 The behaviour of the profit analysis shown in fig.4, the figure indicates that profit of the system goes on decreasing the increase of failure rates. Hence on the basis of the results obtained for a particular case, it is concluded that the system can be made more reliable and profitable when increasing the repair rates, increasing the preventive maintenance rate and revenue per unit up time increases.

Fig.2 Graph between MTSF and Failure rate

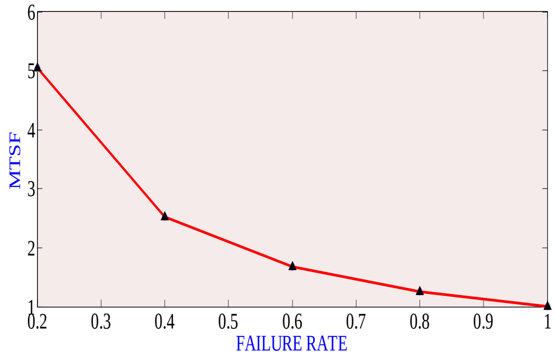


Fig.3 Graph between Availability and failure rate

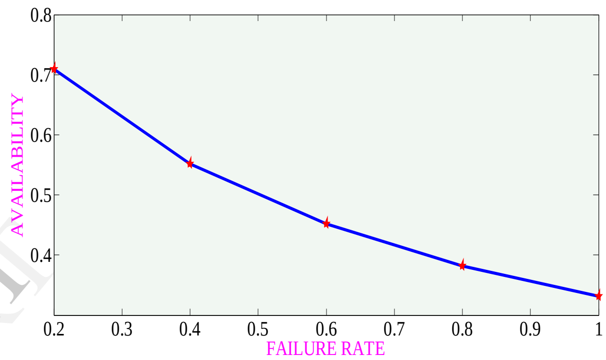
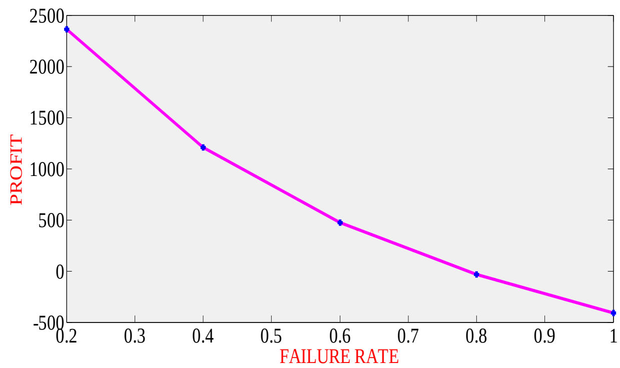


Fig.4 Graph between Profit and failure rate



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