

# Stress Analysis of Mating Involute Spur Gear Teeth

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## ABSTRACT

This paper presented analysis of Bending stress and Contact stress of Involute spur gear teeth in meshing. There are several kinds of stresses present in loaded and rotating gear teeth. Bending stress and contact stress (Hertz stress) calculation is the basic of stress analysis. It is difficult to get correct answer on gear tooth stress by implying fundamental stress equation, such as Lewis formula for bending stress and Hertz equation for contact stress. The detailed gear stressing is the key of this paper. The design of an effective and reliable gearing system is include its ability to withstand RBS (Root Bending Stress) and SCS (Surface Contact Stress). Various research methods such as Theoretical, Numerical and Experimental have been done throughout the years. We primarily prefer Theoretical and Numerical methods because Experimental testing can be expensive. So many researchers have utilized FEM to predict RBS and SCS. In this study we use a 3D model of gear and finite element analysis to conduct RBS and SCS calculation for mating involute spur gears. A pair of involute spur gear without tooth modification and transmission error is define in a CAD system (CATIA V5 and AUTODESK INVENTOR etc.) and FEA is done by using finite element software ANSYS. Obtained FEA results is comparable with theoretical and AGMA standard. It is found that Lewis formula and Hertz equation is used for quick stress calculation for gear, where as the AGMA standards and FEM is used for detailed gear stress calculation for a pair of involute spur gear.

**Keywords:** - Spur Gear, Root Bending Stress, Contact Stress, And Finite Element Method.

## INTRODUCTION

Gears are toothed cylindrical wheels used for transmitting mechanical power from one rotating shaft to another. Spur gear is cylindrical in form and has teeth, which are of involute form in most cases. [1] There are several kinds of stresses present in loaded and rotating gear teeth. We have to consider all the possibilities, so that the gears are proportional to keep all the stresses with in design limit. Generally stresses calculated in gear design formula are not necessary true stress, can make it difficult to get correct answer on gear-tooth stresses, because it may not be known whether load is uniformly distributed across the face width and whether properly shared by the two or more pairs of teeth that are in mesh at the same time. So we have to make right assumption that will allow for things like stress concentration, residual stress, misalignment and tooth error, this means that the calculated stress is probably not a true stress. Each gear tooth may be considering as a cantilever beam, when it transmits the load, it subjected to bending.[1,2] The bending stress is highest at the fillet and can caused breakage or fatigue failure of tooth in root

region. Whereas contact stresses are on the side of the tooth may causes Scoring Wear, Pitting fatigue. Contact stress is a compressive stress occurring at the point of maximum Hertzian stress. [1] Although gear manufacturing has achieved lots of advancement during its evolution, however the failure of gear due to bending and contact stress still remained a challenge for designer and manufacturers. Calculation of RBS and SCS through Lewis formula and Hertz equation, there are certain drawback such as in case of bending the entire load is carried on single tooth and in contact condition of gears are considered as two cylinders are in contact. In order to overcome these factors AGMA came out with several factors, which influence bending and contact stress on the gear. In the gear design a number of parameters involved, due to complex combinations of these parameter conventional design office practice tends to become complicated and time consuming.[1,2] It involves selection of approximate information about materials, dimensions, engineering standards available in engineering/manufacturers catalogue and design handbooks. While if we go to know gear design in detail, there is an acute paucity of research on comparative analysis between various standards and engineering practice. ISO and AGMA are most widely used. [2] Today finite element method is mostly used for predict any kinds of stress, strains and deformation in single parts and assembly. For practical consideration the contact stress on involute spur gear can be better approximated using FEM. It is originated for solving complex elastic and structural analysis problem.

### **BENDING STRESS (LEWIS FORMULA) [2]**

Wilfred Lewis in 1893 provides a formula for estimating the bending stress in a tooth. He modeled a gear tooth taking the full load at its tip as simple cantilever beam. If we substitute a gear tooth for the rectangular beam, we can find the critical point in the root fillet of the gear by inscribing a parabola. Is given by,

$$\sigma_b = \frac{W_t}{F \times m \times Y}$$

Where,  $\sigma_b$  = Root bending stress ( $\text{N/m}^2$ ),  $W_t$  = Transmitted tangential Load (Newton),  $F$  = Face width (m or mm),  $m$  = Module (m or mm),  $Y$  = Lewis form Factor.  $Y$  is the function of number of teeth, pressure angle and an involute depth of the gear.  $Y = 2x/3m$

It is fact that, when teeth mesh, the load is delivered to the teeth with some degree of impact. If we go with simply to calculate bending stress, the velocity factor is should be used in calculation. Now Lewis equation becomes,

$$\sigma_b = \frac{W_t}{K_v \times F \times m \times Y} \quad \text{----- (1)}$$

Where,  $K_v$  is the Velocity Factor.

## CONTACT STRESS (HERTZ EQUATION) [1,2]

The stress on the surface of gear teeth are usually determined by formula derived from the work of H. Hertz's; frequently these stresses are called Hertz stress. Hertz determined the width of the contact band and the stress pattern when various geometric shapes were loaded against each other. The Hertz formula for the width of the band of contact is,

$$B = \sqrt{\frac{16 \times W (K_1 + K_2) R_1 R_2}{L (R_1 + R_2)}}$$

The maximum compressive stress is,

$$\sigma_c = \frac{4 \times W}{L \times \pi \times B}$$

Above Hertz formula can be applied to Spur Gears quite easily by considering that the contact condition of gears are equivalent to those of cylinders having the same radius of curvature at the point of contact as the gear have. P R N Child [2] gives the formula for calculating contact stresses (surface compressive stress) in a pair of mating spur gear teeth. Assuming  $R_1$  and  $R_2$  are the respective radius of curvature of an involute curves at the contact point. Thus,

$$R_1 = r_{p1} \sin \varphi$$

And

$$R_2 = r_{p2} \sin \varphi$$

Where  $r_{p1}$  and  $r_{p2}$  are the pitch radius of the Pinion and Gear respectively and  $\varphi$  is the pressure angle. Now the Hertz equation for contact stresses in the teeth becomes

$$\sigma_c = -C_p \sqrt{\frac{K'_v W_t}{F \cos \varphi} \left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \quad \text{----- (2)}$$

(Negative sign because  $\sigma_c$  is a compressive stress)

**AGMA BENDING STRESS EQUATION:** - The AGMA equation for bending stress is given by, [2]

$$\sigma_b = W_t K_O K'_V K_S \frac{1}{F \times m} \frac{K_H K_B}{Y_J}$$

**AGMA CONTACT STRESS EQUATION:** - The AGMA equation for contact stress is given by, [2]

$$\sigma_c = Z_e \sqrt{W_t K_O K'_V K_S \frac{K_m Z_R}{F \times d} \frac{1}{Z_I}}$$

Following literature review gives a lot of information about stress calculation based on Lewis formula, Hertz theory, AGMA/ANSI (American National Standards Institute) equations and Finite Element Method.

## LITERATURE REVIEW

The comparative analysis of ISO and AGMA standards with Finite element analysis is important for modern design and manufacturing of gears. Current trends in engineering globalization necessitate revisiting various normalized standards to determine their common fundamentals and best approaches needed to develop “Best Practices” in automotive, aerospace and other industries. This can lead to both reduction in redundancies and also cost containment related to needed adjustments between manufactures for missing parts interchangeability and performance due to incompatibility of different standards. The results will allow for a better understanding of existing limitations in the current standards applied in engineering practice as well as provide a basis for future improvement of gear standard.[4] The calculation of tooth bending strength and surface durability of normal and high contact ratio can be enough for preliminary design or standardized purpose, but stress calculation through these simple equations given by the linear theory of elasticity and the Hertzian contact model are not in good agreement with experimental results. This stress calculation has not been studied in depth in the past. [5,13] Stresses of an involute spur gear pair in the external meshing can be estimated theoretically, analytically and numerical method. Wei Yangang gives theoretical research to find location of the maximum contact stress for a pair of involute spur gear, whose contact ratio is larger than 1 and validated by finite element method. And concluded that when the tooth number of small gear is smaller than a certain value, the maximum contact stress in the meshing process is not generated in the inner critical point in the single tooth meshing region of the small gear.[7] So many researchers analyze the contact stress between spur gear teeth using a plane model and validate Hertz stress and AGMA contact stress with finite element contact stress.[5,6,11] and concluded that FEM is able to simulate contact stress in a pair of mating gear, the contact stress is highest at higher point on the involutes and lower where a single pair of teeth assume the full load transmitted and minimal at the pitch point of contact. Ignacio Gonzalez-Peraz developed a finite element model and validated in terms of contact area, pressure distribution and maximum contact pressure for those cases where the Hertz theory can be applied and provide partial crowning to the finite element model where Hertz theory does not work properly, and concluded that analysis results give good agreement with Hertz theory for calculating maximum contact pressure, contact area, and deflection with minute errors.[9] Xianzhang FENG uses a precise model in a large-scale CAD software and defines the stress and displacement field for determining the maximum equivalent stress and maximum displacement. By defining the quasi-static characteristic of finite element analysis, shows model can accurately simulate the distribution of equivalent stress and displacement change in the process of teeth meshing. The results are well agreed with the actual meshing law and not only verify the correctness of model but also expect to dynamic analysis of gear. The loaded contact performance, tooth root bending stress and tooth contact stress for a pair of gears are the important concerned index in the successive process of engagement.[8] S. Sankar using circular root fillet instead of standard trochoidal root fillet in gear. and concluded that the tooth deflection in the circular root fillet is less when compared to the trochoidal root fillet, further there is appreciable reduction in bending stress and contact shear stress for circular

root fillet design in comparison to that of trochoidal root fillet design.[10] ultimately stress calculation for a pair of gears using Lewis formula, Hertz equation and AGMA/ISO standards is comparable with FEA, and for define a safe design, use of CAD software and finite element software is better tool.[3,6,12]

## FEM ANALYSIS TO ESTIMATE THE BENDING AND CONTACT STRESSES

The above literature review concluded that the Finite Element Method is most widely used for stress analysis in a pair of gear. In addition, FEM software has been used for performing meshing simulation. Almost in all of the above cases Contact Stress calculation and Bending Stress calculation play more significant role in the design of gear. This study shows that Hertz theory is the basis of contact stress calculation and Lewis formula is use for calculating bending stress in a pair of gear.

**MODELING OF SPUR GEAR:-**We can use the software of CAD (Computer Aided Design) to complete the three-dimensional parametric modeling of gears. Using the functionality of three-dimensional solid modeling software it is easy to implement parametric gear drive and it also has a good interface with the Finite Element Software. CATIA (Computer Aided Three Dimensional Interactive Application) is able to model the spur gear geometry. The method of gear tooth geometry and parametric modeling is as follow,

- (1) Determining the basic parameter
- (2) Determining the geometrical parameter
- (3) The characteristic parameter

After generating geometrical model by using geometric parameters, we have to establish precise involute gear profile curve,

The parameter equation of the standard involutes curves is expressed as:

$$x = r_b \{ \sin(\pi t) - \cos(\pi t) \pi t \} \quad \text{And} \quad y = r_b \{ \cos(\pi t) + \sin(\pi t) \pi t \}$$

In order to get the correct assembly in CATIA V5:-

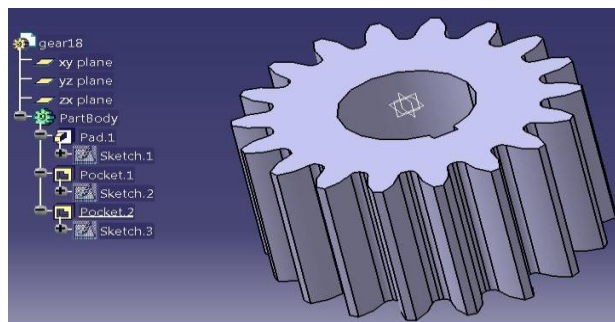
- The axes of pinion and gear must be parallel to each other and the center distance of two gears must be equal to the sum of the two pitch circles radius of gear.
- And at the same time, the tooth surface contact of pinion and gear must be tangential to the point of contact.

The main dimension and parameters for standard 20°, full depth involute spur gear are,

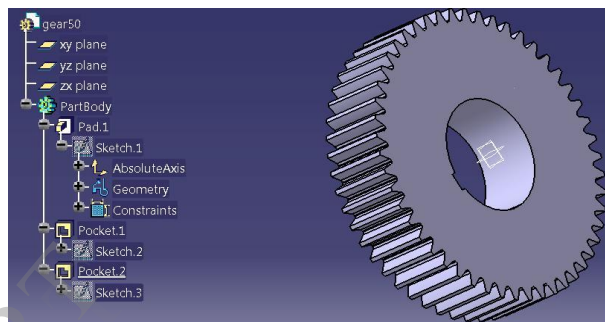
Parameter	Pinion	Gear
Number of teeth	18	50
module	2.5	2.5
Material	Grade 1 steel	Grade 1 steel
Input speed (rpm)	1425	-----
Input Power (Kw)	3	-----

Diameter of pitch circle (mm)	22.5	62.5
Diameter of base circle (mm)	21.15	58.75
Diameter of Addendum circle (mm)	25	65
Diameter of Dedendum circle (mm)	19.375	59.375
Face width (mm)	30	30
Bore diameter (mm)	20	25
Young's modulus (MPa)	$2.1 \times 10^5$	$2.1 \times 10^5$
Poisson's ratio	0.3	0.3

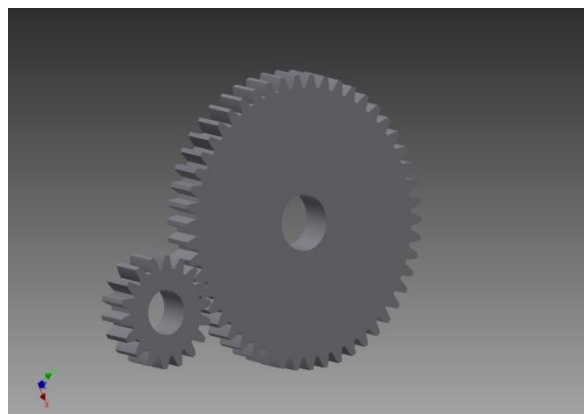
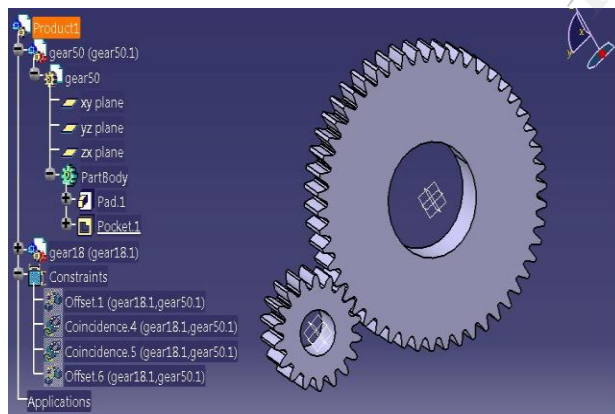
### PARAMETRIC MODELING OF GEARS USING CATIA V5



ASSEMBLY CREATED IN  
CATIA V5



ASSEMBLY CREATED IN  
AUTODESK INVENTOR



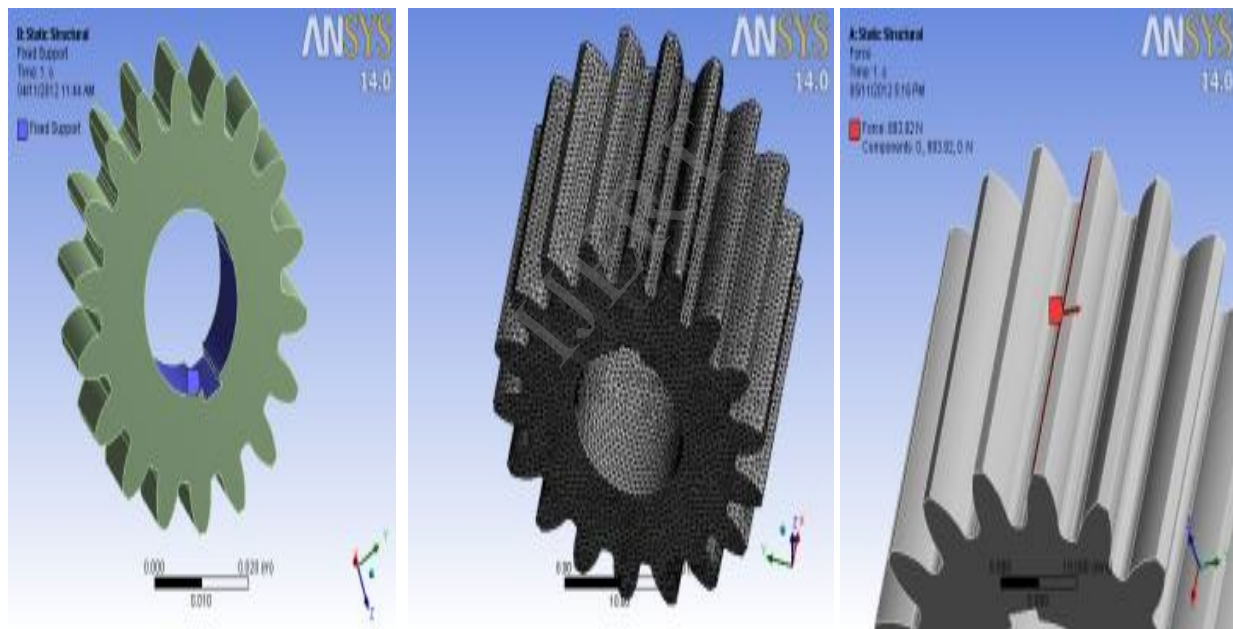
Now, the assembly which was created in CATIA is imported in ANSYS workbench 14 for stress analysis. It is done by saving drawing in STP or IGS file format in CATIA and AUTODESK INVENTOR. Here we also use assembly of spur gear made in Autodesk Inventor. After the assembly is imported in ANSYS workbench 14, as the both teeth are already in contact, our main purpose is to find the root bending stress and contact stress. This is done by using following steps in ANSYS workbench 14.

**3-DIMENSIONAL ANALYSIS OF SPUR GEAR:** - For imported 3-dimensional geometry, firstly we select 3-D and static structural analysis from menu and connecting the geometry to the analysis tab. Then next we enter Young's modulus and Poisson's ratio of the material.

**DEFINING THE CONTACT REGION:** - Once the geometry is attached with static structural analysis tab, we have must define the contact between the two involute teeth, ANSYS has inbuilt option, which automatically reads the attached geometry for any predefined contacts or other boundary definitions. One of the most important things is to change the 'Interface Treatment' to 'Adjust to Touch'. This is helping us to define the kind of contact between the selected bodies.

**MESH GENERATION AND BOUNDARY CONDITION (supports and load):-** A tetrahedron solid elements is used in mesh generations. Boundary condition refers to the external load on the border of the structure. We assumed gear is with fixed support and pinion is subjected to a moment or torque along its axis with frictionless support.

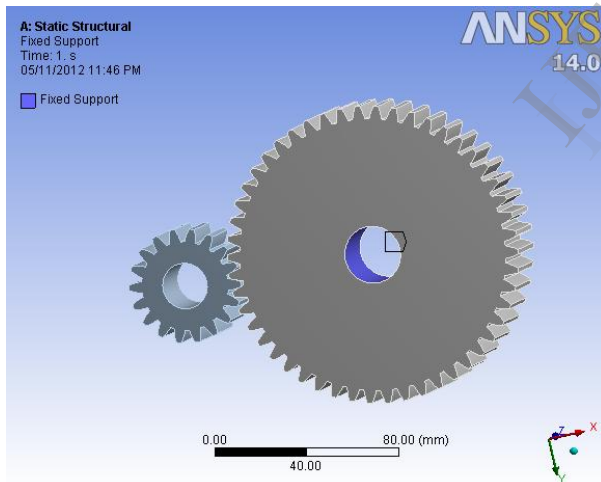
#### MESHING AND BOUNDRY CONDITION FOR PINION



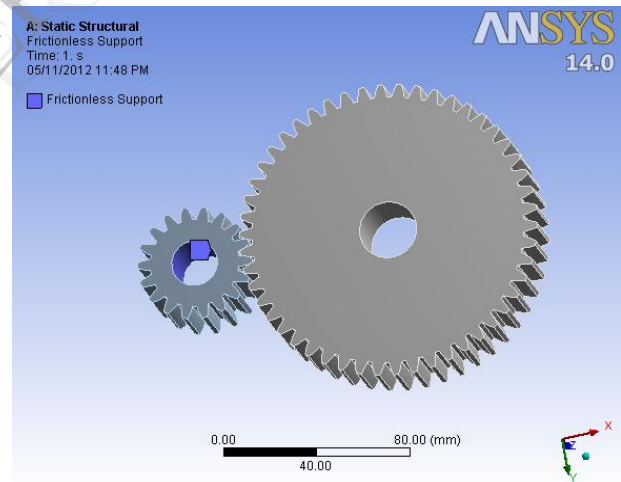
### MESHING AND BOUNDRY CONDITION FOR GEAR



### PINION AND GEAR ASSEMBLY MESHING AND BOUNDRY CONDITION



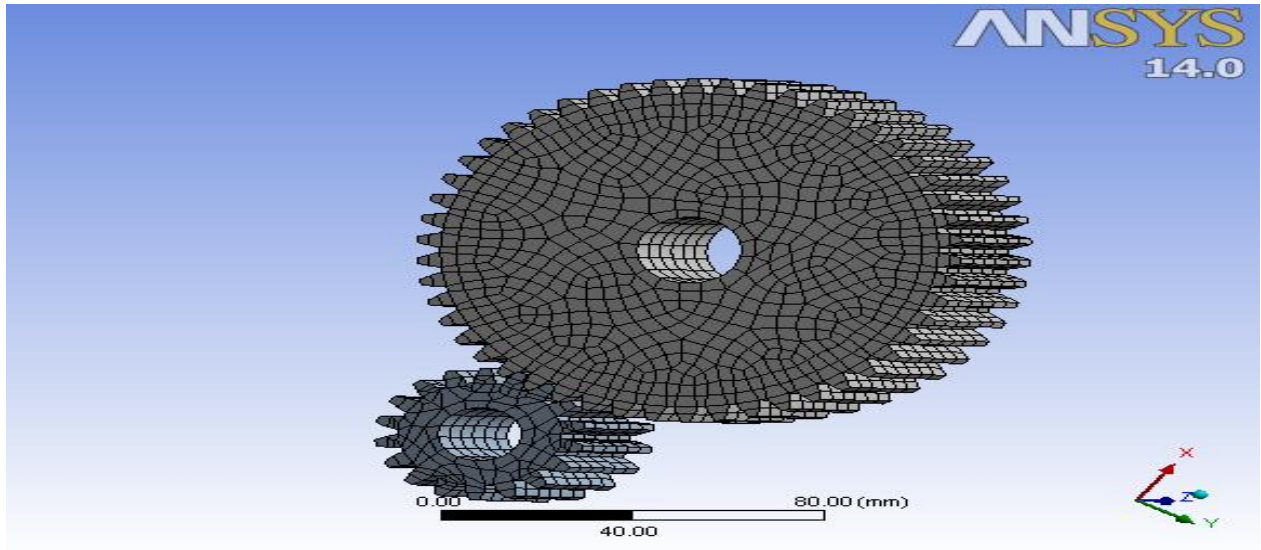
FIXED SUPPORT



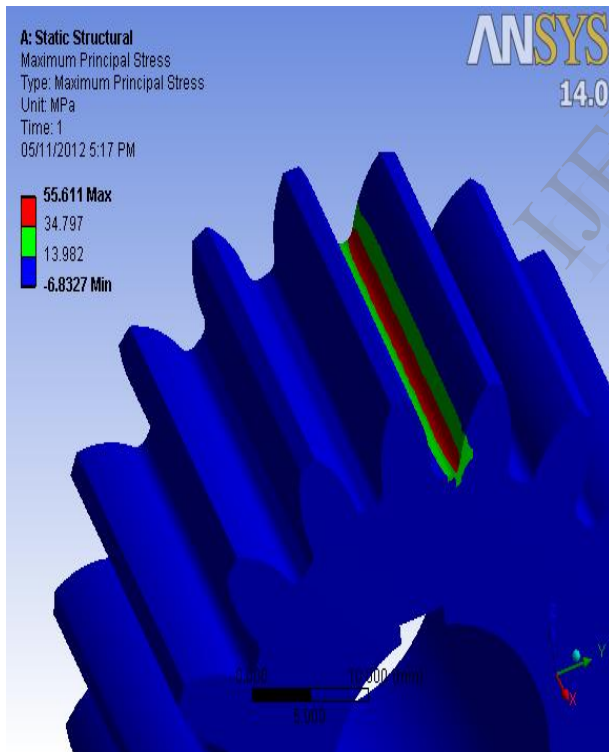
FRICTIONLESS SUPPORT



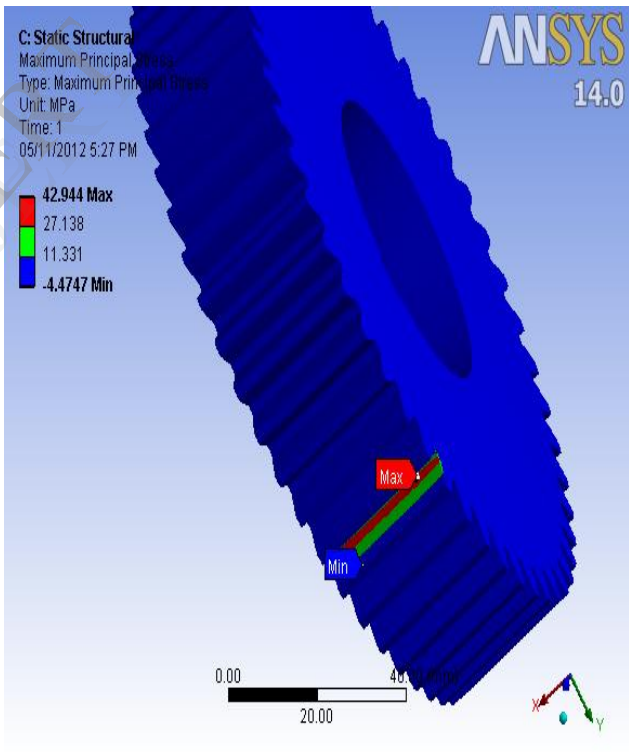
## MESHING



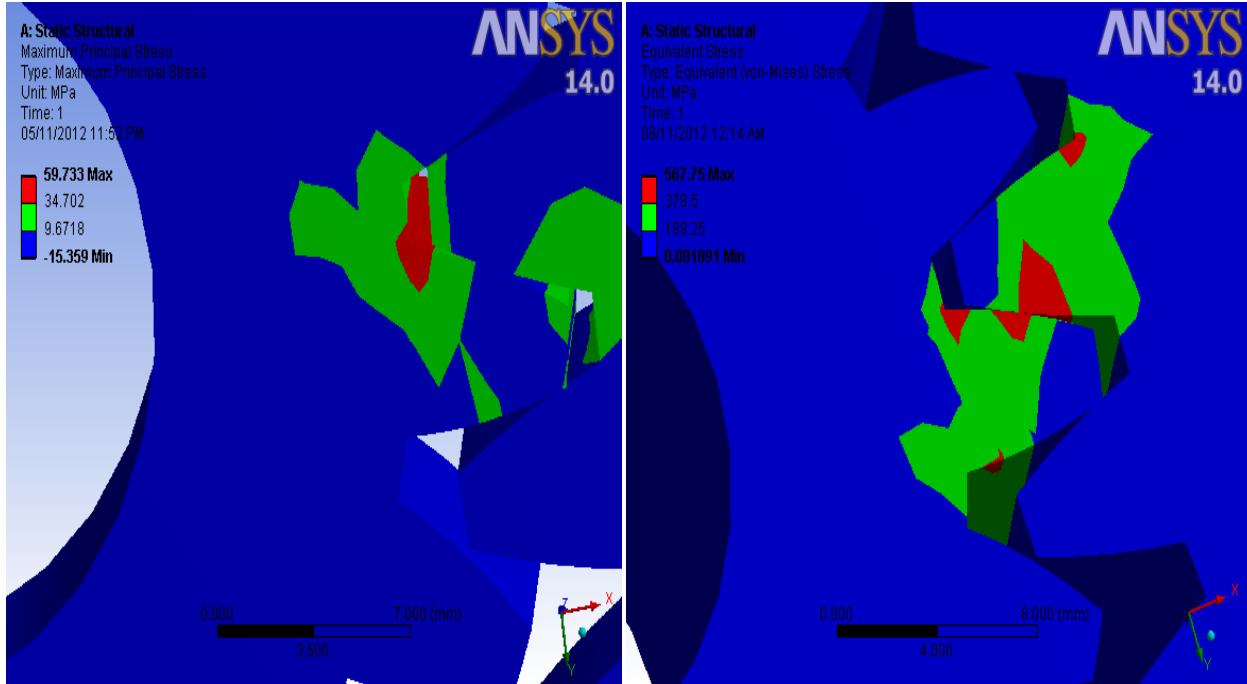
## RESULTS AND DISCUSSION



PINION ROOT BENDING STRESS



GEAR ROOT BENDING STRESS



BENDING STRESS

CONTACT STRESS

**ROOT BENDING STRESS**

	Pinion	Gear
Lewis Formula	63.00 MPa	46.36 MPa
AGMA bending stress	60.39 MPa	48.18 MPa
FEA stress	55.61 MPa	42.94 MPa
FEA (in meshing) Maximum Principle stress	59.73 MPa	

**CONTACT STRESS**

Hertz Equation	-562.27 MPa
AGMA contact stress	572.00 MPa
FEA	567.75 MPa

Theoretically result obtained by Lewis formula and Hertz equation and results found by AGMA/ANSI equations are comparable with Finite Element Analysis of spur gear. Comparing the results of the Root Bending Stress, it is noticeable that the Lewis formula can be used for a quick calculation of the stress on the root of gear tooth. What is more, analyzing by Finite

element method, the stress value is found to be very close to the AGMA standards. The values found for SCS (Surface Contact Stress) is acceptable when compared with the AGMA standard and Hertz equation both.

## CONCLUSION

The above literature review presents that the Finite Element Method is most widely used for stress analysis in a pair of gear. In addition, FEM software has been use for performing meshing simulation. Almost all of the above cases Contact Stress calculation and Bending Stress calculation is play more significant role in the design of gear. This study is shows that Hertz theory is the basis of contact stress calculation and Lewis formula is use for calculating bending stress in a pair of gear. Theoretically result obtained by Lewis formula and Hertz equation and results found by AGMA/ANSI equations are comparable with Finite Element Analysis of spur gear.

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