String Cloud with Quark Matter in Plane Symmetric Space-Time Admitting Conformal Motion

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Abstract: **In this paper, we have examined the string cloud with quark matter in the plane symmetric space- time admitting one-parameter group of conformal motions. Also, we have discussed the properties of the solutions obtained.**

Keywords— Plane symmetry; String cloud; quark matter; conformal motion

I. INTRODUCTION

In general relativity, it is a subject of long- standing interest to look for the exact solutions of Einstein's field equations. To know the exact physical situation at early stage of the formation of our universe is still challenging subject of study. At the very early stages of evolution of the universe, it is generally assumed that during phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as strings, domain walls and monopoles.

Sahoo and Mishra [1] studied plane symmetric space- time with quark matter attached to the string cloud and domain wall in the context of Rosen's biometric theory and observed that, in this theory, string cloud and domain walls do not exist and biometric relativity does not help to describe the early era of the universe. Sahoo and Mishra [2] also studied axially symmetric space- time with strange quark matter attached to the string cloud in Rosen's biometric theory and shown that there is no contribution from strange quark matter and hence vacuum model is presented. Deo [3] studied spherically symmetric Kantowski- Sachs space- time in the context of Rosen's biometric theory with the source matter cosmic strings and domain walls and observed that the space- time does not accommodate the cosmic strings as well as domain walls and it is observed that the resulting space- time represents Robertson-Walker flat space- time which expands according to the signature of the parameter uniformly along the space directions with time. Yilmaz [4] obtained Kaluza-Klien cosmological solutions for quark matter coupled to the string cloud and domain wall in the context of general relativity by using anisotropy feature of the universe. Rao and Neelima [5] studied the anisotropic Bianchi type- VI space-time with strange quark matter attached to string cloud in Barber's second self creation theory and general relativity and noticed that the presence of scalar field does not affect the geometry of the space- time but changes the matter distribution.

General relativity provides a rich arena to use symmetries in order to understand the natural relation between geometry and matter furnished by Einstein equations. Symmetries of geometrical/ Physical relevant quantities of this theory are known as collineations and the most useful collineation is conformal killing vector defined by

$$£_\xi \ g_{ij} = \xi_{i;j} + \xi_{j;i} = \psi \, g_{ij} \,, \ \ \psi = \psi(x^i),$$

where $£_\xi$ signifies the Lie derivative along ξ^i and $\psi = \psi(x^i)$ is the conformal factor. In particular, ξ is a special conformal killing vector, if $\psi_{;ij} = 0$ and $\psi_{,i} \neq 0$. Here (;) and (,) denote covariant and ordinary derivatives respectively.

Conformal killing vectors provide a deeper insight into the space- time geometry and facilitate generation of exact solutions to the field equations. Sharif [6] classified the static plane symmetric space- time according to their matter collineations. Aktas and Yilmaz [7] solved Einstein's field equations for spherical symmetric space- time via conformal motions and examined magnetized quark and strange quark matter in spherical symmetric space- time admitting one-parameter group of conformal motions. Kandalkar, Wasnik and Gawande [8] investigated spherically symmetric string cosmological model with magnetic field admitting conformal motion. Shobhane and Deo [9] examined the wet dark fluid matter in the spherical symmetric space- time admitting one parameter group of conformal motion.

The paper is outlined as follows:

In Sec.2, we have obtained Einstein field equations for string cloud with quark matter in plane symmetric space-time. In Sec.3, the solutions of the field equations are obtained for string cloud with quark matter in plane symmetric space- time admitting one parameter group of conformal motion. In Sec.4, concluding remarks are given.

II. EINSTEIN'S FIELD EQUATIONS

The metric for static plane symmetric space – time is given by

$$ds^2 = e^{\nu} dt^2 - dx^2 - e^{\mu}(dy^2 + dz^2), \qquad (1)$$

where μ and ν are functions of x alone and $x^{1,2,3,4} = x, y, z, t$.

Einstein's field equations can be expressed as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G \, T_{ij}. \qquad (2)$$

Here, we shall use geometrized units so that $8\pi G = c = 1$.

The energy momentum tensor T_{ij} for string cloud is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j , \quad (3)$$

where ρ is the rest energy density for cloud of strings with particles attached to them and λ is the string tension density and they are related by the equation

$$\rho = \rho_p + \lambda \ or \ \rho_p = \rho - \lambda. \quad (4)$$

Here ρ_p is the particle energy density. The string is free to vibrate, and different vibrational modes of the string represent the different particle types, since different modes are seen as different masses or spins. So, we will take quarks instead of particle in the string cloud.

In this case, from (4) , we get

$$\rho = \rho_q + B_c + \lambda \ or \ \rho_q + B_c = \rho - \lambda , \quad (5)$$

where ρ_q is the quark energy density and B_c is the vacuum energy density (called as the bag constant). Further, u_i is the four– velocity of the particles and x_i is the unit spacelike vector representing the string direction in cloud i.e. the direction of anisotropy.

We have

$u^i = \delta_4^i e^{-\nu/2}$ and $x^i = \delta_1^i$.

$\therefore\ u^i u_i = -x^i x_i = 1$ and $u^i x_i = 0$.

Then using (3) we get

$T_{11} = -\lambda$, $T_{22} = T_{33} = 0$ and $T_{44} = \rho e^{\nu}$.

Using (1) and (2), we get

$$\frac{1}{4}\left(\mu'^2 + 2\mu'\nu'\right) = -\lambda, \quad (6)$$

$$2\mu'' + 2\nu'' + \mu'^2 + \nu'^2 + \mu'\nu' = 0 \quad (7)$$

and

$$\frac{1}{4}\left(4\mu' + 3\mu'^2\right) = -\rho, \quad (8)$$

where primes denote differentiation w.r.t. x.

III. SOLUTIONS OF FIELD EQUATIONS

Now, we shall assume that space–time admits a one-parameter group of conformal motions [7] i.e.

$$\pounds_\xi g_{ij} = \xi_{i;j} + \xi_{j;i} = \Psi g_{ij} , \quad (9)$$

where $\pounds_\xi$ signifies the Lie derivative along ξ^i and Ψ is an arbitrary function of x. In particular, ξ is a special conformal killing vector, if $\Psi_{;ij} = 0$ and $\Psi_{,i} \neq 0$. Here (;) and (,) denote covariant and ordinary derivatives respectively.

Using (1) and (9), by virtue of plane symmetry, we get following expressions:

$$\xi^1_{,1} = \frac{\Psi}{2}, \quad (10)$$

$$\xi^4 = c \text{ (constant)}, \quad (11)$$

$$\xi^1 \mu' = \Psi , \quad (12)$$

$$\xi^1 \nu' = \Psi , \quad (13)$$

and

$$\xi^2 = \xi^3 = 0. \quad (14)$$

Using (12) and (13), we get

$$\mu' = \nu'. \quad (15)$$

On integration, we get

$$\mu = \nu + c_1 \ \text{or} \ e^{\mu} = c_2 e^{\nu} , \quad (16)$$

where c_1 and c_2 (> 0) are the constants.

Using (10) and (12), we get

$$\frac{d\xi^1}{\xi^1} = \frac{\mu'}{2}\, dx. \quad (17)$$

The solution of (17) is given by

$$\xi^1 = c_3\, e^{\mu/2} ,\ c_3 > 0. \quad (18)$$

Using (7) and (15), we get

$$4\mu'' + 3\mu'^2 = 0 . \quad (19)$$

Setting $\mu' = \frac{1}{n}$, we get

$$\frac{dn}{dx} = \frac{3}{4} \ \text{i.e.}\ n = \frac{3}{4}x + k_1, \quad (20)$$

where k_1 is the constant of integration.

$$\Rightarrow \frac{d\mu}{dx} = \frac{4}{3x + 4k_1}.$$

On integration, we get

$$e^{\mu} = [k_2(3x + 4k_1)]^{4/3} = c_2 e^{\nu}. \quad (21)$$

Using (12), (18) and (21), we get

$$\Psi = \frac{k}{(3x + 4k_1)^{1/3}} , \tag{22}$$

where $k = 4c_3 (k_2)^{2/3} > 0$ is the constant.

Using (6), (8), (15) and (21), we get

$$\lambda = -\frac{12}{(3x + 4\,k_1)^2} \tag{23}$$

and

$$\rho = 0. \tag{24}$$

As pointed by Letelier [10], [11], the string tension density λ may be positive or negative.

Using (5), we get

$$\rho_p = \rho_q + B_c = -\lambda = \frac{12}{(3x + 4\,k_1)^2} \tag{25}$$

$$\rho_q = \frac{12}{(3x + 4\,k_1)^2} - B_c \tag{26}$$

Assuming that the quarks are massless and non interacting , then we have the quark pressure [4]

$$p_q = \frac{\rho_q}{3} = \frac{4}{(3x + 4\,k_1)^2} - \frac{B_c}{3} . \tag{27}$$

Then Total particle pressure is given by

$$p_p = p_q - B_c = 4\left[\frac{1}{(3x + 4\,k_1)^2} - \frac{B_c}{3}\right]. \tag{28}$$

Using (1), space-time geometry of string cloud with quark matter is given by

$$ds^2 = \frac{e^{\mu(x)}}{c_2}\, dt^2 - dx^2 - e^{\mu(x)}(dy^2 + dz^2), \tag{29}$$

where $e^{\mu(x)} = [k_2(3x + 4k_1)]^{4/3}$.

For c_2=1, above line element reduces to the anti-De Sitter metric in unusual form [12]:

$$ds^2 = e^{\mu(x)}(dt^2 - dy^2 - dz^2) - dx^2, \tag{30}$$

where $e^{\mu(x)} = e^{\nu(x)} = [k_2(3x + 4k_1)]^{4/3}$.

IV. CONCLUSION

From (23), we observed that the string tension density is negative i.e. $\lambda < 0$. From (23) and (25), we get $\rho_p \gg \lambda$. The energy condition $\rho \geq 0$ and $\rho_p \geq 0$ are satisfied. Further when $x \to -4k_1/3$, $\lambda \to \infty$, $\rho_p \to \infty$ and when $x \to \infty$, $\lambda \to 0$, $\rho_p \to 0$ and $\rho_q \to -B_c$.

ACKNOWLEDGEMENT

We would like to thank Prof. Dr. K. D. Patil, BDCOE, Wardha for his valuable suggestions.

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