Super Efficiency with 2- Stage DEA Model

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***Abstract*—DEA model estimate a set of evaluated DMU and use to estimate the efficiency score by evaluating each DMU in a data set. This research determined the new scheme of 2-stage DEA model analysis in obtain of new efficiency score, called super efficiency DEA model. We then extended the DEA model that was formulated by considering input-ouput oriented for each used data. The model formulated by a linear program and gives three major solutions: (1) an alternative new scheme of the 2-stage of DEA model, (2) super efficiency scores for a given data and (3) DMU ranking based on each its super efficiency score.**

Keywords—Data Envelopment Analysis (DEA), linear program, super efficiency, ranking

I. INTRODUCTION

Data Envelopment Analysis (DEA) model is a method that used to estimate a frontier to evaluate the performances or the efficiency of all of the entities that are to be evaluated. In some previous studies on 2-stage DEA model was developed into some of applications. Banker and Natarajan [1] developed 2-stage DEA model by using linear regression analysis, Monte-Carlo simulation, that obtained 2-stage DEA estimator for a certain variable context with definite constraint in input vector on model. Simar and Wilson [2] extended the maximum likelihood method in order to determine 2-stage regression into DEA model that produced a DEA estimator for 2-stage DEA model as a result.

Andersen et al. [3] developed a radial super-efficiency measure called AP model. This model was comparing the DMU that evaluated with a linear combination of other DMUs, while excluding the observations of the DMU being evaluated. This model then was extended by Tone [4] by considering the input-output slacks of a non-radial super efficiency called SBM model. Castelli et al. [5] proposed a comprehensive categorized overview of methods and models for different multi-stage production architectures in DEA model. Seiford and Shu [6] was studied a production process in banking sector by treating the two stages, independently, without assuming any relationship between the two stages. A novel approach then developed by Kao and Hwang [7] that consider a series relationship of the two stages and provide a model that estimates the overall efficiency of the production process. This approach is based on the reasonable assumption that the immediate measures of the value is same without consider whether as outputs or input in the first stage. Chen et al. [8] then introduced the additive efficiency decomposition in two-stage process under the assumption of series relationship. Their modeling approach facilitates the linearization of a non-linear mathematical program and based on the assumption that the weight of the two stages is extendable to Variable Returns to Scale (VRS) assumption.

In this paper we present an alternative new scheme of the 2-stage under the assumptions of the series relationship between the two stages. We then formulated the model by considering input-output oriented for each given data. We select some inputs orientation that produce the efficiency score for the first stage, e^1. Then, outputs orientation for the second stage that produce e^2, the efficiency score for the second stage. Thus, the overall efficiency score can obtained by a simple division rule.

This paper unfolds as follows. Section II we review some background information on the study area and outline that adopted the 2-stage analysis. Section III we propose an alternative new scheme of 2-stage DEA model. In Section IV, we presents the result in comparison of the new model with a given set data studied by Wang et al. [9]. Finally, conclusion and future research are provided in the last section. The result of super efficiency scores and DMU ranking also provided in the Appendix section.

II. DATA ENVELOPMENT ANALYSIS (DEA)

DEA model can be shown in two general forms, linear program and linear regression form. DEA model using linear program method which the weighted score as decision variable and produce the efficiency score for each Decision Making Units (DMUs) as solutions of DEA model (see Seiford and Thrall [10]; Lovell [11]; Cooper et al. [12] and Thanassoulis [13]). Charnes et al. [14] showed that DEA is a multi-factor productivity analysis model that used to estimate the efficiency relative score (E) of homogeny set of DMU that formulated as

$$E = \frac{\text{sum of output weight}}{\text{sum of input weight}} \times 100\% \qquad (2.1)$$

Assume n evaluated DMUs, $\text{DMU}_j\,(j = 1, \dots, n)$ for each using m inputs, $x_{ij}\,(i = 1, \dots, m)$, that produce r outputs, $y_{kj}\,(k = 1, \dots r)$, respectively. The efficiency score for DMU l can be defined as sum of all outputs weight divided by sum of all inputs weight where single input, $v_i(i = 1, \dots, m)$ and single output, $u_k(k = 1, \dots, r)$. Mathematically, it can be formulated as follows

$$\theta_l = \frac{\sum_{k=1}^{r} u_k y_{kl}}{\sum_{i=1}^{m} v_i x_{il}} \qquad (2.2)$$

DEA model estimate a set of evaluated DMU and use to estimate the efficiency score by evaluating each n DMU in a data set. It is done by estimating a frontier point which gives interval of efficiency score, $0 \le E \le 1$ to each n evaluated DMU. This efficiency score was obtained by comparing DMU's performances to all of evaluated DMU's performances in a certain data set. The obtained efficiency score by using DEA model gives the highest efficiency relative to interval $0 \le E \le 1$ for each $\text{DMU}_j\,(j = 1, \dots, n)$.

As linear regression form, DEA model is a nonparametric tool in analyzing efficiency score with multiple inputs and outputs that consider both qualitative and quantitative in a data set. Also, DEA is a linear programming model that calculate multiple inputs and outputs and evaluate DMUs both qualitatively and quantitatively by a linear program form. Generally, there are two basic DEA model as follows.

A. Charnes, Cooper and Rhodes (CCR) Model

DEA model as originally proposed by Charnes et al. [14] namely Charnes, Cooper and Rhodes (CCR) model to produce the efficiency frontier based on concept of Pareto optimum. This model was built on the assumption of Constant Returns to Scale (CRS) for the production frontier in the single input and single ouput case. More generally, this model assumed that the production possibility set

$$P = \{(x, y) \mid x \ge X\lambda, y \le Y\lambda, \lambda \ge 0\}$$

with the pairs of positive inputs and outputs vectors, $(x_j, y_j)(j = 1, \dots, n)$ belongs to P of n DMUs. Thus, we can assume that such a pair of semi positive input-outputs, $x \in R^m$ and $y \in R^n$. Charnes et al. [14] developed CCR basic model input oriented DEA which contains of objective function, maximizing DMU_lefficiency score with constraint that efficiency score for all DMU less than or equal to 1 as follows

$$\max \sum_{r=1}^{s} u_r y_{ro} \qquad (2.3)$$

$$\text{s. t} \quad \sum_{i=1}^{m} v_i x_{io} = 1 \qquad (2.4)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (2.5)$$

for $j = 1, \dots, n$; $r = 1, \dots, s$; $i = 1, \dots, m$ and $u_k, v_i \ge 0$. For (x,y) in P, any semi positive activity $(\bar{x}, \bar{y})$ with $x \ge \bar{x}$ and $\bar{y} \le y$ is included in P. Thus, any activity with input of no less than x in any component and with output no greater than y in any component is feasible.

B. Banker, Charnes and Cooper (BCC) Model

The extended of CCR model then studied by Banker et al. [15]. This model has its production frontiers spanned by convex hull of existing DMUs that leads to Variable Returns to Scale (VRS) assumption characterizations. Banker et al. [15] published the BCC model whose production possibility set which defined by

$$P = \{(x, y) \mid x \ge X\lambda, y \le Y\lambda, e\lambda = 1, \lambda \ge 0\}$$

where $X = (x_j) \in R^{m \times n}$ dan $Y = (y_j) \in R^{s \times n}$ are a given data set, $\lambda \in R^n$ and e is a row vector with all elements equal to 1. The BCC model differs from the CCR model that developed by adding convexity constraint $\sum_{j=1}^{n} \lambda_j = 1$. Mathematically, BCC model can be formulated as follows

$$\max \sum_{r=1}^{s} u_r y_{ro} + w \qquad (2.6)$$

$$\text{s. t} \quad \sum_{i=1}^{m} v_i x_{io} = 1 \qquad (2.7)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + w \le 0 \qquad (2.8)$$

for $j = 1, \dots, n$; $r = 1, \dots, s$; $i = 1, \dots, m$ and $u_k, v_i \ge 0$.

III. 2-STAGE DEA MODEL WITH INPUT-OUTPUT ORIENTED

This research based on 2-stage DEA model developed by Despotis et al. [16]. We then extended the model that was formulated by considering input-ouput oriented for each used data. The objective of DEA model with input oriented is to minimize input which produced at least or equal to total output of given data. Whereas the aim of output oriented model is to maximize the output which obtained not greater than total input of a given data.

Despotis et al. [16] developed an additive decomposition model into 2-stage DEA model to estimate the efficiency score. The model is under the CRS assumptions by considering input-output oriented for each evaluated data. Fig. 1 shows scheme of 2-stage DEA model by Despotis et al. [16] as follows

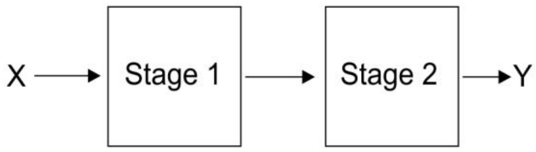

Fig 1. 2-stage DEA model by Despotis et al. [16]

Assume there exists n $\text{DMU}_j\,(j=1,\dots,n)$ where each uses m inputs, $x_{ij}\,(i=1,\dots,m)$, that produces s outputs, $y_{rj}\,(r=1,\dots,s)$. For each efficiency score of evaluated DMU can be obtained by 2-stage DEA model that produce e_{jk}^1 and e_{jk}^2, respectively

Stage 1. Output oriented

$$\frac{1}{e_{jk}^1} = \min \sum_{i=1}^{m} v_i x_{ij_k} \qquad (2.9)$$

$$\text{s. t} \sum_{p=1}^{q} w_p z_{pj_k} = 1 \qquad (2.10)$$

$$\sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (2.11)$$

Stage 2. Input oriented

$$\frac{1}{e_{jk}^2} = \max \sum_{r=1}^{s} u_r y_{rj_k} \qquad (2.12)$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_k} \qquad (2.13)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{p=1}^{q} w_p z_{pj} \le 0 \qquad (2.14)$$

$$w_p, u_r \ge 0$$

Furthermore, 2-stage DEA model are obtained for each input-output oriented as follows:

Stage 1. Output oriented

$$\min \sum_{i=1}^{m} v_i x_{ij_k} \qquad (2.15)$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_k} = 1 \qquad (2.16)$$

$$\sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (2.17)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{p=1}^{q} w_p z_{pj} \le 0 \qquad (2.18)$$

$$v_i, w_p, u_r \ge 0$$

Stage 2. Input oriented

$$\max \sum_{r=1}^{s} u_r y_{rj_k} \qquad (2.19)$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_k} = 1 \qquad (2.20)$$

$$\sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (2.21)$$

$$v_i, w_p, u_r \ge 0$$

Model (2.9 - 2.11) was derived the following combined to be a bi-objective linear program with the aim is to maximize the overall efficiency score from 2-stage DEA model that formulated as follows

$$\max F = \max \sum_{r=1}^{s} u_r y_{rj_k} - \sum_{i=1}^{m} v_i x_{ij_k}$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_k} = 1$$

$$\sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad (2.22)$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{p=1}^{q} w_p z_{pj} \le 0$$

$$v_i, w_p, u_r \ge 0$$

And 2-stage DEA model for estimate the overall efficiency score are

$$\min \theta$$

$$\text{s.t} \sum_{j=1}^{n} \lambda_j z_{pj} - \sum_{j=1}^{n} \mu_j z_{pj} + \theta_{zpj_k} \ge 0$$

$$\sum_{j=1}^{n} \mu_j y_{rj} \ge y_{rj_k} \qquad (2.23)$$

$$\lambda_j, \mu_j \ge 0$$

for $i = 1,\dots,m$; $p = 1,\dots,q$; $r = 1,\dots,s$.

IV. 2-STAGE DEA MODEL WITH INPUT-OUTPUT ORIENTED

This research shows the extended of multi-stage process of DEA model, so we obtained an alternative 2-stage DEA model. Let us consider that there are n $\text{DMU}_j\,(j=1,\dots,n)$ where each using m inputs, $x_{ij}\,(i=1,\dots,m)$. It produce l outputs, $z_{kj}\,(k=1,\dots,l)$ for Stage 1. Then, consider that we have $h_{rj}\,(r=1,\dots,s)$ as inputs of Stage 2 that produce final outputs, $y_{pj}\,(p=1,\dots,q)$. Fig. 2 shows the new scheme of the extended of 2-stage DEA model that we used.

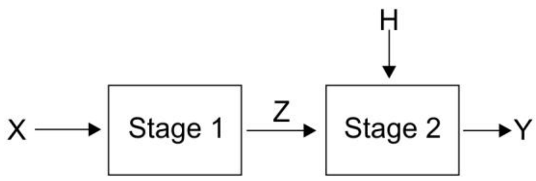

Fig. 2 Extended of 2-stage DEA model

Notice that Stage 1 produce outputs, z_{kj}, that been processed by inputs, x_{ij}. Since the definition of efficiency score in Equation (1), we obtain e_{jk}^{1} as the efficiency score of Stage 1. Afterward for Stage 2, the efficiency score is the estimated of ratio or comparison between the final outputs (y_{pj}) and inputs of Stage 2, (h_{rj}). We then obtained e_{jk}^{2} as the efficiency score of Stage 2. So that, here we obtained DEA model Stage 1 and Stage 2 based on input-output oriented as follows.

Stage 1. Output oriented

$$e_{jo}^{1} = \frac{\sum_{p=1}^{q} w_p z_{pj_o}}{\sum_{i=1}^{m} v_i x_{ij_o}} \tag{2.24}$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_o} = 1 \tag{2.25}$$

$$\sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \tag{2.26}$$

$$v_i, w_p \ge 0$$

Stage 2. Input oriented

$$e_{jo}^{2} = \frac{\sum_{r=1}^{s} u_r y_{rj_o}}{\sum_{k=1}^{l} t_k h_{kj_o}} \tag{2.27}$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_o} = 1 \tag{2.28}$$

$$\sum_{p=1}^{q} u_r y_{rj} - \sum_{k=1}^{l} t_k h_{kj} \le 0 \tag{2.29}$$

$$\sum_{k=1}^{l} t_k h_{kj} - \sum_{p=1}^{q} w_p z_{pj} \le 0 \tag{2.30}$$

$$t_k, u_r, v_i, w_p \ge 0$$

Then, under the CRS assumptions, we can formulated the 2-stage DEA model as

$$\max \sum_{r=1}^{s} u_r y_{rj_o} - \sum_{i=1}^{m} v_i x_{ij_o} \tag{2.31}$$

$$\text{s.t} \sum_{p=1}^{q} w_p z_{pj_o} = 1 \tag{2.32}$$

$$\sum_{p=1}^{q} w_p z_{pj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \tag{2.33}$$

$$t_k, u_r, v_i, w_p \ge 0$$

By a simple division rule, we obtained the overall efficiency score by

$$e^{o} = \frac{e^{1} + e^{2}}{2} \tag{2.34}$$

that then referred as super efficiency score of 2-stage DEA model for each evaluated DMU.

V. COMPUTATIONAL RESULTS

The extended 2-stage DEA model (2.24 – 2.33) obtained then we apply to a data set that studied by Wang et al. [8] that given in Table 1 (see Appendix). We used three outputs: Deposits (z_1), Fixed assets (z_2) and IT data (z_3) that has been processed by an input: Number of employees (x_1) which produce e^1 as the efficiency score of DEA model Stage 1. On Stage 1, we obtained weighted output and weighted input respectively that gives e^1 scores. For DEA model Stage 2, we use an input: Profit (h_1) that produce an output: Loans recovered (y_1) and gives e^2 as the efficiency score of DEA model Stage 2. Therefore, the overall efficiency score obtained by model (2.34) that referred as super efficiency score.

Table 2 (see Appendix) reports the efficiency scores obtained by applying model (2.24 – 2.33) on the data of Table 1. Table 2 shows the comparison results of the efficiency score between model that developed by Despotis et al. [16] and model (2.24 – 2.33). From the results, we obtained input and output weight of DEA model Stage 1 and Stage 2, respectively. From each super efficiency score of each evaluated DMU, e^o, we determined DMU ranking that given in Table 3 (see Appendix) that summarizes the results obtained from model (2.24 – 2.33).

VI. CONCLUSIONS AND FUTURE RESEARCH

This research introduced a new alternative scheme of 2-stage DEA model to obtain super efficiency score of evaluated DMU in a data set. 2-stage DEA model then was formulated into linear program for an based on a new scheme that showed in Fig. 2. This model was extended of CCR model by considering input-output oriented in a data set. The basic idea of this model based on input-output oriented on Stage 1 and output oriented on Stage 2, so that super efficiency score obtained from model (2.34). Testing our models with data sets taken from previous studies [9], shows that results obtained are comparable to those reported in literature as given in Table 2. In future research, we will extend a new alternative scheme of 2-stage DEA model by considering input and output interval in a data set.

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APPENDIX

Table 1. IT data (Source: Wang et al. [9])

DMU$_j$	Fixed assets ($ billions)	IT budget ($ billions)	Number of Employees (thousand)	Deposit ($ billions)	Profit ($ billions)	Loans recovered ($ billions)
1	0.713	0.150	13.3	14.478	0.232	0.986
2	1.071	0.170	16.9	19.502	0.340	0.986
3	1.224	0.235	24.0	20.952	0.363	0.986
4	0.363	0.211	15.6	13.902	0.211	0.982
5	0.409	0.133	18.4	15.206	0.237	0.984
6	5.846	0.497	56.4	81.186	1.103	0.955
7	0.918	0.060	56.4	81.186	1.103	0.986
8	1.235	0.071	12.0	11.441	0.199	0.985
9	18.12	1.500	89.51	124.072	1.858	0.972
10	1.821	0.120	19.80	17.425	0.274	0.983
11	1.915	0.120	19.80	17.425	0.274	0.983
12	0.874	0.050	13.10	14.342	0.177	0.985
13	6.918	0.370	12.50	32.491	0.648	0.945
14	4.432	0.440	41.90	47.653	0.639	0.979
15	4.504	0.431	41.10	52.63	0.741	0.981
16	1.241	0.110	14.40	17.493	0.243	0.988
17	0.450	0.053	7.60	9.512	0.067	0.980
18	5.892	0.345	15.50	42.469	1.002	0.948
19	0.973	0.128	12.60	18.98	0.243	0.985
20	0.444	0.055	5.90	7.546	0.153	0.987
21	0.508	0.057	5.70	7.595	0.123	0.987
22	0.370	0.098	14.10	16.906	0.233	0.981
23	0.395	0.104	14.60	17.264	0.263	0.983
24	2.680	0.206	19.60	36.43	0.601	0.982
25	0.781	0.067	10.50	11.58	0.120	0.987
26	0.872	0.100	12.10	22.207	0.248	0.972
27	1.757	0.010	12.70	20.67	0.253	0.988

Table 2. Results of IT data

Despotis et al. [16]				Model (2.24 – 2.33)							
DMU$_j$	Θ^1	Θ^2	Θ^o	Weighted output (Stage-1)	Weighted input (Stage-1)	e^1	Weighted output (Stage-2)	Weighted input (Stage-2)	e^2	e^o
1	0.639	0.746	0.692	1.000	2.692	0.371	1.218	0.961	1.266	0.819
2	0.651	0.782	0.716	1.352	3.421	0.395	1.326	1.295	1.023	0.709
3	0.518	0.773	0.645	1.433	4.859	0.295	1.349	1.367	0.986	0.640
4	0.599	0.714	0.656	0.942	3.518	0.298	1.193	0.923	1.292	0.795
5	0.556	0.724	0.640	1.031	3.742	0.275	1.221	1.009	1.209	0.742
6	0.760	0.576	0.668	5.707	11.423	0.499	2.058	5.392	0.381	0.440
7	1.000	0.576	0.788	5.441	11.423	0.476	2.089	5.392	0.387	0.431
8	0.535	0.825	0.680	0.826	2.429	0.340	1.184	0.759	1.558	0.949
9	0.625	0.635	0.630	9.217	18.122	0.508	2.830	8.240	0.343	0.426
10	0.496	0.719	0.607	1.255	4.008	0.313	1.257	1.157	1.086	0.699
11	0.495	0.719	0.607	1.260	4.008	0.314	1.257	1.157	1.086	0.700
12	0.668	0.595	0.632	0.999	2.652	0.376	1.162	0.952	1.219	0.798
13	0.949	0.858	0.903	2.530	2.530	1.000	1.593	2.157	0.738	0.869
14	0.588	0.578	0.583	3.403	8.483	0.401	1.618	3.164	0.511	0.456
15	0.658	0.603	0.631	3.738	8.321	0.449	1.722	3.495	0.492	0.470
16	0.665	0.643	0.654	1.228	2.915	0.421	1.231	1.161	1.059	0.740
17	0.718	0.788	0.753	0.656	1.538	0.426	1.047	0.631	1.657	1.041
18	1.000	1.000	1.000	3.138	3.138	1.000	1.950	2.820	0.691	0.845
19	0.814	0.593	0.703	1.313	2.551	0.514	1.228	1.261	0.973	0.744
20	0.693	1.000	0.847	0.525	1.194	0.439	1.140	0.501	2.274	1.357
21	0.707	0.994	0.850	0.531	1.154	0.460	1.110	0.504	2.200	1.330
22	0.794	0.641	0.717	1.142	2.854	0.400	1.214	1.122	1.081	0.740
23	0.780	0.699	0.740	1.167	2.955	0.395	1.246	1.146	1.086	0.740
24	0.930	0.714	0.822	2.563	3.968	0.646	1.583	2.419	0.654	0.650
25	0.627	0.652	0.639	0.811	2.125	0.381	1.107	0.769	1.439	0.910
26	1.000	0.515	0.758	1.521	2.449	0.621	1.220	1.474	0.827	0.724
27	1.000	0.564	0.782	1.467	2.571	0.570	1.241	1.372	0.903	0.737

Table 3. DMU ranking based on its super efficiency score

DMU$_i$	e^1	e^2	e^0	Ranking
20	0.439	2.274	1.357	1
21	0.460	2.200	1.330	2
17	0.426	1.657	1.041	3
8	0.340	1.558	0.949	4
25	0.381	1.439	0.910	5
13	1.000	0.738	0.869	6
18	1.000	0.691	0.845	7
1	0.371	1.266	0.819	8
12	0.376	1.219	0.798	9
4	0.298	1.292	0.795	10
19	0.514	0.973	0.744	11
5	0.275	1.209	0.742	12
16	0.421	1.059	0.740	13
22	0.400	1.081	0.740	14
23	0.395	1.086	0.740	15
27	0.570	0.903	0.737	16
26	0.621	0.827	0.724	17
2	0.395	1.023	0.709	18
11	0.314	1.086	0.700	19
10	0.313	1.086	0.699	20
24	0.646	0.654	0.650	21
3	0.295	0.986	0.640	22
15	0.449	0.492	0.470	23
14	0.401	0.511	0.456	24
6	0.499	0.381	0.440	25
7	0.476	0.387	0.431	26
9	0.508	0.343	0.426	27