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# Structural Topology Optimization using Integrated Multi-Point Approximation

Abdurahman M. Hassen [1], Mohammed A. Hjaji [2]
Mechanical and Industrial Engineering
University of Tripoli
Tripoli, Libya

Hai Huang
Mechanics of Materials
BUAA
Beijing, China

***Abstract*—The two-level multipoint approximation concept was successfully combined with genetic algorithm. The topology variables of the trusses are optimized, through genetic algorithm in the external layer of the first level approximation, while the cross-sectional areas of bars are optimized in the internal layer, which is solved by the dual method in the second level approximation. To avoid singularity of the multipoint approximation, the new study Integrated Multipoint Approximation is proposed, using two approximate functions in two different specified domains, the new function to be used for both topology and sizing. Its accuracy is already studied, results were satisfying, for both topology optimization and sizing; examples of truss structure are demonstrated to show the validity and the efficiency of the proposal.**

***Keywords—Structural Optimization, Multi-point approximation, genetic algorithms, Topology Optimization***

## I. INTRODUCTION

Generally, on discrete structures such as trusses, the topology optimization is concerned with finding an optimal configuration of structure, within a specified domain. The weight of a structure is often taken as objective function. The most difficulty in such problem is that there may exist many local optimal solutions, as well as a singularity problem. The two-level multipoint approximation concepts were successfully combined with genetic algorithm, (GA) [1]. Hajela & Lee [2] developed an approach based on a two level genetic algorithm; in one level they satisfy kinematics stability constraints, followed by response constraints at the second level, to generate near optimal structural topologies. Global search algorithm by Ringertz [3] based on the branch and bound algorithm, effective for a problem with multiple local optimal solution. Sankaranarayana & Haftka [4] used the simultaneous analysis and design (SAND) approach. The SAND approach treats the equilibrium equations as equality constraints, with the nodal displacements used as design variables, in addition to the cross sectional areas of truss members, as a result of the method the design variables increase substantially. KanGAL [5] used GA based optimization, with fixed-length vector of design variables, representing member areas and change in nodal coordinates, this mostly leading to near optimum. Sakamoto [6] used hybrid method composed by the genetic algorithm, to optimize the layout and the cross-sectional area of truss members, but this method not suitable for large structures, because required a large number of function evaluations and structural analysis. In the recent work [7], an exponent modified function is introduced to original MA; GA and two-level multipoint approximation (MA) by Huang [1] are coupled. The method is to process the multi-point approximate function into two levels, with a layered optimization strategy. The topology variables of the trusses are optimized, through GA in the external layer, of the first-level approximation that avoids the use of repeated finite element analysis, while the cross-sectional areas of bars are optimized in the internal layer, which is solved by the dual method, in the second level approximation. The original MA concept shows high quality approximation in sizing, results of application examples are highly competitive, but singularity may take place, when design variable approaches to zero, and adaptive parameters are negative values. In this study, the integrated multipoint approximation (IMA) is proposed. IMA uses two approximate functions (MA and its modified), each of them implemented in different specified domain, to avoid the singularity of the MA, and to gain the best features of both approximate functions; as a result, the high quality of IMA is increased, as unity function to be used for topology and sizing. A series of classical truss examples including some ground truss structure are used to verify the efficiency of the proposal.

## II. IMA FUNCTION USED IN STRUCTURAL TOPOLOGY OPTIMIZATION

First, IMA [8] is a result of gathering the original MA with its modification, in one integrated model that can be reliable and effective, for both sizing and topology optimization. The accuracy of the proposed IMA was tested, through a series of explicit and implicit functions [8]. The results were comparable, and concluded that IMA can integrate the accuracy of original MA in a domain $x_i \epsilon (x_{it}, x_u]$, and that of the modified MA (MMA) in a domain $x_i \epsilon [0, x_{it})$; IMA can also have its independent accuracy, when some components from both domains are shared in one iteration. Where $x_i$ is the design variable; $x_{it}$ is the expansion point, and $x_u$ is the upper limit. At p-th stage of the first-level approximate problem is presented as follows: -

$$\begin{cases} \quad find \quad X = \{x_1, x_2,,, x_n\}^T \\ \quad\quad \alpha = \{\alpha_1, \alpha_2,,, \alpha_n\}^T \\ \quad \min \quad \mathrm{f(x)} = \sum_{i=1}^{n} f_i(X) \\ St \quad \tilde{g}_j^{(p)}(X) \le 0 \qquad j = 1,,, J_1 \\ \alpha_i x_{i(p)}^L + (1-\alpha_i) x_i^b \le x_i \qquad i = 1,,, n \\ x_i \le \alpha_i x_{i(p)}^U + (1-\alpha_i) x_i^b \\ \quad \alpha_i = 0 \quad or \quad \alpha_i = 1 \\ \quad x_{i(p)}^U = min\{x_i^U, \tilde{x}_{i(p)}^U\} \\ \quad x_{i(p)}^L = max\{x_i^L, \tilde{x}_{i(p)}^L\} \end{cases} \tag{1}$$

Where X and α are the vectors of cross-sectional size variables, and topology variables, respectively; $j_1$ is the number

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of active constraint; n is the group number of linked bars; $x_{i(p)}^U$ and $x_{i(p)}^L$ are the upper and lower bounds of the size variables; $x_i^b$ is a small value to substitute the cross sectional size of the removed bar; $f_i(X)$ is the weight of bars in a group of $\alpha_i$; $\tilde{x}_{i(p)}^U$and $\tilde{x}_{i(p)}^L$ are the move limits; $\tilde{g}_j^{(p)}(X)$ represents the approximated constraint function, which is stable even $x_i$ reaches zero. The functions are summarized here as follows: -

$$\tilde{g}_j^{(p)}(X) = \sum_{t=1}^{H} \{g(X_t) + \tilde{g}_{IMA}(X)\} h_t(X) \qquad (2)$$

Where,

$$\tilde{g}_{IMA}(X) = \tilde{g}_{MA}(X) + \tilde{g}_{MMA}(X) \qquad (3)$$

$$\begin{cases} \tilde{g}_{MA}(X) = \dfrac{1}{r_{to}} \displaystyle\sum_{\substack{i=1 \\ i \neq k}}^{n} \dfrac{\partial g(X_t)}{\partial x_i} X_{it}^{1-r_{to}} \left( x_i^{r_{to}} - x_{it}^{r_{to}} \right) \\ where, \qquad x_i \geq x_{it} \\ \tilde{g}_{MMA}(X) = \dfrac{1}{r_{tM}} \displaystyle\sum_{\substack{k=1 \\ k \neq i}}^{n} \dfrac{\partial g(X_t)}{\partial x_i} \left( 1 - e^{-r_{tM}(x_k - x_{it})} \right) \\ and \qquad x_k < x_{it} \end{cases} \qquad (4)$$

Where, $x_{it}(t = 1, \dots, H;\ i = 1, \dots, n)$ are the known points; H is the number of points to be counted; n is the number of design variables in a point; $g(X_t)$ is the function values. And, $h_t(X)$ is the weighting function, which can be determined as: -

$$h_t(X) = \frac{\bar{h}_l(X)}{\sum_{l=1}^{H} \bar{h}_l(X)} \quad , \quad t = 1, \dots, H \qquad (5)$$

$$\bar{h}_l(X) = \prod_{\substack{S=1 \\ S \neq l}}^{H} (X - X_S)^T (X - X_S) \quad , \quad l = 1, \dots, H \qquad (6)$$

The exponent $r_{to}$ and $r_{tM}$ are the adaptive parameters, to control the non-linearity of the approximation, to be found from the following equations respectively: -

$$f(r_{to}) = g(X_H) - \left[ g(X_t) + \frac{1}{r_{to}} \sum_{i=1}^{n} \frac{\partial g(X_t)}{\partial x_i} x_{it}^{1-r_{to}} \left( x_{iH}^{r_{to}} - x_{it}^{r_{to}} \right) \right] \qquad (7)$$

$$f(r_{tM}) = g(X_H) - \left[ g(X_t) + \frac{1}{r_{tM}} \sum_{i=1}^{n} \frac{\partial g(X_t)}{\partial x_i} \left( 1 - e^{-r_{tM}(x_{iH} - x_{it})} \right) \right] \qquad (8)$$

Where, $X_H$ is the present point; usually a domain for $r_{to}$ & $r_{tM}$ $\in [a, 0)$ $and$ $r_{to}$ & $r_{tM}$ $\in (0, b]$ should be given, $a$ & $b$ are lower and upper limitations for $r_{to}$ and $r_{tM}$. In this study the upper and lower limits are defined as –3.5 and 3.5 respectively.

*A. The layered strategy*

Because of mixed variables, a layered strategy was introduced. The topology variables of the trusses are optimized using GA technique, in the external layer, where the finite element is avoided; using this technique, the problem in (1) is transferred into minimum problem with penalty function R.

$$\min F_1 = (X) + R \sum_{j=1}^{j_1} \left[ max \left( \tilde{g}_j^{(p)}(X), 0 \right) \right]^2 \qquad (9)$$

Only the topology variables α are optimized in this layer. Then the problem transferred into internal layer, where the cross-sectional areas of the topology bars are optimized, through second-level approximation, and solved by the duel method.

## III. NUMERICAL EXAMPLES

Before different examples from literature are chosen to demonstrate the validity and to compare the efficiency of the IMA. With the most well-known examples 10-bar and the 72-bar trusses, the comparison is made on two aspects, cross-sectional sizing, and topology optimization. Other examples are ground truss structure; results were compared on topology optimization.

*A. Example 1. The Ten-Bar Truss*

For the structure response the 10-bar truss fig. 1. The optimized Ten-bar truss is shown in fig. 2. The iteration history data of topology as well as sizing are tabulated in table I. Comparison is made with [7] for topology optimization, and for sizing with [1]. Table II shows the final optimum design variable results. Apparently, results for both aspects are clearly mutual. Moreover, present study has less iteration for topology.

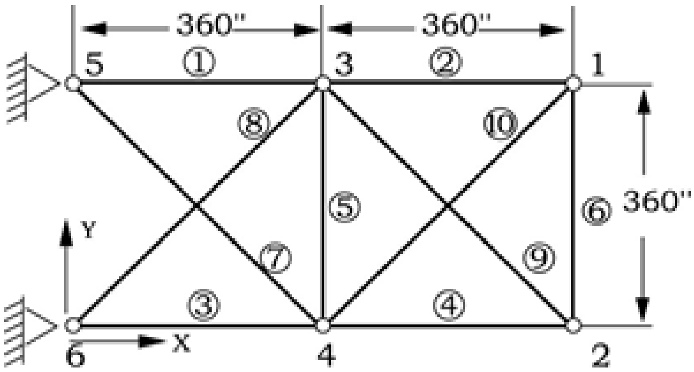


Fig. 1. The Ten Bar Truss

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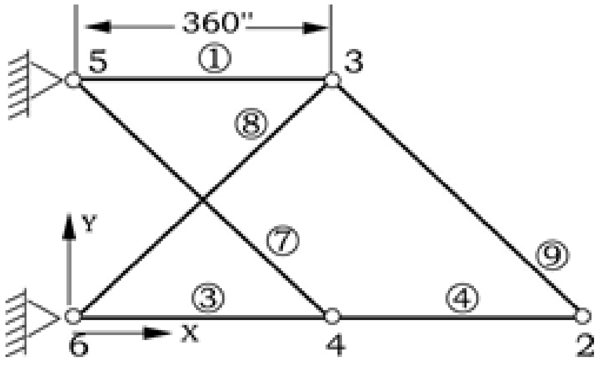


Fig. 2. The Optimized Ten-Bar Truss

TABLE I. ITERATION HISTORY DATA FOR 10-BAR TRUSS

| No. Of Analysis | Weights (lbs) | | | |
|---|---|---|---|---|
| | Present study | | Ref. [1] | Ref. [7] |
| | Sizing | Topology | Sizing | Topology |
| 1 | 12589.40 | 12589.403 | 8266.1 | 12589.4 |
| 2 | 3963.83 | 3858.319 | 6061.3 | 6082.559 |
| 3 | 6682.63 | 4813.135 | 5816.3 | 5687.487 |
| 4 | 5959.35 | 6041.306 | 5482 | 5215.432 |
| 5 | 5928.46 | 5970.800 | 5540.1 | 4851.243 |
| 6 | 5798.99 | 5891.545 | 5106.2 | 4816.312 |
| 7 | 5579.61 | 5721.890 | 5262 | 4882.064 |
| 8 | 5252.91 | 5407.933 | 5076.9 | 4867.044 |
| 9 | 5130.36 | 5229.040 | 5065.1 | 4884.602 |
| 10 | 5100.00 | 5028.617 | 5075.1 | 4873.683 |
| 11 | 5074.42 | 4910.501 | 5062.7 | 4881.59 |
| 12 | 5068.69 | 4928.812 | 5067.4 | 4896.1 |
| 13 | | 4898.899 | | 4895.039 |
| 14 | | 4899.682 | | 4897.345 |
| … | | | | … |
| 19 | | | | 4899.39 |

TABLE II. FINAL OPTIMUM DESIGN VARIABLE RESULTS, FOR10-BAR TRUSS.

| No. Of Variables | Final Optimum Design (in$^2$) | | | |
|---|---|---|---|---|
| | Present study | | Ref. [1] | Ref. [7] |
| | Sizing | Topology | Sizing | Topology |
| 1 | 30.79 | 30.1107 | 30.62 | 30.2897 |
| 2 | 0.0935 | 0 | 0.1 | 0 |
| 3 | 23.154 | 22.1317 | 23.28 | 21.4207 |
| 4 | 15.086 | 15.0522 | 15.13 | 15.1451 |
| 5 | 0.161 | 0 | 0.1 | 0 |
| 6 | 0.670 | 0 | 0.529 | 0 |
| 7 | 7.3 | 6.0724 | 7.503 | 6 |
| 8 | 21.327 | 21.2948 | 21.1 | 21.4184 |
| 9 | 21.334 | 21.2871 | 21.4 | 21.4184 |
| 10 | 0.130 | 0 | 0.1 | 0 |

| No. Of Analysis | 12 | 14 | 12 | 19 |
|---|---|---|---|---|
| Weights [lbs] | 5068.69 | 4899.68 | 5067.4 | 4899.39 |

## B. *Example 2. The 72-Bar Truss*

For the structure response the 72-bar truss fig. 3. The Optimized 72-bar truss is shown in fig. 4. The iteration history data are tabulated in table III, and the final optimum design variable results are in table IV. Clearly, the results are mutual, from the point of view of the function value for both cross section sizing and topology. The present study has better number of analysis for topology, but for cross-section sizing a little higher than [1], also the final optimal design variable results have considerable precise agreement.

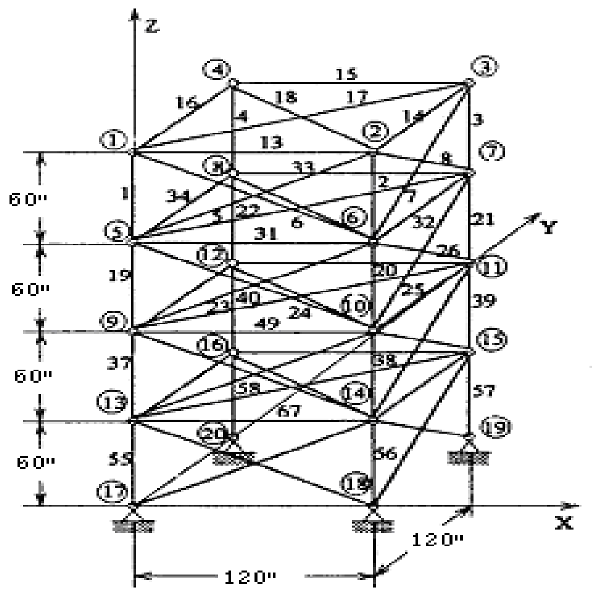


Fig. 3. The 72-Bar Truss

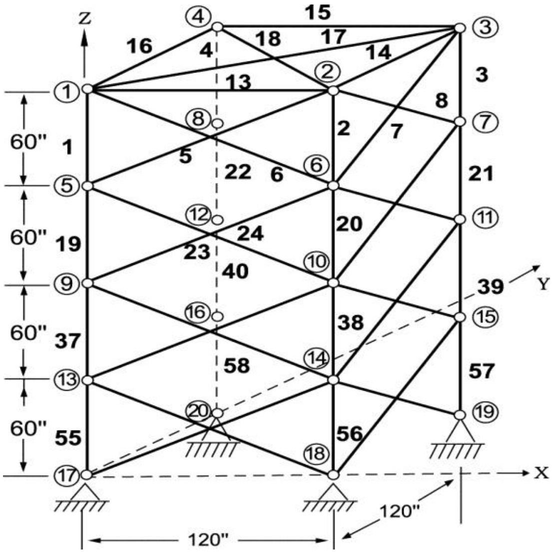


Fig. 4. The optimized 72-Bar Truss

TABLE III. ITERATION HISTORY DATA FOR 72-BAR TRUSS

| No. Of Analysis | Weights (lbs) | | | |
|---|---|---|---|---|
| | Present study | | Ref. [1] | Ref. [7] |
| | Sizing | Topology | Sizing | Topology |
| 1 | 853.09 | 853.09 | 656.77 | 853.09 |
| 2 | 345.19 | 154.68 | 386.4 | 650.83 |
| 3 | 558.57 | 267.28 | 368.17 | 518.23 |
| 4 | 428.51 | 386.88 | 364.82 | 452.33 |
| 5 | 409.39 | 322.48 | 364.69 | 438.97 |
| 6 | 393.32 | 339.74 | | 420.77 |
| 7 | 384.64 | 352.56 | | 414.75 |
| 8 | 388.69 | 356.75 | | 403.64 |
| 9 | 375.11 | 365.11 | | 360.90 |
| 10 | 370.01 | 366.12 | | 327.15 |
| 11 | 367.42 | 362.93 | | 375.01 |
| 12 | 365.85 | 365.95 | | 368.34 |
| 13 | 364.69 | 362.41 | | 362.54 |
| 14 | | 364.12 | | 360.64 |
| … | | … | | 362.64 |
| 20 | | 362.36 | | 362.58 |
| 21 | | | | 362.35 |
| … | | | | … |
| 28 | | | | 362.30 |

TABLE IV. FINAL OPTIMUM DESIGN VARIABLE RESULTS FOR 72-BAR TRUSS

| No. Of Variables | Final Optimum Design (in$^2$) | | | |
|---|---|---|---|---|
| | Present study | | Ref. [1] | Ref. [7] |
| | Sizing | Topology | Sizing | Topology |
| 1 | 0.168 | 0.167 | 0.158 | 0.167 |
| 2 | 0.535 | 0.535 | 0.537 | 0.535 |
| 3 | 0.434 | 0.452 | 0.412 | 0.452 |
| 4 | 0.593 | 0.571 | 0.562 | 0.572 |
| 5 | 0523 | 0.519 | 0.508 | 0.519 |
| 6 | 0.519 | 0.517 | 0.520 | 0.517 |
| 7 | 0.0219 | 0 | 0.1 | 0 |
| 8 | 0.0747 | 0.129 | 0.1 | 0.128 |
| 9 | 1.285 | 1.29 | 1.280 | 1.293 |
| 10 | 0.516 | 0.517 | 0.515 | 0.517 |
| 11 | 0.0213 | 0 | 0.1 | 0 |
| 12 | 0.0155 | 0 | 0.1 | 0 |
| 13 | 1.892 | 1.885 | 1.899 | 1.8846 |
| 14 | 0.516 | 0.517 | 0.516 | 0.517 |
| 15 | 0.0213 | 0 | 0.1 | 0 |
| 16 | 0.0155 | 0 | 0.1 | 0 |
| No. Of Analysis | 13 | 20 | 5 | 28 |
| Weights [lbs] | 364.695 | 362.364 | 364.69 | 362.302 |

*C. Example 3. The Ten-Node, 2D Truss ground structure*

IMA is applied to the ten-node truss ground structure fig. 5, with ground structure of all possible interconnection a total of 34 members; parameters details in [5].

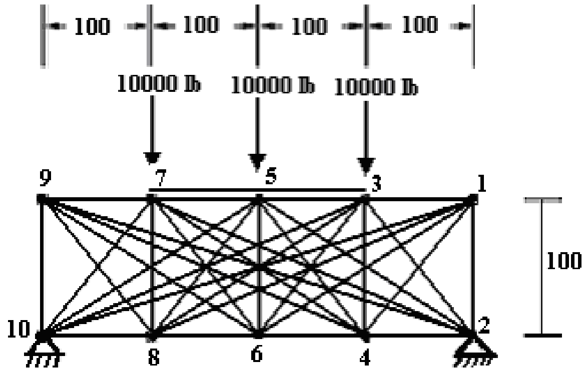


Fig. 5. Ten-Node truss ground structure

The Optimized Ten-node truss ground structure is shown in fig. 6, as well as the optimized solution from [5]. For the objective function and cross-sectional area of members of the optimized truss are tabulated in table V. The optimized solution fig. 6 shows present study has almost same topology as [5], the number of members are less from those in [5], and no overlapping members as they do in their optimum solution. The cross-sectional areas are different for those overlapping members, but are almost identical for others, and the objective function is comparable.

TABLE V. CROSS-SECTIONAL AREA OF THE OPTIMIZED TEN-NODE TRUSS GROUND STRUCTURE

| Member No. | Cross-sectional area (in$^2$) | |
|---|---|---|
| | Present study | Ref. [5] |
| 1 | 0.446612343 | 0.477 |
| 2 | 0.446612343 | 0.477 |
| 3 | 0.565546022 | 0.566 |
| 4 | 0.565546022 | 0.566 |
| 5 | 0.399648979 | 0.082 |
| 6 | 0.399648979 | 0.082 |
| 7 | 0 | 0.321 |
| Weights [lb] | 44.2708 | 44.033 |

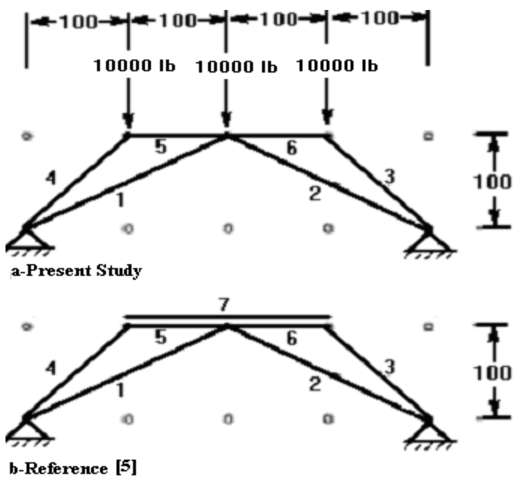


Fig. 6. The Optimized Ten-Node Truss ground structure

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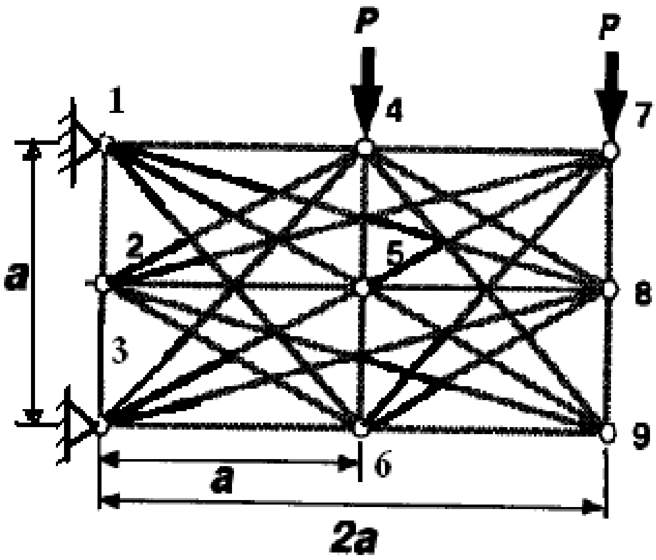
### *D. Example 4. The ground structure of a 9-nodes Truss*

The truss ground structure of nine nodes is shown in fig. 7, for details see [6]. The Optimized Nine-node truss ground structure is shown in fig. 8, as well as the topology-optimized solution from [6], for the objective function and cross-sectional areas of members of the optimized truss are tabulated in table VI. The topology-optimized solution fig. 8 shows the proposed IMA has same topology as [6], and the cross-sectional areas of the members are slightly different.




The Nine-Node Truss ground structure

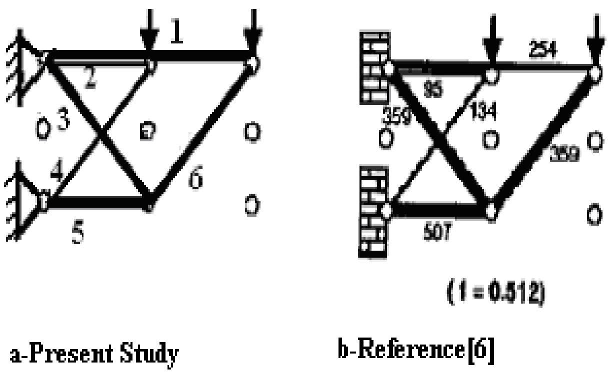



Fig. 7. The Optimized Nine-Node Truss ground structure

TABLE VI. CROSS-SECTIONAL AREA OF THE OPTIMIZED NINE-NODE TRUSS GROUND STRUCTURE

| Member No. | Cross-sectional area ($mm^2$) | |
|---|---|---|
| | Present Study | Ref. [6] |
| 1 | 264.156 | 254 |
| 2 | 105.1604 | 95 |
| 3 | 379.5246 | 359 |
| 4 | 137.4901 | 134 |
| 5 | 539.5367 | 507 |
| 6 | 377.2747 | 359 |
| Weights [kg] | 0.5459 | 0.502 |

## IV. CONCLUSION

Apparently and as discussed in each example the results show the proposed IMA is very satisfying. First from the classical 10-bar, and 72-bar trusses examples bring out that the IMA results, compared with the published one are comparable and satisfying, for both topology optimization as well as cross sectional sizing, also it is noticeable that the proposed method has less iterations for topology analysis. Moreover, for the ground trusses structures results assure that the IMA results are satisfying and comparable for topology optimization as seen from the optimized figures the proposed method has very good fitting with published results with no overlapping. However, the IMA can be very useful for both sizing and topology optimization.

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