

Structural Damping in a Pedestrian Footbridge Under Controlled Traffic Density

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Abstract— In this paper we present the study of the displacement due to the movement between the pedestrian traffic and the concrete footbridge of a pedestrian bridge located in the Matilde Town, Hidalgo State, México, in which, by means of a portable equipment, the acceleration of the pedestrian's center of gravity is measured when walking. The pedestrian is idealized in a dynamic scheme consisting of a mass that produces a temporal stimulation corresponding to the potential response of the system, the load due to the step density of the basic mechanics of walking is simulated in a mathematical scheme, considering the vertical and lateral displacement on the catwalk, formulating matrix with the equation of movement through Lagrangean mechanics, four scenarios are studied, where the simulations of the energy contribution of the periodic load of six pedestrian masses are presented, with obtained results that show the structural behavior, which help to identify the service conditions of the structure of the pedestrian bridge.

Keywords— pedestrian; Pedestrian bridge; pedestrian density; accelerometer; Lagrangean mechanics.

I. INTRODUCTION.

The pedestrian bridges at every moment are subject to external forces[1], as the intensity of pedestrian traffic, what causes variation in the traffic of the footbridge[2], said traffic is regulated according to pedestrian locomotion standards, of the dimensions of the catwalk and the speed of walking, this last stochastic condition varies according to time, and its effect causes levels of charges congestion[3].

Human-structure interaction models on bridges, are used to assess the vibration, the methodology to monitor long-term said vibrations at present is very sophisticated, and it serves to identify and know the effects that the mobile force produces general dependent on time, where the pedestrian is idealized in a force with his mass moving[4].

The applications to identify scenarios are five and their regulatory classification used in[5], shows four according to traffic density and one considering fifteen pedestrians transiting. In this paper we will analyze the four densities according to the way in which the pedestrian load is presented in a determined surface with a controlled crowd, therefore, in the first (Tc_1), Weak traffic, in an area of five square meters, in the second (Tc_2), Dense traffic, in two square meters, in the third (Tc_3), Very dense traffic, in one square meter and the fourth (Tc_4), Exceptionally dense traffic, in sixty-seven square centimeters [3][5].

By analyzing the sample, the interaction between the crowd and the structure of the bridge is quantified, obtaining a normal distribution, where the average stimulation frequency and its standard deviation, maintain known criteria as in[6], when considering only one phase angle, it is taken into account that, the arrival and the momentum, maintains a surface of the pedestrian walk, represented imaginarily, through a circle, such form who, the magnitude of the radius, corresponds to the density of pedestrian traffic, and plus much, there is a control so that adjacent pedestrians do not invade said space, so then, the crowd moves along the bridge, timing the acceleration, until the determination of the average transit speed of 1.25 m/s[7].

One of the factors to be taken into account in the sample, is the variability of the weight of each pedestrian, which contributes to causing phase differences between pedestrians, effects of uncontrolled traffic density, lack of synchronization of walking between pedestrians and the decompensated vibration of the structure [8], this variability is highly related to probability, generates a problem whose solution is not feasible that can be addressed by deterministic terms [9].

Pedestrians are more susceptible to horizontal vibration than to vertical vibration, vertical displacements are needed so that pedestrians can be bothered, because the human movement and the internal forces associated with this movement are reciprocal to the generation induced by man, these internal forces in the structure support activities such as walking, run or jump, on the other hand, these conditions of life reveal to us that the mechanics of human locomotion have been the subject of a high level of research [10]. Therefore, The forces produced by rhythmic activities can be represented by considering the sum of the dynamic component, represented with the Fourier series of forced rhythm, in this way, the normalized function of the applied static weight is determined [11].

II. DYNAMIC SYSTEM

The dynamic system approach of the present work, consider two schemes, The first consists of a physical model, and the second in a mathematical model, both of them, help define the conditions to determine the variables that intervene in the use and service of the structure [12], that complies with the principles of cutting-edge structural dynamics, achieving compliance with globalized standards, and with it this system contemplates satisfying the current design codes [13][14], in

these conditions it is essential to perform vibration tests on the catwalk considering the necessary scenarios, and detect experimentally the initiation of vibration problems [15].

Dynamic stimulation and evaluation of your response shows the service capacity, in accordance with the built walkways [2], for this reason, the incorporation of the frequencies of the different phenomena that produce accelerations that lead to the initiation of a limit state of service or failure was unified in a load model [3].

In this work we analyze the cyclic loading pattern of the stimulation from walking on a footbridge of a pedestrian bridge, where the structure is subjected to a controlled dynamic system according to the regulations established by current codes [13].

The basic pedestrian activity that governs the present model, is determined by the common gait [9], being often that, the daily activity presents a pattern of stimulation of the pedestrian load that is associated to a dynamic representation of a common structural system [16].

The forces generated from the pedestrian-structure contact, they vary with respect to the kinematics of the center of mass during the whole cycle of the walking of the pedestrian who intervenes in the sample [17], what it shows, the complexity of the analysis, incorporating multiple masses.

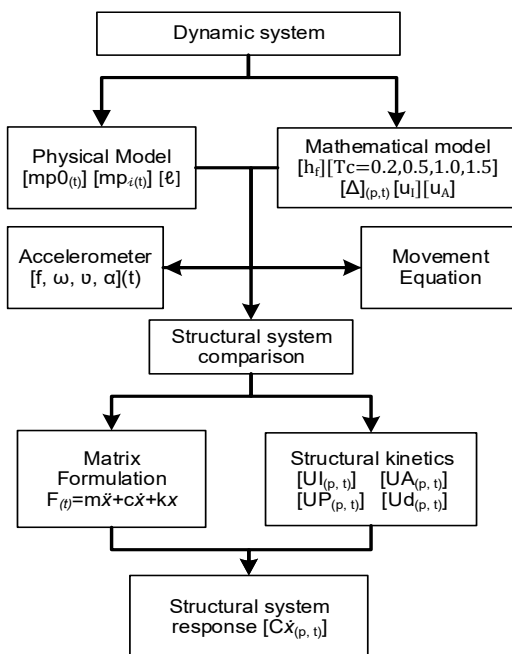


Figure 1 Dynamic system block diagram

The block diagram of figure 1, presents the structural system dynamic response, which is obtained considering the catwalk structural kinetics applying the matrix formulation, proposing the movement equation in Lagrangean mechanics [18], measuring by means of a portable instrument the patterns of the cyclic stimulation behavior, raised at the conceptual level in a mathematical model, so that, study the contact forces at every moment, control its density and transit speed, at the same time, controls the energy of contribution to the vibration [19], in a useful task, but in turn it is necessary, understanding that, by knowing the periodic load experimentally, we obtain the conditions of the pedestrian bridge [20].

A. Structural modeling of pedestrians on a bridge.

The gangway acts rigidly by being supported laterally, and in turn, It is built on secondary beams of common structural profiles [2], so much as, it is coupled to a flexible system, so that, the kinetic energy is acquired by the vibration due to the movement of the pedestrian [3].

this structure, It has elasticity and bending of the structural continuity, for such a situation, a linear system is constituted subjected to a forced excitation, what produces a second order analysis [21], one of the techniques used to solve said system requires advanced numerical solutions, to determining the conformation of the temporary periodic load according to the cyclic stimulation pattern [16].

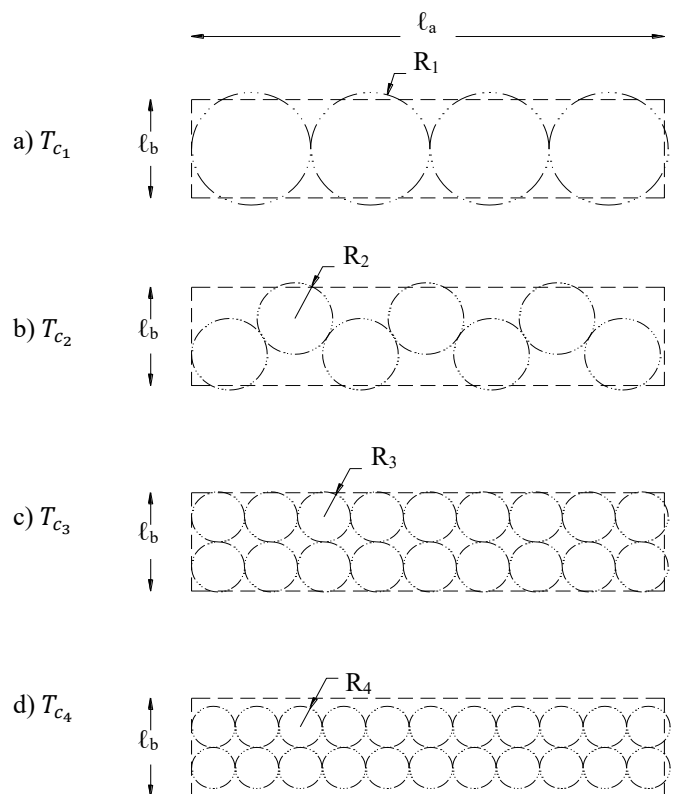


Figure 2 Graphic representation of pedestrian traffic.

In the scheme of figure 2, the characteristics of pedestrian traffic are observed (T_c) [5], representing with radio circles R_a , the space in which the pedestrian develops his walked [9], these idealized circular spaces are used in the present study, considering a long pedestrian crossing l_a and wide l_b , noting that the traffic T_{c1} , Consider a model with four circles that denote the free circulation of each pedestrian, where the radio R_1 Acting has a magnitude of 1.26 m, therefore, in the model T_{c2} , the radio $R_2=0.80$ m., as well as, T_{c3} , $R_3=0.56$ m. and finally in T_{c4} , $R_4=0.46$ m[13].

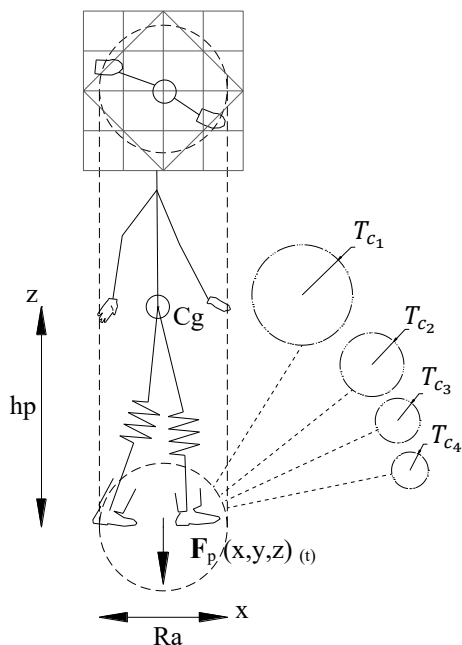


Figure 3 Graphical representation of the characteristics of the acting radius (Ra) of the pedestrian.

The scheme of figure 3, shows the degrees of freedom [22], the pedestrian is modeled considering a concentrated mass (m_p), in the center of gravity (Cg)[23].

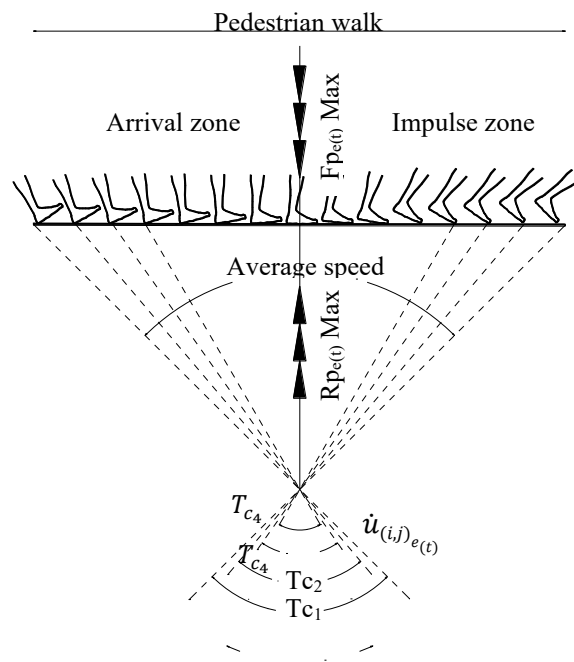


Figure 5 Graphical representation of the basic characteristics of pedestrian walking.

Figure 5 shows the speed of pedestrian traffic [16], where the periodic load of the mass of the pedestrian, has contact with the catwalk [8], shown in figure 6.



Figure 4 Graphic representation of the lateral characteristics of the pedestrian bridge

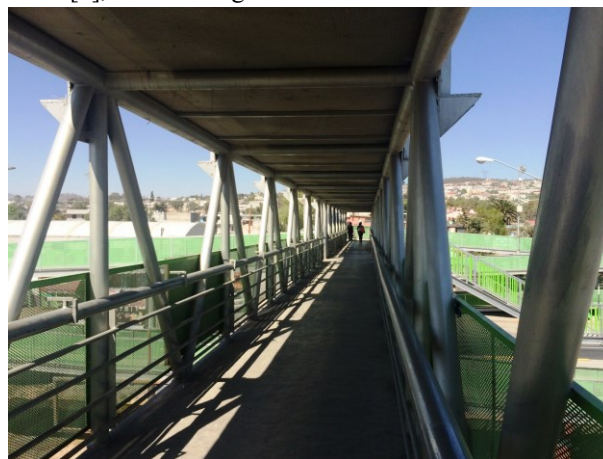


Figure 6 Graphical representation of the interior characteristics of the pedestrian bridge

The bridge built with metallic structural elements, as shown in figure 4, It has vertical, horizontal and diagonal bars, that form envelope the transit board, whose footbridge is designed with a plate of reinforced concrete, supported on secondary traves [2], stairs access or pedestrian exit are connected [11], where the dynamic stimulation of the scenarios under study is born [14], this is where, pedestrian traffic is controlled according to the density of movement of the walk, at par, with the designation of the sample [15]. The two legs represent two stiffness springs K_p , and the passage has two periods: the impulse phase and the arrival phase [24], as observed on the figure 5, in this way the response of the temporary periodic load is represented [25], called $R_{p_e(t)}$.

Speed (\dot{u}), average pedestrian [7], can be controlled in the four study scenarios, If and only if, the zones interfere with the frequency of passage [26], just like that, the radio R_a defined and the idealized zones have dominant geometry conditions [27], thereupon, the changes in the response (Rp), affect the potential energy of the catwalk, and its magnitude is determined with the impulse force [21], the arrival zone, being the initial part, begins with the support of the heel and ends when the sole touches the surface of the catwalk in its entirety, at this moment the periodic charge usually reaches its maximum magnitude, because the impulse phase of the next step begins [3].

B. Matrix formulation.

It is assumed that the sample maintains contact with the surface of the footbridge of the bridge under study [5], as shown in figure 5, likewise, it is assumed that portable measuring equipment captures displacement data $(u_x, u_y, u_z)_{(t)}$ [17], that has the center of gravity of the sample selected when walking [20], the response that the sensor emits, is considered with a duration of eight seconds (Seg.), since it is necessary to travel the distance la [15], In addition, it is considered a time required to standardize the average speed, so the time $(t_a)_i$ represented in each simulation will be at least fifteen seconds [17].

$$m \ddot{u} + \xi \dot{u} + k u = F(t) \quad e \rightarrow \{x, y, z\}$$

$$u_{(i,j)_{e(t)}} = \begin{bmatrix} u_{(1,1)} & \dots & u_{(4,j)} \\ \vdots & \ddots & \vdots \\ u_{(i,6)} & \dots & u_{(i,j)} \end{bmatrix}_{e(t)} \quad (1)$$

$$u_{(i,j)_{e(t)}} = \int_{t_0}^{t_a} \dot{u}_{(i,j)} d(t) \Big|_e$$

$$u_{(i,j)_{e(t)}} = \iint_{t_0}^{t_a} \ddot{u}_{(i,j)} d^2(t) \Big|_e$$

$$m_{(i,j)} = \begin{bmatrix} m_{(1,1)} & \dots & m_{(4,j)} \\ \vdots & \ddots & \vdots \\ m_{(i,6)} & \dots & m_{(i,j)} \end{bmatrix}$$

$$\xi_{(i,j)} = \begin{bmatrix} \xi_{(1,1)} & \dots & \xi_{(4,j)} \\ \vdots & \ddots & \vdots \\ \xi_{(i,6)} & \dots & \xi_{(i,j)} \end{bmatrix} \quad (2)$$

$$k_{(i,j)} = \begin{bmatrix} k_{(1,1)} & \dots & k_{(4,j)} \\ \vdots & \ddots & \vdots \\ k_{(i,6)} & \dots & k_{(i,j)} \end{bmatrix}$$

In each scenario there is an induced temporary load proportional to the mass, to the damping and the stiffness, in longitudinal, transverse and vertical direction [22], In addition, equation 1 determines the movement that governs the system [23], and the dynamic characteristics in matrix form are determined with equation 2, these are used to determine the kinetic energy (Ec) of each pedestrian in the time domain, shown in equation 3 and 4, likewise, the potential energy (Ep) determined in equation 5 and equation 6 of the temporal system and the periodic charge is determined with equation 7, as determined in [28].

C. Dynamic stimulation

The sample that provides energy and the movement of the center of gravity (Cg) will have generalized coordinates, whose displacement will be in the direction $(u_x, u_y, u_z)_{(t)}$ [29], horizontal and vertical, due to the energy used to reproduce the march of the pedestrian, this movement, to maintain the cycle, yields energy to the slab of the footbridge, which also partially dissipates by to damping $(\xi_{(i,j)_x}^e, \xi_{(i,j)_y}^e, \xi_{(i,j)_z}^e)_{(t)}$, [30].

One of the main characteristics of the loads produced by pedestrians is the low intensity [31]. When it is applied to structures with great mass and its high rigidity would hardly make them vibrate significantly [32], however, pedestrian bridges are light structures compared to other civil structures

[33], being designed and built with high sensitivity to dynamic loads [2][6].

The cyclic force is determined considering Table 1, of the sample, where, P-1, P-2, P-3, are of the feminine gender and the rest masculine [34], both maintain similar characteristics with respect to the dynamic load factor (DLF=0.371), the phase presents a value of $\phi_i = 1.57$, [35]

Table 1 Sample characteristics

No.	Stature [m]	Wp [N]	fp	Fp0	Wp(1,i)(t)	Wp(2,i)(t)	Wp(3,i)(t)	Wp(4,i)(t)
			[S/s]	[N]	[N/m2]	[N/m2]	[N/m2]	[N/m2]
P-1	1.52	530	1.441	713.27	142.65	356.64	713.27	1069.91
P-2	1.55	569	1.412	748.65	149.73	374.32	748.65	1122.97
P-3	1.61	608	1.441	818.25	163.65	409.12	818.25	1227.37
P-4	1.71	755	1.382	961.58	192.32	480.79	961.58	1442.38
P-5	1.74	804	1.529	1,097.34	219.47	548.67	1,097.34	1646.02
P-6	1.80	942	1.706	1,037.38	207.48	518.69	1,037.38	1556.08

In the present study a sample is taken with six pedestrians [36], where the values corresponding to the temporal force of stimulation are listed (Fp), which is determined with equation 8, as shown in Figure 5, and highlighting the role played by each of the structural parameters involved in the measurement process and portable instrumentation [34].

In general, the loads produced by pedestrians are variable, temporary, stochastic and generate periodic loads, calculated with equation 7, where the mass and the frequency of passage intervene. [37].

$$Ec_{(p,t)} = \begin{bmatrix} Ec_{x(p,t)} & 0 & 0 \\ 0 & Ec_{y(p,t)} & 0 \\ 0 & 0 & Ec_{z(p,t)} \end{bmatrix} \quad (3)$$

$$Ec_{e(p,t)} = \frac{1}{2} Fp_{(t)} (\dot{u}_{e(t)})^2 \quad (4)$$

$$Ep_{(p,t)} = \begin{bmatrix} Ep_{x(p,t)} & 0 & 0 \\ 0 & Ep_{y(p,t)} & 0 \\ 0 & 0 & Ep_{z(p,t)} \end{bmatrix} \quad (5)$$

$$Ep_{e(p,t)} = \frac{1}{2} K_p (u_{e(t)})^2 + Fp_{(t)} gh \cos(\theta_{e(t)}) \quad (6)$$

$$Fp_{(t)} = W_p + \sum_{i=1}^n W_p \alpha_i \text{sen}(2\pi f_p t - \phi_i) \quad (7)$$

D. Dynamic response of the structural system

The load $W_{P(i,j)(t)} (Tc_i, m_{P_j})_{(t)}$ As shown in figure 7, it represents the points A, B and C, which correspond to the idealization of the supports of the pedestrian bridge, as was done in [36], when incorporating boundary conditions, the displacements at the ends are nullified.

$$F_{P(i,j)(t)} = \left[W_{P_j} + W_{P_j} \alpha_{P_j} \text{sen}(2\pi f_{P_j}) \right]_{e(t)} \quad (8)$$

$$W_{P_j(t)} = \prod_{(i,j)(t_0)}^{(4,6)(t_a)} [\varphi_{i_{e(t)}}] [F_{P(i,j)(t)}] [\hat{e}]$$

$$[\varphi]_{e(t)}; [i = 1; \varphi(t) = 0.2], [i = 2; \varphi(t) = 0.5]$$

$$[i = 3; \varphi(t) = 1.0], [i = 4; \varphi(t) = 1.5]$$

When applying the method of superposition, two beams are identified, the first is a beam simply supported on A and C, with a uniformly distributed gravitational load [38], whose magnitude corresponds to the periodic load generated $Tc_{i_{e(t)}}$, with their respective densities of transit and displacement in the coordinate axes (e), whose mathematical scheme is shown in figure 7, and the second beam simply supported on A and C, with a point load formed by the magnitude of the redundant load $B_{i_{e(t)}}$, of the response in the support B[14].

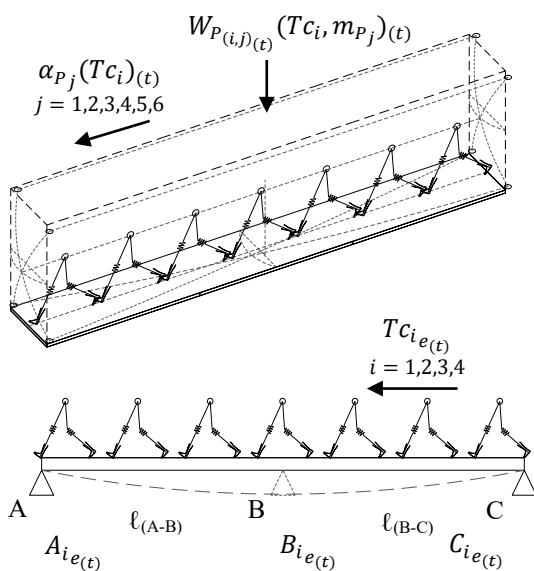


Figure 7 Graphical representation of the characteristics of the traffic temporary density.

The stimulation generated, three men and three women, $j = 1, 2, 3, 4, 5, 6$, are captured with the portable equipment [30], where the measurements collected represent the acceleration of the individual center of gravity [27], In addition, the instrumentation and measurement of the sample collects information for each pedestrian, controlling the average speed, with the traffic direction, $i = 1, 2, 3, 4$, as is done in [26], observing in figure 5 said control, determined with equation 8.

The sample passes through a walkway with two longitudinally continuous sections, with $l_{(A-B)} = 21.04$ m. y $l_{(A-B)} = 24.39$ m. of long correspondingly, both sections have, and $l_b = 2.20$ m. wide [2][13], whose, free body diagram is observed in figure 7, where the space $\varphi_{i_{e(t)}}$, used for each pedestrian traffic.

The analysis of the weight $W_{P_j(t)}$, of the human body, which forms a complex non-linear system, for structural engineering purposes it is treated as a simplified single system of two degrees of freedom, in the plane $e \rightarrow \{x, z\}$, $e \rightarrow \{y, z\}$ [26], however, the vibration specifically does not propagate in the same linearity, and the measurement of the instrument confirms

that the forces of the foot-gangway contact in a section affect the entire bridge, $e \rightarrow \{x, y, z\}$, where the step frequency varies directly proportional to the Stature [39].

The starting position $(i, j)_{(t_0)}$ it's random, likewise, the first step is random Tc_0 , which causes pedestrians to approach or leave, limiting themselves to $(i = 4, j = 6)_{(t_a)}$. So this aspect is controlled taking into account the traffic, Tc_i and the density of traffic D_i in case of approach and withdrawal, $Tc_i = D_i$, with the parameters of the equation 8 [34].

$$R_{p_{e(t)}} = R_{k_{e(t)}} + R_{f_{e(t)}} + R_{c_{e(t)}}$$

$$R_{k_{e(t)}} = \frac{1}{2} K_p (u_{a_{e(t)}} - u_{0_{e(t)}})^2$$

$$R_{f_{e(t)}} = F_{p_{e(t)}} g h_p \cos(\theta_{e(t)})$$

$$R_{c_{e(t)}} = \frac{1}{2} EI \int_{l_0}^{l_{\Delta x}} (u''_{e(t)})^2 d(e) \quad (9)$$

The analysis with equation 9, determines the reaction $R_{p_{e(t)}}$, by the amount of movement of the potential energy capable of balancing the system in each analysis scenario [22], for this rigidity is used $R_{k_{e(t)}}$, the force of action $R_{f_{e(t)}}$ and the restitution of the system $R_{c_{e(t)}}$.

E. Checking the structural system

The periodic study of the load varies permanently $Fp(t)$, likewise, in the present experiment, the magnitudes of the acting external forces are disclosed, so that this causes the interacting behavior to be predictable [40].

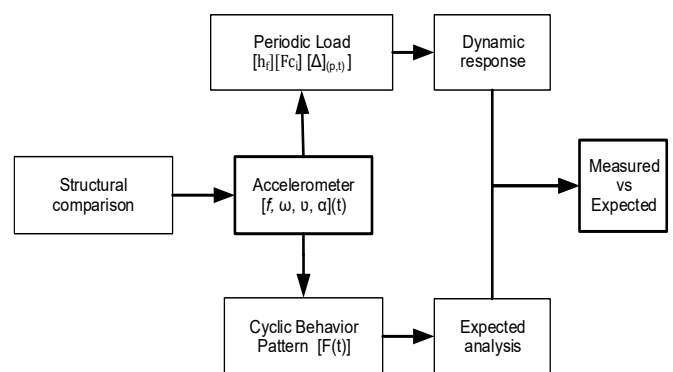


Figure 8 block diagram of the structural comparison of the system.

The initiation of a methodology of comparison of the experimental part and the analytical one is fundamental to assimilate the measurement of the dynamic response $a_{(i,j)_{e(t)}}$, this acceleration is the product of the stimulation and the configuration in which the system adapts to said response, on the other hand, the objective of determining the cyclical behavior pattern is primarily to use it to find an expected structural finding, and infer in its measured result[21].

$$m \ddot{u}_{(i,j)e(t)} + \xi \dot{u}_{(i,j)e(t)} + k u_{(i,j)e(t)} = F_{(i,j)e(t)} \quad (10)$$

$$\xi_{(i,j)e(t)} = \left[\frac{W_{p(i,j)}(Tc_i, m_{p_j}) \hat{e} - k_{p_j} u_{p_j} \hat{e} - m_{p_j} \ddot{u}_{p_j} \hat{e}}{\dot{u}_{p_j} \hat{e}} \right]_{e(t)} \quad (11)$$

$$\hat{e} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \|\hat{e}\| = \sqrt{x^2 + y^2 + z^2}$$

The kinetic energy that produces from the force $F_{p(i,j)e(t)}$, which is calculated with equation 8, and the potential energy $E_{p(i,j)e(p,t)}$, or response, $R_{p(i,j)e(t)}$, it is calculated with equation 9, where, in this last equation, the term of the damping, $\xi_{(i,j)e(t)}$, is determined with the equation 11, and equation 12,

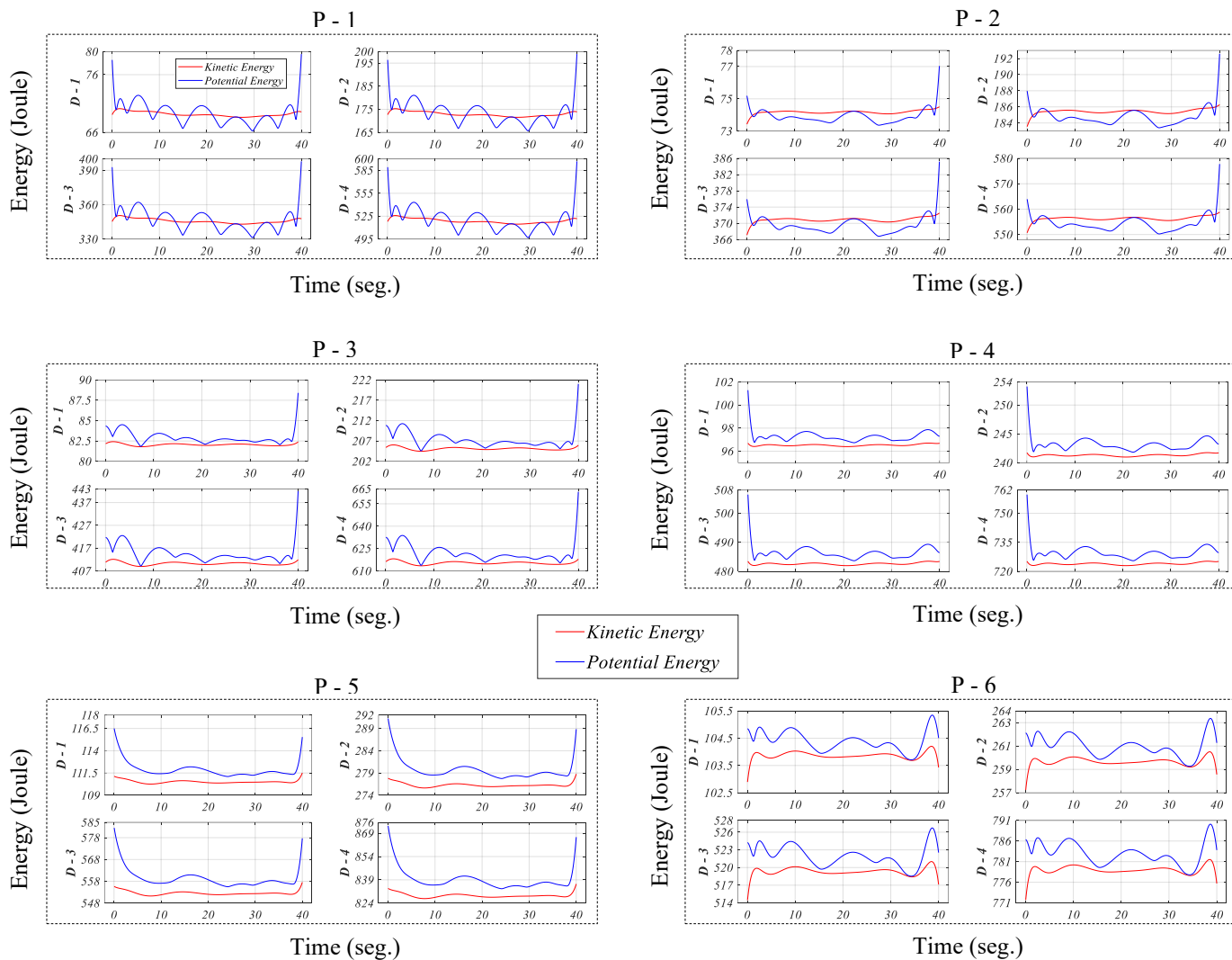


Figure 9 Graphical representation of the kinetic energy of the pedestrian and potential energy of the bridge.

To determine the pattern of cyclic behavior, the displacement variable obtained as a function of the increasing deformation in the catwalk was selected, this magnitude represents the amplitude [7].

If the mass, the speed and the step frequency remain constant, the periodic load produces the response of the gateway $R_{p(t)}$, that varies incrementally according to the mass moves away from the supports, thus, the magnitude of the structural dynamics of the system $F_{(i,j)e(t)}$, it are geted with the equation 10.

The instrument simultaneously measures the cyclic effects on the catwalk, which are taken to determine the expected analysis using equation 8, as shown in the figure 7.

$$R_{k(i,j)e(t)} = \frac{1}{2} K_p (u_{a(i,j)e(t)} \hat{e} - u_{0(i,j)e(t)} \hat{e})^2 \quad (12)$$

$$K_p = \left\| K_{p(i,j)e(t)} \hat{e} \right\|$$

The linear displacement $\Delta_{B i e(t)}$, in the redundant of the catwalk is proportional to the variation of potential energy, with respect to the kinetic energy [28], therefore, both magnitudes are determined in the whole experiment [34].

III. ANALYTICAL PROCESS

When determining the force and velocity in equation 4 and equation 6, the magnitude of the kinetic energy is obtained, $E_{c(i,j)e(p,t)}$ and potential energy $E_{p(i,j)e(p,t)}$, these magnitudes are observed in the graphical representation of figure 9, which are used from equation 13 and equation 14, to find the amount of work that the pedestrian in each scenario Tc_i of analysis generates as a result of its movement [28].

The concentration of forces resulting from the randomness of traffic is repetitive in the center of the clearing of the bridge [21], for this reason, the information obtained by the double integration method is analyzed [41], and the measured acceleration [22], using equation 11, therefore, based on said measured acceleration [38], after determining the damping represented in figure 10, where, in each graph the magnitude normalized by the largest one is represented, it is illustrated that the ratio of the pedestrian traffic speed to the speed measured by the portable instrument does not exist [34], where the force exerted by the cyclic periodic charge manifests itself as an underdamped system [42], proven this theory with figure 10, the friction is weak or nonexistent [6].

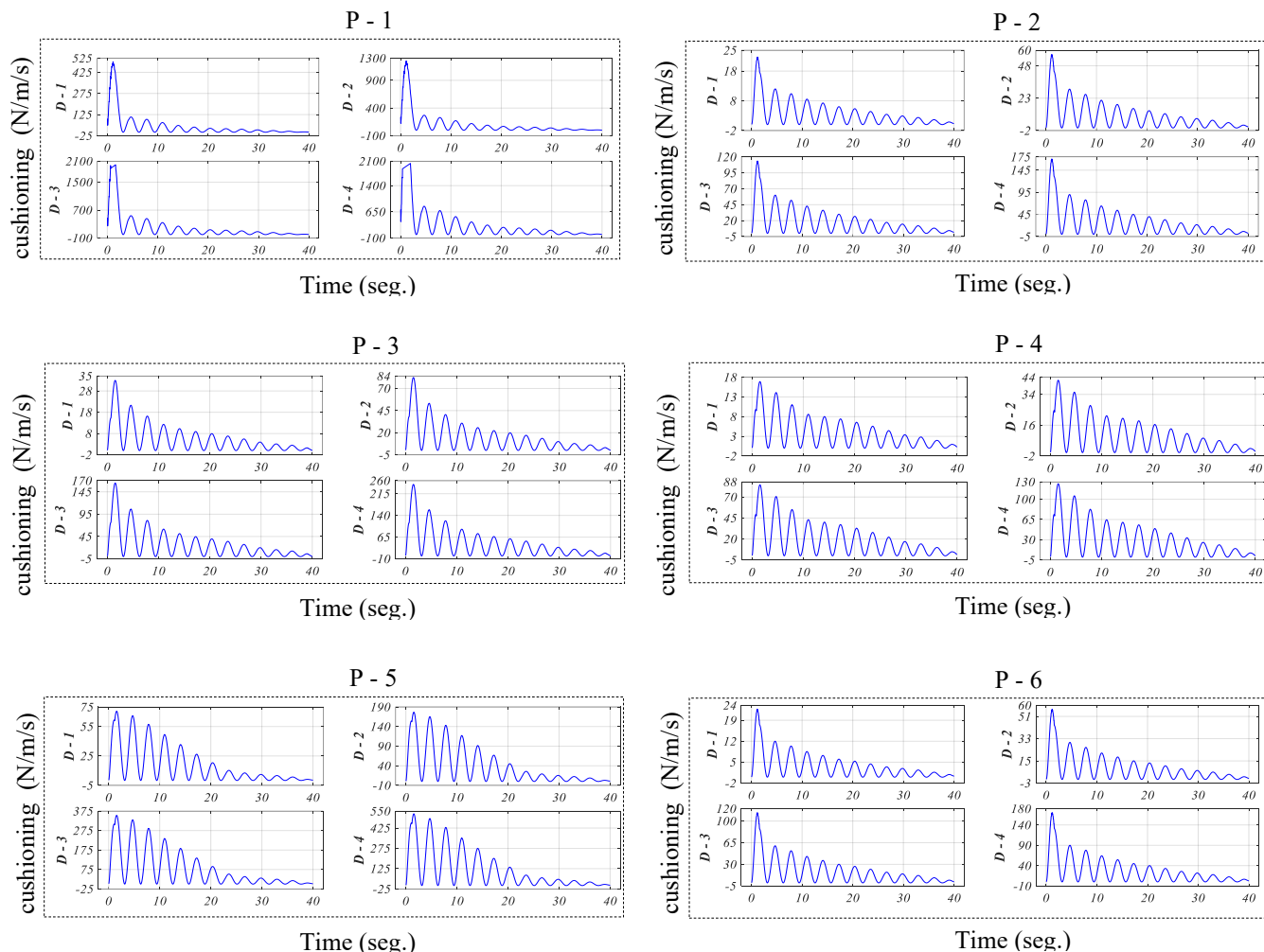


Figure 10 Graphical representation of the damping pattern on the walkway of the pedestrian bridge.

The amount of movement, $\xi \dot{u}_{(i,j)e(t)}$, in each scenario, it produces potential energy corresponding to the inertia, (I), and to the elasticity, (E), of the material that forms the footbridge of the bridge, calculating the restoration $R_{c e(t)}$ using the equation 9.

$E_{c(i,j)(t)} = \left\ \frac{1}{2} Tc_P(i,j) (\dot{u}_{p(i,j)})^2 \right\ _{e(t)} \quad (13)$
$E_{p(i,j)(t)} = \left\ R_{f(i,j)} + R_{c(i,j)} \right\ _{e(t)} \quad (14)$

The effect of apparent synchronization of the restitution of the footbridge will be through a positive weakening pattern, which infers in the return positions according to the initial conditions and to the diminished amplitude in the time domain, given for each pedestrian $[P_j]_{i=1}^4(t)$ and in each scenario Tc_i , the previous thing, it rejects the analysis to determine in the present work the degree of structural constriction, because, the elastic component of the maximum deformation expected for alternative load cycles, is more relevant than the plastic obtained from the material of the footbridge [43], when kinetic energy $E_{c(i,j)(t)}$

and potential energy $E_{p(i,j)(t)}$, are counteracted, then then, life to fatigue is controlled mainly by resistance $R_{f(i,j)e(t)}$ of the materials, determined with the equation 15 and for structural capacity of construction system [2] [4], as seen in the figure 11.

$$R_{f(i,j)e(t)} = F_{p(i,j)e(t)} g h_p \cos(\theta_{(i,j)e(t)} \hat{e})$$

$$F_{p(i,j)e(t)} = \left\| (Fp_{(i,j)x}, Fp_{(i,j)y}, Fp_{(i,j)z})(t) \hat{e} \right\| \quad (15)$$

deformation with the measured data $\delta_{m(i,j)(t)}$ and the expected deformation with deterministic data $\delta_{s(i,j)(t)}$.

B. Cyclic load Analysis

Deformations are reciprocal due to the selection of the scenario $i = 1,2,3,4$, n the movement evolution when the load produces displacements $(u_x, u_y, u_z)(t)$. Applying the force and substituting in the temporal equation as amplitude, indicated in the equation Asen (ωt) , we obtain a pattern in the behavior of the force.

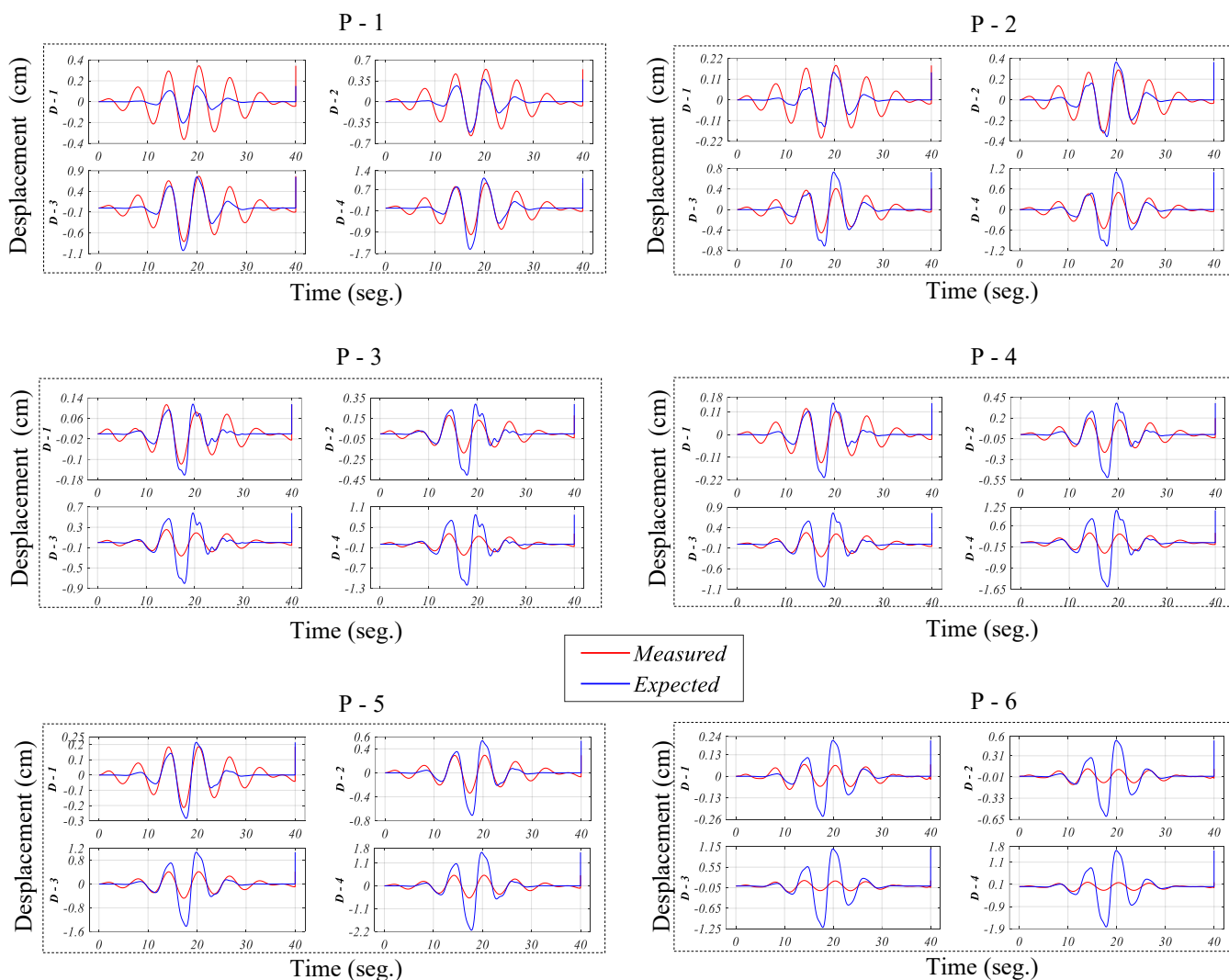


Figure 11 Graphical representation of the deformation obtained by the cyclic periodic load expected in each pedestrian, and that obtained by measuring the acceleration in the pedestrian's center of gravity.

A. Numerical simulation.

The frequency of passage allows the periodic load to have a constant value regardless of the weight of the pedestrian [1][43], and, on the other hand, the energy produced is absorbed by the catwalk and contained until such frequency allows it [6], so that the catwalk behaves permanently elastic [22].

The permanent elasticity allows to have effects corresponding to two degrees of freedom, and to border conditions [38], essentially compressed and stretched, taken from Figure 9, which applied in the equation 16 y 17, determines the

C. Expected analysis and structural check

On the surface of the footbridge of 45.43 meters in length and 2.20 meters in width, can pass in stages D1, D2, D3 and D4: 18, 32, 82 and 100 pedestrians correspondingly, under the conditions of Figure 2, said scenarios of traffic density [44], they transit stochastically and only the speed of the gait is controlled. The step synchronization is random, for this reason the possibility of repetition of the test is statistically high, whether the step frequency characteristic is controlled or not.

$\delta_{m(i,j)(t)} = 2 \left\ \left\ \frac{R_{p(i,j)} - R_{f(i,j)}}{E I} \right\ \right\ _0^{Lx} \quad (16)$	
$\delta_{s(i,j)(t)} = 5 \left\ \left\ \frac{F P_{(i,j)(t)}^e t^4}{384 E I} \right\ \right\ _0^{Lx} \quad (17)$	

The importance of the damping in the dynamic system of the bridge proposes that the number of oscillations in a typical decay time be uniform, in all scenarios, as shown in Figure 10, in such a way that the continuous interacting system is qualified trustworthy.

The comfort of walking is lost with traffic, and it is expected that the effects of density increase, observing figure 10, the pedestrian Number 1 (P-1), generates the effects in magnitude of damping too high with respect to other pedestrians, this is because their walking produces very small speeds captured by the same portable instrument that measures the walk of the total sample, when the damping is determined (c), of equation 7, the numerator is the derivative of the displacement with respect to time, which potentiates the resulting value observed in graphs.

The simulation of figure 11 presents the analysis of the expected deformations, according to the cyclic loads, considering the damping present in figure 10, comparing said results, with those obtained from the present energy stimulation as seen in figure 9, of the data obtained from the portable instrument.

IV. CONCLUSIONS

In this research work was made the measurement of the traffic of the sample, walking on the surface of the footbridge, as well as the movement of the mass concentrated in the center of gravity of the pedestrian, in both cases the deformation of the footbridge was determined, applying the methodology of the double integration and the coupling of the equation of movement through Lagrange respectively. Obtaining results by implementing the methodology concludes with the following advantages:

1. The magnitude of the potential energy is greater than the kinetic energy, which corresponds to discern, on the elastic structural capacity, then then, a behavior is summarized where the deformation is not permanent because they are below the elastic limit.
2. When the magnitude of the amplitude in all instants is the value of the mass, the responsiveness of the gangway, replaces the deformation with damping, which, by combining the stimulating and restoring effects of elasticity, transform the energy provided, in cushioned movement, directly dependent on the physical characteristics of the pedestrian and its locomotor movement.
3. The behavior against the deformation of the materials of the footbridge, which are unloaded, from a certain elasto-plastic state, depends on the maximum deformation present in the center of the stretch $B_{i_e(t)}$ of the bridge, these deformations are representative of low cycle effect fatigue, and present a degree of structural constriction less than the capacity of the maximum deformation, product of the cyclic load.

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