Stochastic Models on Time to Recruitment in a Two Grade Manpower System using Univariate Recruitment Policy

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***Abstract*— In this paper for a two grade manpower system involving optional and mandatory exponential thresholds, the analytical results for some performance measures related to time to recruitment are obtained using a univariate policy of recruitment by considering different forms of inter-decision times and loss of manpower in the system.**

Keywords*— *Two grade manpower system; loss of manpower; Univariate policy of recruitment, geometric process; Correlated random variables and Mean time to recruitment.

I. INTRODUCTION

In any organization, depletion of manpower is quite common whenever policy decisions are announced .Frequent recruitment to compensate this depletion is costlier and hence suitable policy decision on recruitment has to be designed. In this context, several authors have obtained performance measures namely mean and variance of time to recruitment using different policies of recruitment based on shock mode approach. Employing the univariate recruitment policy, the expected time to recruitment is obtained under different conditions for several models in [1], [2] and [3].In [5],for a single grade man-power system with a mandatory exponential threshold for the loss of manpower ,the authors have obtained the system performance measures when the inter-decision times form an order statistics .In [2] ,for a single grade manpower system, the author has considered a new recruitment policy involving two thresholds for the loss of man-power in the organization in which one is optional and the other is mandatory and obtained the mean time to recruitment under different conditions on the nature of thresholds ,inter -decision times and loss of man-hours . In [6-9] the authors have extended the results in [2] for a two-grade system according as the thresholds are exponential random variables or extended exponential random variables or SCBZ property possessing random variables or geometric random variables. In [10],the authors have obtained performance measures by assuming that the inter-decision times for the two grades form same geometric process .In [11-19], the authors have extended the results in [5] for a two-grade system involving two thresholds by assuming different distributions for thresholds under different condition of inter-decision time and wastage. The objective of the present paper is to obtain performance measures for a two grade manpower system with exponential thresholds by considering different forms of loss of manpower and inter decision times.

II. MODEL DESCRIPTION AND ANALYSIS OF MODEL –I

Consider an organization taking decisions at random epoch in $(0, \infty)$ and at every decision epoch a random number of persons quit the organization. There is an associated loss of man-hours if a person quits. It is assumed that the loss of man-hours are linear and cumulative. Let Y_1, Y_2 (Z_1, Z_2) denotes the optional (mandatory) thresholds for the loss of man-hours in grades 1 and 2, with parameters $\theta_1,\theta_2,\alpha_1,\alpha_2$ respectively, where $\theta_1,\theta_2,\alpha_1,\alpha_2$ are positive. It is assumed that $Y_1 < Z_1$and $Y_2 < Z_2$. Write $Y = \mathrm{Max}(Y_1, Y_2)$ and $Z=\mathrm{Max}(Z_1, Z_2)$, where Y (Z) is the optional (mandatory) threshold for the loss of man-hours in the organization. The loss of man- hours, optional and mandatory thresholds are assumed as statistically independent. Let T be the time to recruitment in the organization with cumulative distribution function L (.), probability density function l (.), mean E (T) and variance V (T). Let $F_k(.)$ be the k fold convolution of F(.). Let $l^*(.)$ and $f^*(.)$, be the Laplace transform of l (.) and f (.), respectively. Let $V_k(t)$ be the probability that there are exactly k decision epochs in (0, t]. It is known from Renewal theory that $V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$.Let p be the probability that the organization is not going for recruitment whenever the total loss of man-hours crosses optional threshold Y. The Univariate recruitment policy employed in this paper is as follows: If the total loss of man-hours exceeds the optional threshold Y, the organization may or may not go for recruitment. But if the total loss of man-hours exceeds the mandatory threshold Z, the recruitment is necessary.

III. MAIN RESULTS

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i \le Y\right) + p\sum_{k=0}^{\infty} V_k(t) P\left(\sum_{i=1}^{k} X_i > Y\right) \times P\left(\sum_{i=1}^{k} X_i < Z\right) \qquad (1)$$

We now obtain some performance measures related to time to recruitment for different forms of wastage and inter-decision times.

Case (i): Let X_i be the loss of man hours due to the i^{th} decision epoch , i=1,2,3… forming a sequence of independent and identically distributed exponential random variables with mean $\frac{1}{c}$ (c>0), probability density function g(.),U_i are exchangeable and constantly correlated exponential random variables denoting inter-decision time between $(i-1)^{th}$ and i^{th} decision, i=1,2, 3….k. with cumulative distribution function F(.), probability density function f(.) and mean u .

Using law of total probability

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = \int_0^{\infty} (1 - P(Y \le x)) g_k(x) dx = \int_0^{\infty} (1 - H(x)) g_k(x) dx \qquad (2)$$

Since the distribution of thresholds follow exponential distribution it can be shown that

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = \int_0^{\infty} \left(e^{-\theta_1 x} + e^{-\theta_2 x} - e^{-(\theta_1+\theta_2)x}\right) g_k(x) dx \qquad (3)$$

Similarly,

$$P\left(\sum_{i=1}^{k} X_i < Z\right) = \int_0^{\infty} \left(e^{-\alpha_1 x} + e^{-\alpha_2 x} - e^{-(\alpha_1+\alpha_2)x}\right) g_k(x) dx \qquad (4)$$

Using (3) and (4) in (1) and on further simplification we get,

$$\begin{aligned} P(T>t) = & \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left(\left(g^*(\theta_1)\right)^k + \left(g^*(\theta_2)\right)^k - \left(g^*(\theta_1+\theta_2)\right)^k\right) \\ & + p\sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left(1 - \left(\left(g^*(\theta_1)\right)^k + \left(g^*(\theta_2)\right)^k - \left(g^*(\theta_1+\theta_2)\right)^k\right)\right)\left(\left(g^*(\alpha_1)\right)^k + \left(g^*(\alpha_2)\right)^k - \left(g^*(\alpha_1+\alpha_2)\right)^k\right) \end{aligned} \qquad (5)$$

Since $L(t) = 1 - P(T > t)$ from (5)

$$\begin{aligned} L(t) = & \left(1-g^*(\theta_1)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_1)\right)^{k-1} + \left(1-g^*(\theta_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_2)\right)^{k-1} \\ & - \left(1-g^*(\theta_1+\theta_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_1+\theta_2)\right)^{k-1} + p\Bigg(\left(1-g^*(\alpha_1)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\alpha_1)\right)^{k-1} \\ & - \left(1-g^*(\theta_1)g^*(\alpha_1)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_1)g^*(\alpha_1)\right)^{k-1} - \left(1-g^*(\theta_2)g^*(\alpha_1)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_2)g^*(\alpha_1)\right)^{k-1} \\ & + \left(1-g^*(\theta_1+\theta_2)g^*(\alpha_1)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_1+\theta_2)g^*(\alpha_1)\right)^{k-1} + \left(1-g^*(\alpha_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\alpha_2)\right)^{k-1} \\ & - \left(1-g^*(\theta_1)g^*(\alpha_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_1)g^*(\alpha_2)\right)^{k-1} - \left(1-g^*(\theta_2)g^*(\alpha_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_2)g^*(\alpha_2)\right)^{k-1} + \\ & + \left(1-g^*(\theta_1+\theta_2)g^*(\alpha_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\theta_1+\theta_2)g^*(\alpha_2)\right)^{k-1} - \left(1-g^*(\alpha_1+\alpha_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\alpha_1+\alpha_2)\right)^{k-1} \\ & + \left(1-g^*(\alpha_1+\alpha_2)g^*(\theta_1)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\alpha_1+\alpha_2)g^*(\theta_1)\right)^{k-1} + \left(1-g^*(\alpha_1+\alpha_2)g^*(\theta_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\alpha_1+\alpha_2)g^*(\theta_2)\right)^{k-1} \\ & - \left(1-g^*(\alpha_1+\alpha_2)g^*(\theta_1+\theta_2)\right)\sum_{k=1}^{\infty} F_k(t) \left(g^*(\alpha_1+\alpha_2)g^*(\theta_1+\theta_2)\right)^{k-1}\Bigg) \end{aligned} \qquad (6)$$

By assumption $\{U_i\}$ is a sequence of exchangeable and constantly correlated random variable each following the exponential distribution .As in [4] , we get the cumulative distribution function of $\sum_{i=1}^{k} U_i$ as

$$F_k(t) = \frac{1-\rho}{1-\rho+k\rho} \sum_{i=0}^{\infty} \left(\frac{k\rho}{1-\rho+k\rho} \right)^i \left[1 - \sum_{j=0}^{k+j-1} \frac{e^{-y/b} (y/b)^{k+i-j-1}}{(k+i-j-1)!} \right] \qquad (7)$$

Taking Laplace-Stieltjes transform on both sides, we have

$$F_k^*(s) = \frac{(1-\rho)(1+bs)^{1-k}}{(1-\rho)(1+bs)+k\rho bs} \qquad (8)$$

$$\left[\frac{d}{ds} (F_k^*(s)) \right]_{s=0} = -uk \text{ and}$$

$$\left[\frac{d^2}{ds^2} F_k^*(s) \right]_{s=0} = u^2 \{(1+\rho^2)k^2 + (1-\rho^2)k\} \text{ , where } u = \frac{b}{1-\rho} \qquad (9)$$

It is known that

$$E(T) = -\left. \frac{d(l^*(s))}{ds} \right|_{s=0} , E(T^2) = \left. \frac{d^2(l^*(s))}{ds^2} \right|_{s=0} \text{ and } V(T) = E(T^2) - (E(T))^2 \qquad (10)$$

Using (6) and (9) in (10) we get the first two moments

$$E(T) = u \left(\frac{1}{1-g^*(\theta_1)} + \frac{1}{1-g^*(\theta_2)} - \frac{1}{1-g^*(\theta_1+\theta_2)} + p \left(\frac{1}{1-g^*(\alpha_1)} + \frac{1}{1-g^*(\alpha_2)} - \frac{1}{1-g^*(\alpha_1+\alpha_2)} - \frac{1}{1-g^*(\theta_1)g^*(\alpha_1)} - \frac{1}{1-g^*(\theta_2)g^*(\alpha_1)} + \frac{1}{1-g^*(\theta_1+\theta_2)g^*(\alpha_1)} - \frac{1}{1-g^*(\theta_1)g^*(\alpha_2)} - \frac{1}{1-g^*(\theta_2)g^*(\alpha_2)} + \frac{1}{1-g^*(\theta_1+\theta_2)g^*(\alpha_1)} + \frac{1}{1-g^*(\theta_1)g^*(\alpha_1+\alpha_2)} + \frac{1}{1-g^*(\theta_2)g^*(\alpha_1+\alpha_2)} - \frac{1}{1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)} \right) \right) \qquad (11)$$

$$E(T^2) = 2u^2 \left(\frac{1+\rho^2 g^*(\theta_1)}{(1-g^*(\theta_1))^2} + \frac{1+\rho^2 g^*(\theta_2)}{(1-g^*(\theta_2))^2} - \frac{1+\rho^2 g^*(\theta_1+\theta_2)}{(1-g^*(\theta_1+\theta_2))^2} + p \left(\frac{1+\rho^2 g^*(\alpha_1)}{(1-g^*(\alpha_1))^2} + \frac{1+\rho^2 g^*(\alpha_2)}{(1-g^*(\alpha_2))^2} - \frac{1+\rho^2 g^*(\alpha_1+\alpha_2)}{(1-g^*(\alpha_1+\alpha_2))^2} - \frac{1+\rho^2 g^*(\theta_1)g^*(\alpha_1)}{(1-g^*(\theta_1)g^*(\alpha_1))^2} - \frac{1+\rho^2 g^*(\theta_2)g^*(\alpha_1)}{(1-g^*(\theta_2)g^*(\alpha_1))^2} + \frac{1+\rho^2 g^*(\theta_1+\theta_2)g^*(\alpha_1)}{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1))^2} - \frac{1+\rho^2 g^*(\theta_1)g^*(\alpha_2)}{(1-g^*(\theta_1)g^*(\alpha_2))^2} - \frac{1+\rho^2 g^*(\theta_2)g^*(\alpha_2)}{(1-g^*(\theta_2)g^*(\alpha_2))^2} + \frac{1+\rho^2 g^*(\theta_1+\theta_2)g^*(\alpha_1)}{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1))^2} + \frac{1+\rho^2 g^*(\theta_1)g^*(\alpha_1+\alpha_2)}{(1-g^*(\theta_1)g^*(\alpha_1+\alpha_2))^2} + \frac{1+\rho^2 g^*(\theta_2)g^*(\alpha_1+\alpha_2)}{(1-g^*(\theta_2)g^*(\alpha_1+\alpha_2))^2} - \frac{1+\rho^2 g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))^2} \right) \right) \qquad (12)$$

where $g^*(\tau) = \frac{c}{c+\tau}$ for $\tau = \theta_1, \theta_2, \theta_1 + \theta_2, \alpha_1, \alpha_2, \alpha_1 + \alpha_2$.

(11) gives the expected time to recruitment, (11)and (12) together with (10) gives the variance of time to recruitment.

Case (ii):

Let X_i be the loss of man-hours due to the i^{th} decision epoch , i=1,2,3…k Let $X_{(1)}, X_{(2)}, X_{(3)}, \ldots X_{(n)}$ be the order statistics selected from the sample $X_1, X_2, \ldots X_n$ with respective density functions $g_{x(1)}(.), g_{x(2)}(.), \ldots g_{x(n)}(.)$

If U_i be exchangeable and constantly correlated exponential random variables denoting inter-decision times between (i-1) and i th decision epoch ,i=1,2,3….

Suppose $g(t) = g_{x(1)}(t)$.

The first two moments are given by (11) and (12) where

$$g^*_{x(1)}(\tau) = \frac{kc}{kc+\tau}. \qquad (13)$$

Suppose $g(t) = g_{x(k)}(t)$. The first two moments are given by (11) and (12) where

$$g^*_{x(k)}(\tau) = \frac{k!c^k}{(\tau+c)(\tau+2c)\ldots(\tau+kc)} \qquad (14)$$

(14)

If U_i form a geometric process with parameter 'a' , a>0 then we find that

$$f_k^*(s) = \prod_{r=1}^{k} f^*\left(\frac{s}{a^{r-1}}\right), k = 1,2,3\ldots. \qquad (15)$$

Suppose $g(t) = g_{x(1)}(t)$. Since $l^*(s) = \frac{d}{ds}(l(t))$ and using (6),(15) in (10) we get the first two moments of time to recruitment

ISSN: 2278-0181

$$E(T) = aE(U_1)\left[C_1 + C_2 - C_3 + p\left(C_4 + C_5 - C_6 - H_{1,4} - H_{1,5} + H_{1,6} - H_{2,4} - H_{2,5} + H_{2,6} + H_{3,4} + H_{3,5} - H_{3,6}\right)\right] \quad (16)$$

$$\begin{aligned}
E(T^2) = {} & \frac{\sigma_{U1}^2 a^2}{(a^2-1)}\left[1 - \frac{(1-g^*(\theta_1))}{a^2-g^*(\theta_1)} - \frac{(1-g^*(\theta_2))}{a^2-g^*(\theta_2)} + \frac{(1-g^*(\theta_1+\theta_2))}{a^2-g^*(\theta_1+\theta_2)}\right] + \frac{E(U_1)a^2}{(a-1)^2}\left[1 - \frac{2(1-g^*(\theta_1))}{a-g^*(\theta_1)} + \frac{1-g^*(\theta_1)}{a^2-g^*(\theta_1)} - \frac{2(1-g^*(\theta_2))}{a-g^*(\theta_2)}\right. \\
& \left. + \frac{1-g^*(\theta_2)}{a^2-g^*(\theta_2)} + \frac{2(1-g^*(\theta_1+\theta_2))}{a-g^*(\theta_1+\theta_2)} - \frac{1-g^*(\theta_1+\theta_2)}{a^2-g^*(\theta_1+\theta_2)}\right] + p\frac{\sigma_{U1}^2 a^2}{(a^2-1)}\left[-\frac{(1-g^*(\alpha_1))}{a^2-g^*(\alpha_1)} - \frac{(1-g^*(\alpha_2))}{a^2-g^*(\alpha_2)} + \frac{(1-g^*(\alpha_1+\alpha_2))}{a^2-g^*(\alpha_1+\alpha_2)}\right. \\
& + \frac{(1-g^*(\theta_1)g^*(\alpha_1))}{a^2-g^*(\theta_1)g^*(\alpha_1)} + \frac{(1-g^*(\theta_2)g^*(\alpha_1))}{a^2-g^*(\theta_2)g^*(\alpha_1)} - \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1))}{a^2-g^*(\theta_1+\theta_2)g^*(\alpha_1)} + \frac{(1-g^*(\theta_1)g^*(\alpha_2))}{a^2-g^*(\theta_1)g^*(\alpha_2)} + \frac{(1-g^*(\theta_2)g^*(\alpha_2))}{a^2-g^*(\theta_2)g^*(\alpha_2)} \\
& \left. - \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_2))}{a^2-g^*(\theta_1+\theta_2)g^*(\alpha_2)} - \frac{(1-g^*(\theta_1)g^*(\alpha_1+\alpha_2))}{a^2-g^*(\theta_1)g^*(\alpha_1+\alpha_2)} - \frac{(1-g^*(\theta_2)g^*(\alpha_1+\alpha_2))}{a^2-g^*(\theta_2)g^*(\alpha_1+\alpha_2)} + \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))}{a^2-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}\right] \\
& + \frac{pE(U_1)a^2}{(a-1)^2}\left[-\frac{2(1-g^*(\alpha_1))}{a-g^*(\alpha_1)} + \frac{1-g^*(\alpha_1)}{a^2-g^*(\alpha_1)} - \frac{2(1-g^*(\alpha_2))}{a-g^*(\alpha_2)} + \frac{1-g^*(\alpha_2)}{a^2-g^*(\alpha_2)} + \frac{2(1-g^*(\alpha_1+\alpha_2))}{a-g^*(\alpha_1+\alpha_2)} - \frac{1-g^*(\alpha_1+\alpha_2)}{a^2-g^*(\alpha_1+\alpha_2)}\right. \\
& + \frac{2(1-g^*(\theta_1)g^*(\alpha_1))}{a-g^*(\theta_1)g^*(\alpha_1)} - \frac{(1-g^*(\theta_1)g^*(\alpha_1))}{a^2-g^*(\theta_1)g^*(\alpha_1)} + \frac{2(1-g^*(\theta_2)g^*(\alpha_1))}{a-g^*(\theta_2)g^*(\alpha_1)} - \frac{(1-g^*(\theta_2)g^*(\alpha_1))}{a^2-g^*(\theta_2)g^*(\alpha_1)} - \frac{2(1-g^*(\theta_1+\theta_2)g^*(\alpha_1))}{a-g^*(\theta_1+\theta_2)g^*(\alpha_1)} \\
& + \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1))}{a^2-g^*(\theta_1+\theta_2)g^*(\alpha_1)} + \frac{2(1-g^*(\theta_2)g^*(\alpha_2))}{a-g^*(\theta_2)g^*(\alpha_2)} - \frac{(1-g^*(\theta_2)g^*(\alpha_2))}{a^2-g^*(\theta_2)g^*(\alpha_2)} - \frac{2(1-g^*(\theta_1+\theta_2)g^*(\alpha_2))}{a-g^*(\theta_1+\theta_2)g^*(\alpha_2)} \\
& + \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_2))}{a^2-g^*(\theta_1+\theta_2)g^*(\alpha_2)} - \frac{2(1-g^*(\theta_1)g^*(\alpha_1+\alpha_2))}{a-g^*(\theta_1)g^*(\alpha_1+\alpha_2)} + \frac{(1-g^*(\theta_1)g^*(\alpha_1+\alpha_2))}{a^2-g^*(\theta_1)g^*(\alpha_1+\alpha_2)} - \frac{2(1-g^*(\theta_2)g^*(\alpha_1+\alpha_2))}{a-g^*(\theta_2)g^*(\alpha_1+\alpha_2)} \\
& \left.\left. + \frac{(1-g^*(\theta_2)g^*(\alpha_1+\alpha_2))}{a^2-g^*(\theta_2)g^*(\alpha_1+\alpha_2)} + \frac{2(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))}{a-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)} - \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))}{a^2-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}\right]\right]
\end{aligned} \quad (17)$$

In (16) and (17) $E(U_1) = -f^{*'}(0), \sigma_{U1}^2 = f^{*''}(0) - (f^{*'}(0))^2$,

$$C_c = \frac{1}{a - D_c} \text{ and } H_{b,d} = \frac{1}{a - D_a}, \text{ for c=1,2,3…6 , b=1,2,3 and d=4,5,6.} \quad (18)$$

$D_1 = g^*(\theta_1), D_2 = g^*(\theta_2), D_3 = g^*(\theta_1 + \theta_1), D_4 = g^*(\alpha_1), D_5 = g^*(\alpha_2), D_6 = g^*(\alpha_1 + \alpha_2)$ .are given by (13)

(16) gives the expected time to recruitment, (16) and (17) together with (10) gives the variance of time to recruitment.

Suppose $g(t) = g_{x(k)}(t)$. The first two moments are given by (16), (17) and (14).

Case (iii):

Let X_i be a exponential random variable denoting the loss of man-hours due to i[th] decision epoch, i=1,2,3….with cumulative distribution function G (.) and probability density function g(.)

If U_i are exchangeable and constantly correlated exponential random variables

Considering the first term of (1) and conditioning upon y we get

$$P\left(\sum_{i=1}^{k} X_i \le Y\right) = \int_0^\infty P\left(\sum_{i=1}^{k} X_i \le Y\right) h(y)\,dy = \int_0^\infty G_k(y)\,h(y)\,dy$$

Since X_i's are assumed to be identical constantly correlated and exchangeable exponential random variables with parameter α , c.d.f of the partial sum $S_k = X_1 + X_2 + \ldots\ldots + X_k$ is given by Gurland(1955) as

$$G_k(y) = (1-\rho)\sum_{i=0}^{\infty} \frac{(k\rho)^i \ \phi\left(k+i, \frac{y}{b}\right)}{(1-\rho+k\rho)^{i+1}(k+i-1)!} \qquad (19)$$

where ρ is the constant correlation between X_i and X_j , $i \neq j$.

$$\phi\left(k+i, \frac{y}{b}\right) = \int_0^{\frac{y}{b}} e^{-z} z^{k+i-1} dz \ \text{ and } b = \alpha(1-\rho)$$

Since the thresholds follow exponential distribution

$$h(y) = \theta_1 e^{-\theta_1 y} + \theta_2 e^{-\theta_2 y} - (\theta_1 + \theta_2) e^{-(\theta_1+\theta_2) y} \qquad (20)$$

Using (19),(20) and on further simplification we get

$$\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] P\left(\sum_{i=1}^{k} X_i \le Y\right) = (1-\rho)\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \left[W_{1k} + W_{2k} - W_{3k}\right] \qquad (21)$$

Similarly,

$$\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] P\left(\sum_{i=1}^{k} X_i > Y\right) = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \left[1 - (1-\rho)\left[W_{1k} + W_{2k} - W_{3k}\right]\right] \qquad (22)$$

$$P\left(\sum_{i=1}^{k} X_i < Z\right) = (1-\rho)\left[W_{4k} + W_{5k} - W_{6k}\right] \qquad (23)$$

(23)

$$W_{1k} = \frac{1}{(b\theta_1+1)^{k-1}\left[(1-\rho+k\rho)(b\theta_1+1) - k\rho\right]}, \quad W_{2k} = \frac{1}{(b\theta_2+1)^{k-1}\left[(1-\rho+k\rho)(b\theta_2+1) - k\rho\right]}$$

$$W_{3k} = \frac{1}{(b(\theta_1+\theta_2)+1)^{k-1}\left[(1-\rho+k\rho)(b(\theta_1+\theta_2)+1) - k\rho\right]}, \quad W_{4k} = \frac{1}{(b\alpha_1+1)^{k-1}\left[(1-\rho+k\rho)(b\alpha_1+1) - k\rho\right]}$$

$$W_{5k} = \frac{1}{(b\alpha_2+1)^{k-1}\left[(1-\rho+k\rho)(b\alpha_2+1) - k\rho\right]}, \quad W_{6k} = \frac{1}{(b(\alpha_1+\alpha_2)+1)^{k-1}\left[(1-\rho+k\rho)(b(\alpha_1+\alpha_2)+1) - k\rho\right]}$$

(24)

Using (21) ,(22) and (23) in (1) we get

$$P(T>t) = (1-\rho)\left[\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right]\left[W_{1k} + W_{2k} - W_{3k}\right] + p\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right] \left[W_{4k} + W_{5k} - W_{6k}\right] - (1-\rho)\sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t)\right]\left[W_{1k} + W_{2k} - W_{3k}\right]\left[W_{4k} + W_{5k} - W_{6k}\right]\right] \qquad (25)$$

Since $L(t) = 1 - P(T>t)$ from (25).

$$L(t)=1-(1-\rho)\left[\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left[W_{1k}+W_{2k}-W_{3k}\right]-p\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\right.$$
$$\left.\left[W_{4k}+W_{5k}-W_{6k}\right]+p(1-\rho)\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left[W_{1k}+W_{2k}-W_{3k}\right]\left[W_{4k}+W_{5k}-W_{6k}\right]\right] \quad (26)$$

Proceeding in the same way we get the first two moments

$$E(T)=(1-\rho)u\sum_{k=0}^{\infty}(W_{1k}+W_{2k}-W_{3k})+p(W_{4k}+W_{5k}-W_{6k})$$
$$-p(1-\rho)\,(W_{1k}+W_{2k}-W_{3k})(W_{4k}+W_{5k}-W_{6k}) \quad (27)$$

$$E(T^2)=\frac{2b^2}{1-\rho}\sum_{k=0}^{\infty}\left(k(1+\rho^2)+1\right)\left((W_{1k}+W_{2k}-W_{3k})+p(W_{4k}+W_{5k}-W_{6k})\right.$$
$$\left.-p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(W_{4k}+W_{5k}-W_{6k})\right) \quad (28)$$

(27) gives the expected time to recruitment,(27) and (28) together with (10) gives the variance of time to recruitment.

If U_i form a geometric process with parameter 'a', then the first two moments are given by

$$E(T)=E(U_1)(1-\rho)\sum_{k=0}^{\infty}\left(\frac{1}{a^k}\right)\left[(W_{1k}+W_{2k}-W_{3k})+p(W_{4k}+W_{5k}-W_{6k})\right.$$
$$\left.-p(1-\rho)\,(W_{1k}+W_{2k}-W_{3k})(W_{4k}+W_{5k}-W_{6k})\right] \quad (29)$$

$$E(T^2)=(1-\rho)\sum_{k=0}^{\infty}\left(V(U_1)\left(\frac{1}{a^{2k}}\right)+(E(U_1))^2\left(\sum_{j=1}^{k}\left(\frac{1}{a^{i-1}}\right)^2-\left(\sum_{j=1}^{k}\frac{1}{a^{j-1}}\right)^2\right)\right)\left((W_{1k}+W_{2k}-W_{3k})\right.$$
$$\left.+p(W_{4k}+W_{5k}-W_{6k})-p(1-\rho)(W_{1k}+W_{2k}-W_{3k})(W_{4k}+W_{5k}-W_{6k})\right) \quad (30)$$

(29) gives the expected time to recruitment, (29) and (30) together with (10) gives the variance of time to recruitment.

IV. MODEL DESCRIPTION AND ANALYSIS FOR MODEL-II

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y=\min(Y_A,Y_B)$ and $Z=\min(Z_A,Z_B)$. All the other assumptions and notations are as in model-I.

Case (i): Let X_i be the loss of man hours due to the i^{th} decision epoch , i=1,2,3… forming a sequence of independent and identically distributed exponential random variables with mean $\frac{1}{c}$ (c>0), probability density function g(.),U_i are exchangeable and constantly correlated exponential random variables denoting inter-decision time between $(i-1)^{th}$ and i^{th} decision, i=1,2, 3….k with cumulative distribution function F(.), probability density function f(.) and mean u .

Proceeding as in previous model we get the first two moments

$$E(T) = u\left(\frac{1}{1-g^*(\theta_1+\theta_2)} + p\left(\frac{1}{1-g^*(\alpha_1+\alpha_2)} - \frac{1}{1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}\right)\right) \quad (31)$$

$$E(T^2) = 2u^2\left(\frac{1+\rho^2 g^*(\theta_1+\theta_2)}{(1-g^*(\theta_1+\theta_2))^2} + p\left(\frac{1+\rho^2 g^*(\alpha_1+\alpha_2)}{(1-g^*(\alpha_1+\alpha_2))^2} - \frac{1+\rho^2 g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))^2}\right)\right) \quad (32)$$

(31) gives the expected time to recruitment, (31) and (32) together with (10) gives the variance of time to recruitment.

Case (ii):

Let x_i be the loss of man-hours due to the i[th] decision epoch , i=1,2,3…k Let $x_{(1)}, x_{(2)}, x_{(3)}, \ldots, x_{(n)}$ be the order statistics selected from the sample $X_1, X_2, \ldots X_n$ with respective density functions $g_{x(1)}(.), g_{x(2)}(.), \ldots g_{x(n)}(.)$,

If U_i are exchangeable and constantly correlated exponential random variables.

Suppose $g(t) = g_{x(1)}(t)$

The first two moments are given by (31),(32) and (13).

Suppose $g(t) = g_{x(k)}(t)$

The first two moments are given by (31) ,(32) and (14)

If U_i form a geometric process with parameter 'a'

Suppose $g(t) = g_{x(1)}(t)$

$$E(T) = aE(U_1)\left[C_3 + p(C_6 - H_{3,6})\right] \quad (33)$$

$$E(T^2) = \frac{\sigma_{U1}^2 a^2}{(a^2-1)}\left[1 - \frac{(1-g^*(\theta_1+\theta_2))}{a^2 - g^*(\theta_1+\theta_2)}\right] + \frac{E(U_1)a^2}{(a-1)^2}\left[\left[1 - \frac{2(1-g^*(\theta_1+\theta_2))}{a - g^*(\theta_1+\theta_2)} + \frac{1-g^*(\theta_1+\theta_2)}{a^2 - g^*(\theta_1+\theta_2)}\right]\right.$$
$$+ p\frac{\sigma_{U1}^2 a^2}{(a^2-1)}\left[\frac{-(1-g^*(\alpha_1+\alpha_2))}{a^2 - g^*(\alpha_1+\alpha_2)} + \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))}{a^2 - g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}\right]$$
$$\left. + \frac{E(U_1)a^2}{(a-1)^2}\left[-\frac{2(1-g^*(\alpha_1+\alpha_2))}{a - g^*(\alpha_1+\alpha_2)} + \frac{1-g^*(\alpha_1+\alpha_2)}{a^2 - g^*(\alpha_1+\alpha_2)} + \frac{2(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))}{a - g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)} - \frac{(1-g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2))}{a^2 - g^*(\theta_1+\theta_2)g^*(\alpha_1+\alpha_2)}\right]\right] \quad (34)$$

Where $g^*_{x(1)}(\tau)$ $\tau = \theta_1 + \theta_2, \alpha_1 + \alpha_2$ is given by (13)

(33) gives the expected time to recruitment, (33) and (34) together with (10) gives the variance of time to recruitment.

Suppose $g(t) = g_{x(k)}(t)$

The first two moments are given by (33),(34) and (14).

Case (iii):

Let X_i be a exponential random variable denoting the loss of man-hours due to i^{th} decision epoch, i=1,2,3….with cumulative distribution function G (.) and probability density function g(.)

If U_i are exchangeable and constantly correlated exponential random variables.

The first two moments are given by

$$E(T) = (1-\rho)u\sum_{k=0}^{\infty}\left(W_{3k} + pW_{6k} - p(1-\rho)W_{3k}W_{6k}\right) \quad (35)$$

$$E(T^2) = \frac{2b^2}{1-\rho}\sum_{k=0}^{\infty}\left(k(1+\rho^2)+1\right)\left(W_{3k} + pW_{6k} - p(1-\rho)W_{3k}W_{6k}\right) \quad (36)$$

where W_{ak} ,(a=3,6) are given by (24).

If U_i forms geometric process then the first two moments are given by

$$E(T) = E(U_1)(1-\rho)\sum_{k=0}^{\infty}\left(\frac{1}{a^k}\right)\left(W_{3k} + pW_{6k} - p(1-\rho)W_{3k}W_{6k}\right) \quad (37)$$

$$E(T^2) = (1-\rho)\sum_{k=0}^{\infty}\left(V(U_1)\left(\frac{1}{a^{2k}}\right) + (E(U)_1)^2\left(\sum_{j=1}^{k}\left(\frac{1}{a^{i-1}}\right)^2 - \left(\sum_{j=1}^{k}\frac{1}{a^{j-1}}\right)^2\right)\right)\left(W_{3k} + pW_{6k} - p(1-\rho)W_{3k}W_{6k}\right) \quad (38)$$

where W_{ak} *,(a=3,6) are given by (24).*

V. MODEL DESCRIPTION AND ANALYSIS OF MODEL-III

For this model, the optional and mandatory thresholds for the loss of man-hours in the organization are taken as $Y = Y_1 + Y_2$ and $Z = Z_1 + Z_2$ .All the other assumptions and notations are as in model-I.

$$P\left(\sum_{i=1}^{k} X_i < Y\right) = \int_0^{\infty}\left(\frac{\theta_1 e^{-\theta_2 x}}{\theta_1 - \theta_2} - \frac{\theta_2 e^{-\theta_1 x}}{\theta_1 - \theta_2}\right) g_k(x)\,dx$$

$$\text{(i.e) } P\left(\sum_{i=1}^{k} X_i < Y\right) = \frac{\theta_1}{\theta_1 - \theta_2} g^*(\theta_2)^k - \frac{\theta_2}{\theta_1 - \theta_2} g^*(\theta_1)^k$$

Case (i): Let X_i be the loss of man hours due to the i^{th} decision epoch , i=1,2,3… forming a sequence of independent and identically distributed exponential random variables with mean $\frac{1}{c}$ (c>0), probability density function g(.),U_i are exchangeable and constantly correlated exponential random variables denoting inter-decision time between $(i-1)^{th}$ and i^{th} decision, i=1,2, 3….k with cumulative distribution function F(.), probability density function f(.) and mean u .

Proceeding as in model –I we get the first two moments

$$E(T) = u\left(\frac{A_2}{1-g^*(\theta_2)} - \frac{A_1}{1-g^*(\theta_1)} + p\left(\frac{A_5}{1-g^*(\alpha_2)} - \frac{A_4}{1-g^*(\alpha_1)} - \frac{A_1A_4}{1-g^*(\theta_1)g^*(\alpha_1)} + \frac{A_2A_4}{1-g^*(\theta_2)g^*(\alpha_1)} + \frac{A_1A_5}{1-g^*(\theta_1)g^*(\alpha_2)} - \frac{A_2A_5}{1-g^*(\theta_2)g^*(\alpha_2)}\right)\right) \quad (39)$$

$$E(T^2) = 2u^2\left(A_2\frac{1+\rho^2 g^*(\theta_2)}{(1-g^*(\theta_2))^2} - A_1\frac{1+\rho^2 g^*(\theta_1)}{(1-g^*(\theta_1))^2} + p\left(A_5\frac{1+\rho^2 g^*(\alpha_2)}{(1-g^*(\alpha_2))^2} - A_4\frac{1+\rho^2 g^*(\alpha_1)}{(1-g^*(\alpha_1))^2} - \right.\right.$$
$$\left.\left. - A_1A_4\frac{1+\rho^2 g^*(\theta_1)g^*(\alpha_1)}{(1-g^*(\theta_1)g^*(\alpha_1))^2} + A_2A_4\frac{1+\rho^2 g^*(\theta_2)g^*(\alpha_1)}{(1-g^*(\theta_2)g^*(\alpha_1))^2} + A_1A_5\frac{1+\rho^2 g^*(\theta_1)g^*(\alpha_2)}{(1-g^*(\theta_1)g^*(\alpha_2))^2} - A_2A_5\frac{1+\rho^2 g^*(\theta_2)g^*(\alpha_2)}{(1-g^*(\theta_2)g^*(\alpha_2))^2}\right)\right) \quad (40)$$

In (39) and (40) $A_1 = \frac{\theta_2}{\theta_1-\theta_2}, A_2 = \frac{\theta_1}{\theta_1-\theta_2}, A_4 = \frac{\alpha_2}{\alpha_1-\alpha_2}, A_5 = \frac{\alpha_1}{\alpha_1-\alpha_2}$

(39) gives the mean time to recruitment, (39) and (40) together with (10) gives the variance of the time to recruitment .

Case (ii):

Let x_i be the loss of man-hours due to the i^{th} decision epoch , i=1,2,3…k Let $X_{(1)}, X_{(2)}, X_{(3)},....X_{(n)}$ be the order statistics selected from the sample $X_1, X_2,....X_n$ with respective density functions $g_{x(1)}(.), g_{x(2)}(.),....g_{x(n)}(.)$,

If U_i are exchangeable and constantly correlated exponential random variables.

If $g(t) = g_{x(1)}(t)$

The first two moments are given by (39),(40) and (13) .

If $g(t) = g_{x(k)}(t)$

The first two moments are given by (39) ,(40)and (14)

If U_i form a geometric process with parameter ‘a’, then the first two moments are given by

Suppose $g(t) = g_{x(1)}(t)$

$$E(T) = aE(U_1)\left[A_2C_2 - A_1C_1 + p\left(A_5C_5 - A_4C_4 - A_1A_4H_{1,4} + A_2A_4H_{2,4} + A_1A_5H_{1,5} - A_2A_5H_{2,5}\right)\right] \quad (41)$$

$$E(T^2)=\frac{\sigma_{U1}^2 a^2}{(a^2-1)}\left[A_2\left(1-\frac{(1-g^*(\theta_2))}{a^2-g^*(\theta_2)}\right)-A_1\left(1-\frac{(1-g^*(\theta_1))}{a^2-g^*(\theta_1)}\right)\right]+\frac{E(U_1)a^2}{(a-1)^2}\left[A_2\left[1-\frac{2(1-g^*(\theta_2))}{a-g^*(\theta_2)}+\frac{1-g^*(\theta_2)}{a^2-g^*(\theta_2)}\right]\right.$$
$$-A_1\left[1-\frac{2(1-g^*(\theta_1))}{a-g^*(\theta_1)}+\frac{1-g^*(\theta_1)}{a^2-g^*(\theta_1)}\right]+p\frac{\sigma_{U1}^2 a^2}{(a^2-1)}\left[A_5\left(1-\frac{(1-g^*(\alpha_2))}{a^2-g^*(\alpha_2)}\right)-A_4\left(1-\frac{(1-g^*(\alpha_1))}{a^2-g^*(\alpha_1)}\right)-A_1A_4\left(\frac{(1-g^*(\theta_1)g^*(\alpha_1))}{a^2-g^*(\theta_1)g^*(\alpha_1)}\right)\right.$$
$$\left.+A_2A_4\left(\frac{(1-g^*(\theta_2)g^*(\alpha_1))}{a^2-g^*(\theta_2)g^*(\alpha_1)}\right)+A_1A_5\left(\frac{(1-g^*(\theta_1)g^*(\alpha_2))}{a^2-g^*(\theta_1)g^*(\alpha_2)}\right)-A_2A_5\left(\frac{(1-g^*(\theta_2)g^*(\alpha_2))}{a^2-g^*(\theta_2)g^*(\alpha_2)}\right)\right]$$
$$+\frac{pE(U_1)a^2}{(a-1)^2}\left[\left[A_5\left(1-\frac{2(1-g^*(\alpha_2))}{a-g^*(\alpha_2)}+\frac{1-g^*(\alpha_2)}{a^2-g^*(\alpha_2)}\right)-A_4\left(1-\frac{2(1-g^*(\alpha_1))}{a-g^*(\alpha_1)}+\frac{1-g^*(\alpha_1)}{a^2-g^*(\alpha_1)}\right)\right.\right.$$
$$-A_1A_4\left(1-\frac{2(1-g^*(\theta_1)g^*(\alpha_1))}{a-g^*(\theta_1)g^*(\alpha_1)}+\frac{1-g^*(\theta_1)g^*(\alpha_1)}{a^2-g^*(\theta_1)g^*(\alpha_1)}\right)+A_2A_4\left(1-\frac{2(1-g^*(\theta_2)g^*(\alpha_1))}{a-g^*(\theta_2)g^*(\alpha_1)}+\frac{1-g^*(\theta_2)g^*(\alpha_1)}{a^2-g^*(\theta_2)g^*(\alpha_1)}\right)$$
$$\left.\left.+A_1A_5\left(1-\frac{2(1-g^*(\theta_1)g^*(\alpha_2))}{a-g^*(\theta_1)g^*(\alpha_2)}+\frac{1-g^*(\theta_1)g^*(\alpha_2)}{a^2-g^*(\theta_1)g^*(\alpha_2)}\right)-A_2A_5\left(1-\frac{2(1-g^*(\theta_2)g^*(\alpha_2))}{a-g^*(\theta_2)g^*(\alpha_2)}+\frac{1-g^*(\theta_2)g^*(\alpha_2)}{a^2-g^*(\theta_2)g^*(\alpha_2)}\right)\right]\right]$$

(42)

(41) gives the mean time to recruitment , (41) and (42) together with (10) gives the variance of the time to recruitment .

where $g^*_{x(1)}(\tau)$, $\tau=\theta_1,\theta_2\alpha_1,\alpha_2$ is given by (13).

Suppose $g(t)=g_{x(k)}(t)$.

The first two moments are given by (41),(42) and (14)

Case (iii):

Let X_i be a exponential random variable denoting the loss of man-hours due to i[th] decision epoch, i=1,2,3….with cumulative distribution function G (.) and probability density function g(.)

If U_i form a geometric process with parameter 'a'.

The first two moments of time to recruitment is given by

$$E(T)=(1-\rho)u\sum_{k=0}^{\infty}\left(\frac{1}{k}\right)[(W_{33k}-W_{34k})+p(W_{35k}-W_{36k})$$
$$-p(1-\rho)(W_{33k}-W_{34k})(W_{35k}-W_{36k})] \quad (43)$$

$$E(T^2)=\frac{2b^2}{1-\rho}\sum_{k=0}^{\infty}\left(k(1+\rho^2)+1\right)[(W_{33k}-W_{34k})+p(W_{35k}-W_{36k})$$
$$-p(1-\rho)(W_{33k}-W_{34k})(W_{35k}-W_{36k})]$$

(44)

$$W_{33k}=\frac{\theta_1}{(\theta_1-\theta_2)(b\theta_2+1)^{k-1}[(1-\rho+k\rho)(b\theta_2+1)-k\rho]}$$

$$W_{34k}=\frac{\theta_2}{(\theta_1-\theta_2)(b\theta_1+1)^{k-1}[(1-\rho+k\rho)(b\theta_1+1)-k\rho]}$$

$$W_{35k}=\frac{\alpha_1}{(\alpha_1-\alpha_2)(b\alpha_2+1)^{k-1}[(1-\rho+k\rho)(b\alpha_2+1)-k\rho]} \quad (45)$$

$$W_{36k}=\frac{\alpha_2}{(\alpha_1-\alpha_2)(b\alpha_1+1)^{k-1}[(1-\rho+k\rho)(b\alpha_1+1)-k\rho]}$$

(43) gives the expected time to recruitment, (43) and (44) together with (10) gives the variance of time to recruitment.

If U_i form a geometric process with parameter 'a'

Then the first two moments are given by

$$E(T)=(1-\rho)E(U_1)\sum_{k=0}^{\infty}\left(\frac{1}{a^k}\right)\Big[(W_{33k}-W_{34k})+p(W_{35k}-W_{36k}) -p(1-\rho)(W_{33k}-W_{34k})(W_{35k}-W_{36k})\Big] \quad (46)$$

$$E(T^2)=(1-\rho)\sum_{k=0}^{\infty}\left(V(U_1)\left(\frac{1}{a^{2k}}\right)+(E(U_1))^2\left(\sum_{j=1}^{k}\left(\frac{1}{a^{i-1}}\right)^2-\left(\sum_{j=1}^{k}\frac{1}{a^{j-1}}\right)^2\right)\right)\Big[(W_{33k}-W_{34k}) +p(W_{35k}-W_{36k})-p(1-\rho)(W_{33k}-W_{34k})(W_{35k}-W_{36k})\Big] \quad (47)$$

Where W_{ak}, a=33,34,35,36 are given by (45).

(46) gives the expected time to recruitment, (46) and (47) together with (10) gives the variance of time to recruitment.

VI. NUMERICAL ILLUSTRATIONS

The mean and variance of the time to recruitment for all the models are given in the following tables for the cases (i) and (ii),

Case (i):Table – I(a) (Effect of ρ on performance measures)

$$\theta_1=0.4,\theta_2=0.6,\alpha_1=0.5,\alpha_2=0.8,p=0.8,c=2.5$$

ρ	MODEL I		MODEL II		MODEL III	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.6	26.9969	699.2067	10.8231	139.4303	34.1896	1.3954e+003
0.7	35.9959	1.4628e+003	14.4308	282.2801	45.5861	2.8775e+003
0.8	53.9938	3.8617e+003	21.6462	724.4484	68.3791	7.5047e+003

Case (ii):

Table – II (a) (Effect of ρ and k on performance measures)

$$\theta_1 = 0.4, \theta_2 = 0.6, \alpha_1 = 0.5, \alpha_2 = 0.8, p = 0.8, c = 1.5$$

$$g(t) = g_{x(1)}(t)$$

ρ	k	MODEL I		MODEL II		MODEL III	
		E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.6	2	31.7785	964.2689	12.4203	184.3391	40.4002	1.9427e+003
0.7	2	42.3713	2.0227e+003	16.5604	374.701	53.8669	4.015e+003
0.8	2	63.5569	5.352e+003	24.8405	965.0629	80.8003	1.0491e+004
0.6	1	17.4197	296.0506	7.6103	68.1898	21.7516	570.7102
0.6	2	31.7785	964.2689	12.4203	184.3391	40.4002	1.9427e+003
0.6	3	46.1127	2.1034e+003	17.1967	356.3135	59.0232	4.1246e+003

Table – II (b) (Effect of ρ and k on performance measures)

$$\theta_1 = 0.4, \theta_2 = 0.6, \alpha_1 = 0.5, \alpha_2 = 0.8, p = 0.8, c = 1.5$$

$$g(t) = g_{x(k)}(t)$$

ρ	K	MODEL I		MODEL II		MODEL III	
		E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
0.6	2	11.824	136.893	5.3347	33.1588	14.7110	260.1546
0.7	2	15.7667	280.3428	7.1129	64.8872	19.6147	527.0417
0.8	2	23.65	726.7705	10.6694	161.4127	29.4220	1.3534e+003
0.6	1	17.4197	296.0506	7.6103	68.1898	21.7516	570.7102
0.6	2	11.825	136.893	5.3347	33.1588	14.7110	260.1546
0.6	3	9.8102	94.6229	4.5346	23.7928	12.1695	177.7643

VI. FINDINGS

From the tables we observe the following which agree with reality

case (i) :

- As ρ increases , the mean and variance of time to recruitment increase for all the models .

case (ii)

- As ρ increases , the mean and variance of time to recruitment increase for all models when the probability density function of loss of manpower is the probability density function of first order statistics and k-th order statistics.
- As k increases , the mean and variance of time to recruitment increase for all models when the probability density function of loss of manpower is the probability density function first order statistics and decrease when the probability density function of loss of manpower is the probability density function of k-th order statistics.