

# The Dynamics of Fixed Base and Free-Floating Robotic Manipulator

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**Abstract-** Dynamic modeling means deriving equations that explicitly describes the relationship between force and motion in a system. To be able to control a robot manipulator as required by its operation, it is important to consider the dynamic model in design of the control algorithm and simulation of motion.

In general there are two approaches available; the Euler-Lagrange formulation and the Newton-Euler formulation. This thesis investigates the Lagrange-Euler method in detail. A complete derivation of the method is done using two degree of freedom serial manipulator with revolute joints in the presence and absence of gravitational force. The mathematical model for the Dynamic behaviour of the two degree of freedom manipulator is developed. The dynamic parameters of the system are estimated, and the validity of the resulting dynamic model is verified by several simulations, that describe the dynamic response of the manipulator to input actuator torques. A suggestion for future work is performing thorough dynamic parameter identification. An improved model can ultimately be implemented in the controller of the manipulator, and optimized for a specific job task.

**Keywords-** Torque, Dynamics, manipulator, force, motion

## I-INTRODUCTION

During working cycle a manipulator must accelerate, move at constant speed and decelerate. The time varying position and orientation of the manipulator is termed as its dynamic behaviour. Time varying torques is applied at the joints to balance out internal and external force.

Dynamics is a huge field of study devoted to studying the forces required to cause motion [1], [2]. The dynamic motion of the manipulator arm in a robotic system is produced by the torques generated by the actuators. This relationship between the input torques and the time rates of change of the robot arm components configurations, represent the dynamic modeling of the robotic system which is concerned with the derivation of the equations of motion of the manipulator as a function of the forces and moments acting on. So, the dynamic modeling of a robot manipulator consists of finding the mapping between the forces exerted on the structures and the joint positions, velocities and accelerations [3]. A good model has to satisfy two conflicting objectives. It must include enough detail to represent the real behaviour of the robot with sufficient accuracy, and it should permit an efficient, stable evaluation not only of the model equations but also of their derivatives that are needed in optimization. The availability of the dynamic model is very useful for mechanical design of the structure, choice of actuators, determination of control strategies, and computer simulation manipulator motion.

A robot manipulator is basically a positioning device. To control the position we must know the dynamic properties of the manipulator in order to know how much force to exert on it to cause it to move: too little force and the manipulator is slow to react; too much force and the arm may crash into objects or oscillate about its desired position.

A significant amount of research has been reported concerning optimal trajectory planning using evolutionary methods for an industrial manipulator system. Research made the use of cubic spline curves to generate the trajectory between the intermediate points of the path. The problem of kinematics is solved for two-degree-of-freedom linear industrial manipulators. The Newton-Euler technique is used for the formulation of the dynamic equations of the manipulator. The effectiveness of the proposed method is verified through MATLAB simulations in [7], it has shown a trajectory generation for a two link robot manipulator without the use of the inverse Jacobian matrix, in which the cubic spline approach was employed.

Robotic manipulators will play important roles in future space missions. The control of such space manipulators poses planning and control problems not found in terrestrial fixed-base manipulators due to the dynamic coupling between space manipulators and their spacecraft. A number of control techniques for such systems have been proposed; these schemes can be classified in three categories. In the first category, spacecraft position and attitude are controlled by reaction jets to compensate for any manipulator dynamic forces exerted on the spacecraft. In this case, control laws for earth-bound manipulators can be used, but the utility of such systems may be limited because manipulator motions can both saturate the reaction jet system and consume relatively large amounts of attitude control fuel, limiting the useful life of the system [10], [11], and [14]. In the second category, the spacecraft attitude is controlled, although not its translation, by using reaction wheels or attitude control jets [12]. The control of these systems is somewhat more complicated than that of the first category, although a technique called the Virtual Manipulator (VM) can be used to simplify the problem [10], [11], [12], [13], and [14]. The third proposed category assumes a free-floating system in order to conserve fuel or electrical power [10], [15], and [16]. Such a system permits the spacecraft to move freely in response to manipulator motions. These too can be modelled using the VM approach [15]. Work was done on control algorithm for free-floating space manipulators in [14]. It was shown that nearly and algorithm that can be applied to conventional

fixed-based manipulator can be directly applied to free-floating manipulators, with few additional conditions.

and equilibrium of forces and moments, strictly hold true during the operation.

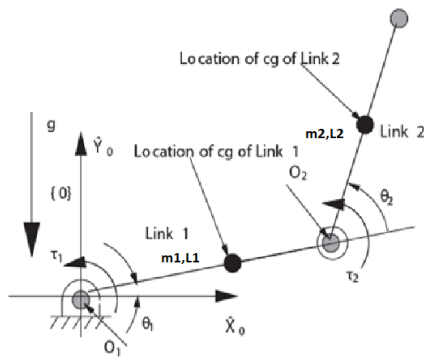


Fig. 1 A 2-DOF planar articulated (RR) arm with fixed base

These include the measurement or estimation of a spacecraft's orientation and the avoidance of dynamic singularities.

Significant amount of work is done on the Dynamic Control of a Space Robot system with no Thrust Jets controlled base [22]. In this paper he discusses dynamic control of a free-flying space robot system where the base attitude is not controlled by thrust jets. Without external forces and moments, the system is governed by linear and angular momentum conservation laws. he first derive the system dynamic formulations in joint space and in inertia space, based on Lagrangian dynamics. Then discuss the fact that dynamics of a space robot system cannot be linearly parameterized, as opposed to the case of a fixed-based robot.

## II-BASIC FORMULATION

In this paper we are considering two cases, one for fixed base manipulator and another for the free-floating manipulator, the end-effector is made to move in a circular trajectory figure 1 shows fixed base manipulator, figure 2 shows free-floating manipulator. The time varying torques is compared for both.

### A. Assumptions

In this paper author assume a simple model of a robot satellite which has an articulated manipulator system, in order to clarify the point at issue, they make the following assumptions [19]

- 1) It is a 2-DOF manipulator with two links, articulated robotic arm.
- 2) Control technique used is FREE-FLOATING.
- 3) Mass of the spacecraft is assumed to very high when compared with the mass of the links.
- 4) Gravitational force is zero.
- 5) Robot arms are not affected by the friction and disturbance.
- 6) There are no mechanical restriction nor external forces and torques, so that momentum conservation,

### B. Nomenclature

The symbols are defined as

$\tau$	torque
$F$	force
$\theta, q$	joint angle
$\dot{\theta}, \dot{q}$	angular velocity
$\ddot{\theta}, \ddot{q}$	angular acceleration
$L$	Lagrangian
$l_1, a_1, L1$	length of link 1
$l_2, a_2, L2$	length of link 2
$m_1, m_2$	mass of link 1 and link 2
$\dot{x}$	linear velocity in x-direction
$\dot{y}$	linear velocity in y-direction
$\ddot{x}$	linear acceleration in x-axis
$\ddot{y}$	linear acceleration in y-direction

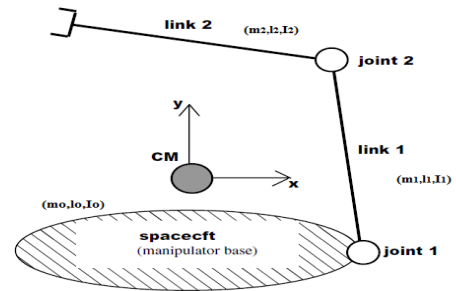


Fig.2 Model of a 2-DOF planar robot without fixed base

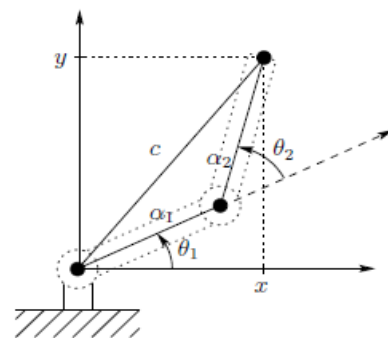


Fig.3 Solving for the joint angles of a two-link planar arm

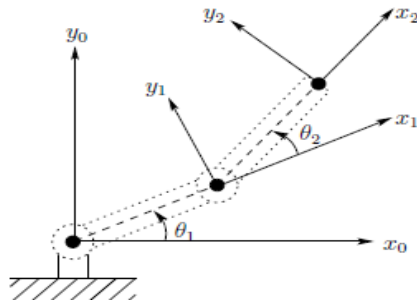


Fig.4 Coordinate frames for two-link planar robot

### C. fundamental equation

Inverse kinematics of manipulators, in order to command the end-effector to move to a particular position we need the angles of both the links  $\theta_1$  and  $\theta_2$

$$\cos\theta_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}; = D. \quad (1)$$

$$\sin(\theta_2) = \pm\sqrt{1 - D^2} \quad (2)$$

$$\theta_2 = \tan^{-1} \frac{\pm\sqrt{1-D^2}}{D} \quad (3)$$

$$\theta_1 = \tan^{-1}(y/x) - \tan^{-1} \left( \frac{a_2 \sin\theta_2}{a_1 + a_2 \cos\theta_2} \right) \quad (4)$$

Velocity Kinematics of manipulator, coordinates (x, y) of the tool are expressed in this coordinate frame as

$$x = x_2 = a_1 \cos\theta_1 + a_2 \cos(\theta_1 + \theta_2) \quad (5)$$

$$y = y_2 = a_1 \sin\theta_1 + a_2 \sin(\theta_1 + \theta_2) \quad (6)$$

In order to follow a contour at constant velocity, or at any prescribed velocity, we must know the relationship between the velocity of the tool and the joint velocities. In this case we can differentiate Equations (5) and (6) to obtain

$$\dot{x} = -a_1 \sin\theta_1 \cdot \dot{\theta}_1 - a_2 \sin(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \quad (7)$$

$$\dot{y} = a_1 \cos\theta_1 \cdot \dot{\theta}_1 - a_2 \cos(\theta_1 + \theta_2) (\dot{\theta}_1 + \dot{\theta}_2) \quad (8)$$

$$dX = J(q)d\theta$$

Inverse Velocity and Acceleration,

$$\dot{X} = J(q)\dot{q} \quad (9)$$

$$\ddot{X} = J(q)\ddot{q} + \left( \frac{d}{dt} J(q) \right) \dot{q}. \quad (10)$$

$$b = J(q)\ddot{q}$$

$$b = \ddot{X} - \left( \frac{d}{dt} J(q) \right) \dot{q}.$$

$$\dot{q} = J(q)^{-1} \dot{X}$$

$$\ddot{q} = J(q)^{-1} \ddot{X}$$

Trajectory generation, a common way of causing a manipulator to move from here to there in a smooth, controlled fashion is to cause each joint to move as specified by a smooth function of time. Commonly, each joint starts and ends its motion at the same time, so that manipulator motion appears coordinated. Exactly how to compute these motion functions is the problem of trajectory generation. Often, a path is described not only by a desired destination but also by some intermediate locations, or via points, through which the manipulator must pass en route to the destination. In such instances the term spline is sometimes used to refer to a smooth function that passes through a set of via points. In order to force the end-effector to follow a straight line (or other geometric shape) through space, the desired motion must be converted to an equivalent set of joint motions [2].

Equations of Motion, The purpose of a robot manipulator is to position and interface its end-effector with the working object. The equations of motion are important to consider in the design of robots, as well as in simulation and animation, and in the design of control algorithms. Equation of motion can be described by a set of differential or difference equations. The equation set consists of two parts, the kinematics equations and the dynamic equation. Robot arm kinematics deals with the geometry of robot arm motion as a function of time (position, velocity, and acceleration) without regards to the forces and moments that cause it.

### D. Dynamic Equations

Dynamics of robot is the study of motion with regard to forces (the study of the relationship between forces/torques and motion). A dynamic analysis of a manipulator is useful for the following purposes:

- 1) It determines the joint forces and torques required to produce specified end-effector motions (the direct dynamic problem).
- 2) It produces a mathematical model which simulates the motion of the manipulator under various loading conditions (the inverse dynamic problem) and/or control schemes.
- 3) It provides a dynamic model for use in the control of the actual manipulator.

## III-DYNAMIC FORMULATION

In this section we derive a general set of differential equations that describe the time evolution of mechanical

systems. These are called the Euler-Lagrange equations of motion.

Lagrangian formulation, which describes the behavior of a dynamic system in terms of work and energy stored in the system rather than of forces and moments of the individual members involved. The constraint forces involved in the system are automatically eliminated in the formulation of Lagrangian dynamic equations. The closed-form dynamic equations can be derived systematically in any coordinate system.

For control design purposes, it is necessary to have a mathematical model that reveals the dynamical behavior of a system; we derive the dynamical equations of motion for a robot manipulator. Our approach is to derive the kinetic and potential energy of the manipulator and then use Lagrange's equations of motion.

#### A. Lagrange's Equations of Motion

Lagrange's equation of motion for a conservative system are given by

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tau \quad (11)$$

where  $q$  is an  $n$ -vector of generalized coordinates  $q_i$ ,  $\tau$  is an  $n$ -vector of generalized forces  $\tau_i$ , and the Lagrangian is the difference between the kinetic and potential energies

$$L = K - P. \quad (12)$$

In our usage,  $q$  will be the joint-variable vector, consisting of joint angles  $\theta_i$  (in degrees or radians) and joint offsets  $d_i$  (in meters). Then  $\tau$  is a vector that has components  $n_i$  of torque (Newton-meters) corresponding to the joint angles, and  $f_i$  of force (Newtons) corresponding to the joint offsets. We shall use Lagrange's equation to derive the general robot arm dynamics. Let us first get a feel for what is going on by considering some examples.

#### B. Two degree of freedom manipulator with fixed base

The Lagrangian requires kinetic and potential energies of the manipulator. The kinetic energy of a rigid body (a link) is given by

$$K = \frac{1}{2} m v^2 + \frac{1}{2} I w^2 \quad (13)$$

Where  $v$  is the linear velocity,  $w$  is the angular velocity,  $m$  is the mass, and  $I$  is the moment of inertia of the rigid body at its centre of mass. linear velocity  $v_1 = \frac{1}{2} L_1 \dot{\theta}_1$ , angular velocity  $w_1 = \dot{\theta}_1$ , moment of inertia  $I_1 = \frac{1}{12} m_1 L_1^2$

$$K_1 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_1 w_1^2 \quad (14)$$

After substituting the values

$$K_1 = \frac{1}{6} m_1 L_1^2 \dot{\theta}_1^2 \quad (15)$$

And the potential energy is given by

$$P_1 = \frac{1}{2} m_1 g L_1 \sin \theta_1 \quad (16)$$

Where  $g$  is the magnitude of acceleration due to gravity in the negative  $y$ -axis direction.

For link 2

$$x_2 = L_1 \cos \theta_1 + \frac{1}{2} L_2 \cos (\theta_1 + \theta_2) \quad (17)$$

$$y_2 = L_2 \sin \theta_1 + \frac{1}{2} L_2 \sin (\theta_1 + \theta_2) \quad (18)$$

Differentiating equation (17) and (18) will give velocity component of link 2  $\dot{x}$  and  $\dot{y}$ , from these component, the square of the magnitude of velocity of the end of link 2 is

$$v_2^2 = \dot{x}_2^2 + \dot{y}_2^2$$

The kinetic energy of link 2 with  $w_2 = \dot{\theta}_1 + \dot{\theta}_2$  and  $I_2 = \frac{1}{2} m_2 L_2^2$  is

$$K_2 = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 w_2^2 \quad (19)$$

The potential energy of link 2

$$P_2 = m_2 g L_1 S_1 + \frac{1}{2} m_2 g L_2 S_{12} \quad (20)$$

Where  $C_1 = \cos \theta_1$ ,  $S_1 = \sin \theta_1$ ,  $C_{12} = \cos (\theta_1 + \theta_2)$  and  $S_{12} = \sin (\theta_1 + \theta_2)$

The Lagrangian  $L = K - P = K_1 + K_2 - P_1 - P_2$  is obtained by substituting values of equation (15), (16), (19) and (20) in above equation of Lagrangian give the value of  $L$ . The derivation is in detail in ([2], [3])

The Lagrangian-Euler formulation for link1 equation (11) gives the torque  $\tau_1$  at joint 1 as

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = \tau_1 \quad (21)$$

For link 2 it gives torque  $\tau_2$  at the joint 2 as

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} = \tau_2 \quad (22)$$

The Lagrangian value is differentiated w r t  $\theta_1$  and  $\dot{\theta}_1$  and substituted in equation (21) and differentiated w r t  $\theta_2$  and  $\dot{\theta}_2$  and substituted in equation (22) which gives

$$\begin{aligned} \tau_1 &= H_{11} \ddot{\theta}_1 + H_{12} \ddot{\theta}_2 + C_1 + G_1 \\ \tau_2 &= H_{21} \ddot{\theta}_1 + H_{22} \ddot{\theta}_2 + C_2 + G_2 \end{aligned} \quad (23)$$

Where

$$H_{11} = \left[ \left( \frac{1}{3} m_1 + m_2 \right) L_1^2 + \frac{1}{3} m_2 L_2^2 + m_2 L_1 L_2 C_2 \right]$$

$$\begin{aligned}
 H_{12} &= H_{21} = m_2 \left[ \frac{1}{3} L_2^2 + \frac{1}{2} L_1 L_2 C_2 \right] \\
 H_{22} &= \frac{1}{3} m_2 L_2^2 \\
 C_1 &= -m_2 L_1 L_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - \frac{1}{2} m_2 L_1 L_2 S_2 \dot{\theta}_2^2 \\
 C_2 &= \frac{1}{2} m_2 L_1 L_2 S_2 \dot{\theta}_1^2 \\
 G_1 &= \left( \frac{1}{2} m_1 + m_2 \right) L_1 C_1 + \frac{1}{2} m_2 L_2 C_{12} g \\
 G_2 &= \frac{1}{2} m_2 L_1 C_{12} g
 \end{aligned}$$

These coefficients are defined as iterations

$M_{ii} = H_{ii}$  = effective inertia,  $M_{ij} = H_{ij}$  = effective coupling inertia,

$C_i$  = centrifugal and Coriolis acceleration forces

The general form of equation is

$$[M(q)]\ddot{q} + [V(q, \dot{q})]\dot{q} + G(q) = \tau \quad (24)$$

- 1)  $[M(q)]$  -  $n \times n$  mass matrix
- 2)  $[V(q, \dot{q})]$  -  $n \times n$  matrix and  $[V(q, \dot{q})]\dot{q}$  is an  $n \times 1$  vector of centripetal and Coriolis terms
- 3)  $G(q)$  -  $n \times 1$  vector of gravity terms, and
- 4)  $\tau$  -  $n \times 1$  vector of joint torque or forces
  - a. It can also be written as

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (25)$$

this is the final form of the robot dynamical equation we have been seeking.

The units of elements of  $M(q)$  corresponding to revolute joint variables  $q_i = \theta_i$  are kg-m<sup>2</sup>. The units of the elements of  $M(q)$  corresponding to prismatic joint variables  $q_i = d_i$  are kilograms. The units of elements of  $V(q)$  and  $G(q)$  corresponding to revolute joint variables are kg-m<sup>2</sup>/s<sup>2</sup>. The units of elements of  $V(q)$  and  $G(q)$  corresponding to prismatic joint variables are kg-m/s<sup>2</sup>.

The final EOM (dynamic model) is

$$\tau_i = \sum_{j=1}^n M_{ij}(q) \ddot{q}_j + \sum_{j=1}^n \sum_{k=1}^n C_{ijk} \dot{q}_j \dot{q}_k + G_i$$

For  $i = 1, 2, \dots, n$

The physical meaning of above equation is as follows.

1. The coefficients of the  $\ddot{q}_j$  terms in these equations represent inertia. It is known as effective inertia when acceleration of joint  $i$  cause a torque at joint  $i$ . and coupling inertia when acceleration at joint  $j$  causes a torque at joint  $i$ . In other words, the coefficient  $M_{ij}$  is related to acceleration of the joint and represents inertia loading of the actuator, In summary
  - $M_{ii}$  = effective inertia at the joint  $i$  where the driving torque  $\tau_i$  acts.
  - $M_{ij}$  = coupling inertia between joint  $i$  and joint  $j$ . It is reaction torque  $M_{ij} \ddot{q}_j$  at joint  $i$  induced by acceleration at joint  $j$ . Reverse applies equally with torque  $M_{ji} \ddot{q}_j$  as torque at joint  $j$  due to acceleration of joint  $i$ .
  - Since  $\text{Tr}(A) = \text{Tr}(A^T)$ , it can be shown that  $M_{ij} = M_{ji}$ .

2. The coefficient  $C_{ijk}$  represents the velocity induced reaction torque at joint  $i$ , the first index. The indices  $j$  and  $k$  are related to velocities of joint  $j$  and joint  $k$ , whose dynamic interplay induces a reaction torque at joint  $i$ . In particular, a term of the form  $C_{ijj} \dot{q}_j^2$  is the centrifugal force acting at joint  $i$  due to velocity at joint  $j$  and a term of the form  $C_{ijk} \dot{q}_j \dot{q}_k$  is shown as the Coriolis force acting at joint  $i$  due to velocities at joint  $j$  and  $k$ . In particular
  - $C_{ijk}$  = Coriolis force coefficients generated by the velocities of joint  $j$  and joint  $k$  and "felt" at joint  $i$ . Coriolis force acting at joint  $i$  due to velocities at joint  $j$  and joint  $k$  is a combination term of  $C_{ijk} \dot{q}_j \dot{q}_k + C_{ijk} \dot{q}_k \dot{q}_j$ .
  - $C_{ijj}$  = Centrifugal force coefficient at joint  $i$  generated due to angular velocity at joint  $j$ . the centrifugal force acting at joint  $i$  due to velocity at joint  $j$  is given by  $C_{ijj} \dot{q}_j^2$ .
  - $C_{ijk} = C_{ikj}$  because the Coriolis force acts at joint  $i$  due to velocities of joints  $j$  and  $k$  and suffix order does not matter.
  - $C_{iii} = 0$ , Coriolis force at joint  $i$  is not due to joint velocity itself

3. The term involving gravity  $g$  represents the gravity generated moment at joint  $i$ . The coefficient  $G_i$  is the gravity loading force as joint  $i$  due to the links  $i$  to  $n$ . The gravity term is a function of the current position.
4.  $\tau_i$  = generalized force applied at joint  $i$  due to motion of links.
5.  $q_i$  = joint displacement of joint  $i$ .
6.  $\dot{q}_i$  = velocity of joint  $i$ .
7.  $\ddot{q}_i$  = acceleration of joint  $i$ .

In dynamic model equation inertia and gravity terms are significant in manipulator control as they affect positioning accuracy and servo stability, which in turn determine the repeatability of the manipulator. The Coriolis and centrifugal forces are significant for high-speed motion of the manipulator.

### C. Two degree of freedom manipulator with no fixed base (Free-floating manipulator)

We have considered smaller systems installed on a free-floating unmanned satellite, considered a manipulator placed on a satellite having its mass and inertia much larger than the manipulator, and considered it similar to fixed based manipulator without considering the gravitational effect. [16] Any movement of the manipulator on satellite will not disturb the position or attitude of the satellite.

The kinetic energy both the links remains same as the one for the fixed base manipulator but the potential energy for both the link will be zero.

$$K_1 = \frac{1}{6} m_1 L_1^2 \dot{\theta}_1^2$$

$$K_2 = \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I_2 \omega_2^2$$

$$P1=0$$

$$P2=0$$

The Lagrangian-Euler formulation for link1 gives the torque  $\tau_1$  at joint 1 is given by equation (21) and for link 2 it gives torque  $\tau_2$  at the joint 2 is given by equation (22).for derivation refer Appendix 1.

The Lagrangian value is differentiated w.r.t  $\theta_1$  and  $\dot{\theta}_1$  and substituted in equation (21) and differentiated wrt  $\theta_2$  and  $\dot{\theta}_2$  and substituted in equation (22) which gives

$$\tau_1 = H_{11}\ddot{\theta}_1 + H_{12}\ddot{\theta}_2 + C_1 \quad (26)$$

$$\tau_2 = H_{21}\ddot{\theta}_1 + H_{22}\ddot{\theta}_2 + C_2$$

The general form of equation is

$$[M(q)]\ddot{q} + [C(q, \dot{q})]\dot{q} = \tau \quad (27)$$

$$H_{11} = \left[ \left( \frac{1}{3} m_1 + m_2 \right) L_1^2 + \frac{1}{3} m_2 L_2^2 + m_2 L_1 L_2 C_2 \right]$$

$$H_{12} = H_{21} = m_2 \left[ \frac{1}{3} L_2^2 + \frac{1}{2} L_1 L_2 C_2 \right]$$

$$H_{22} = \frac{1}{3} m_2 L_2^2$$

$$C_1 = -m_2 L_1 L_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - \frac{1}{2} m_2 L_1 L_2 S_2 \dot{\theta}_2^2$$

$$C_2 = \frac{1}{2} m_2 L_1 L_2 S_2 \dot{\theta}_1^2$$

#### IV-CASE STUDY

Let us consider the Inverse kinematic problem, given two link manipulator as shown in the figure 1, having mass, inertia and geometry as shown in table 1, chosen circular trajectory of end-effector as  $x=a + r\cos(\phi)$ ,  $y=b + r\sin(\phi)$ , where  $r = 0.2$ ,  $a = 1.2$ ,  $b = 1.2$  and makes this circular rotation from 0 to  $2\pi$  in 10 seconds.

We will find the torque  $\tau(t)$  applied at both the joints at different time intervals

Table 1  
2R Manipulator parameters[8]

Link	Length (m)	Mass (kg)	C.G (m)	Inertia (kg m <sup>2</sup> )
1	1.0	12.456	.773	1.042
2	1.0	12.456	.583	1.042

Let us consider the point to point trajectory tracking where end-effector moves from 0 to 360 degrees in 10 seconds. Here

(28)

we find the orientation for every time, consider the cubic polynomial equation [2],

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

deriving equation (28) will give the joint velocity, and deriving again will give joint acceleration.

$$\dot{q}(t) = a_1 + 2a_2 t + 3a_3 t^2 \quad (29)$$

$$\ddot{q}(t) = 2a_2 + 6a_3 t \quad (30)$$

To solve these equations we require atleast four constraints, we consider initial and final velocity as zero, and starting and goal-point values.

$$q(0) = q^s$$

$$q(t_g) = q^g$$

$$\dot{q}(0) = 0$$

$$\dot{q}(t_g) = 0$$

Where  $q^s$ ,  $q^g$  starting and goal-point position and  $t_s$  and  $t_g$  are initial and final time.

Applying constraints in equation (28), (29) and (30) gives following set of equation,

$$a_0 = 0$$

$$a_1 = 0$$

$$a_2 = \frac{3}{t_g^2} (q^g - q^s)$$

$$a_3 = -\frac{2}{t_g^3} (q^g - q^s)$$

the value of  $a_2$  and  $a_3$  after substituting the values are

$$a_2 = 0.1885$$

$$a_3 = -0.0125$$

Equation (28) becomes

$$q(t) = 0.1885(t^2) - 0.0125(t^3)$$

calculate  $q(t)$  for every change of time and substitute in given trajectory

$x=a + r\cos(\phi)$ ,  $y=b + r\sin(\phi)$ , considering  $q(t) = \phi$ , we get  $x$  and  $y$  values

The fig.4 generated by MatLab shows the trajectory taken by end-effector.

Now consider the two links and find the initial angle  $\theta_1$  and  $\theta_2$  using equation (1) – (4). Which gives two cases elbow-up and elbow-down position,

Case 1:  $\theta_1 = 0.3109$  and  $\theta_2 = 0.7953$ .

Case 2:  $\theta_1 = 1.1056$  and  $\theta_2 = -0.7953$

We consider case 2 condition, and we get the following results as shown in the figures. Using the values of  $x$  and  $y$  find the value of  $\dot{x}$  ie  $[\Delta x \ \Delta y]$  which actually gives the linear velocity.

$$\Delta x = x_2 - x_1, \Delta y = y_2 - y_1$$

We know the Jacobian and using equation (9) find the value of  $\dot{q}$  or  $\dot{\theta}$ , i.e.

$$\dot{\theta} = J^{-1} \dot{x}$$

Fig.6 gives the angular velocity path

Find  $\ddot{q}$  using the equations (9) to (10)

$$\ddot{X} = J(q)\ddot{q} + \left(\frac{d}{dt}J(q)\right)\dot{q}$$

Where  $\ddot{X}$  is acceleration which is change in velocity.

$$\ddot{X} = \dot{x}_2 - \dot{x}_1$$

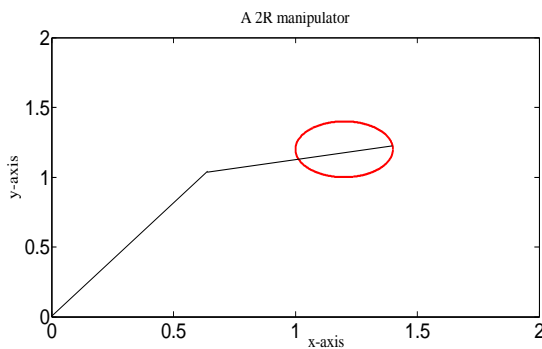


Fig.5 Path taken by the end-effector

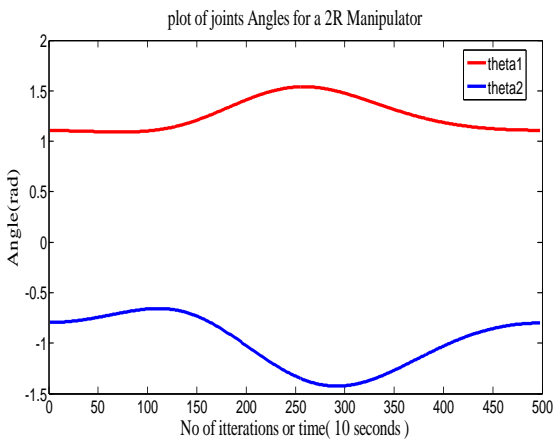


Fig.6 Computed  $\theta_1$  and  $\theta_2$  using inverse kinematics

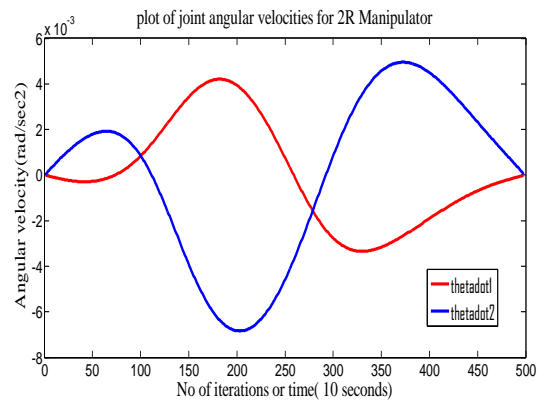


Fig.7 computed  $\dot{\theta}_1$  and  $\dot{\theta}_2$  using inverse Jacobian

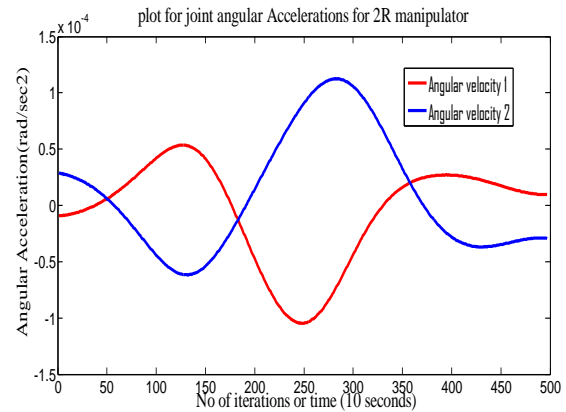


Fig.8 Computed  $\ddot{\theta}_1$  and  $\ddot{\theta}_2$

Using equation (23) we find the values of  $\tau$  of both the joints.

$$\tau_1 = (26.2128 + 14.52 \cdot \cos(\theta_2)) \ddot{\theta}_1 + (5.275 + 7.26 \cdot \cos(\theta_2)) \ddot{\theta}_2 - (7.26 \cdot \sin(\theta_2) \cdot \dot{\theta}_1 \cdot \dot{\theta}_2) - (7.26 \cdot \sin(\theta_2) \cdot \dot{\theta}_2^2) + 9.81 \cdot (22.084 - \cos(\theta_1) + 7.26 \cdot C_{12})$$

$$\tau_2 = (5.275 + 7.26 \cdot \cos(\theta_2)) \ddot{\theta}_1 + 5.275 \cdot \ddot{\theta}_2 - (7.26 \cdot \sin(\theta_2) \cdot \dot{\theta}_1^2) + (71.238 \cdot C_{12})$$

The torque is also calculated without the gravitational force, using equation (26) below shows the equations obtained

$$\tau_1 = (26.2128 + 14.52 \cdot \cos(\theta_2)) \ddot{\theta}_1 + (5.275 + 7.26 \cdot \cos(\theta_2)) \ddot{\theta}_2 - (7.26 \cdot \sin(\theta_2) \cdot \dot{\theta}_1 \cdot \dot{\theta}_2) - (7.26 \cdot \sin(\theta_2) \cdot \dot{\theta}_2^2)$$

$$\tau_2 = (5.275 + 7.26 \cdot \cos(\theta_2)) \ddot{\theta}_1 + 5.275 \cdot \ddot{\theta}_2 - (7.26 \cdot \sin(\theta_2) \cdot \dot{\theta}_1^2)$$

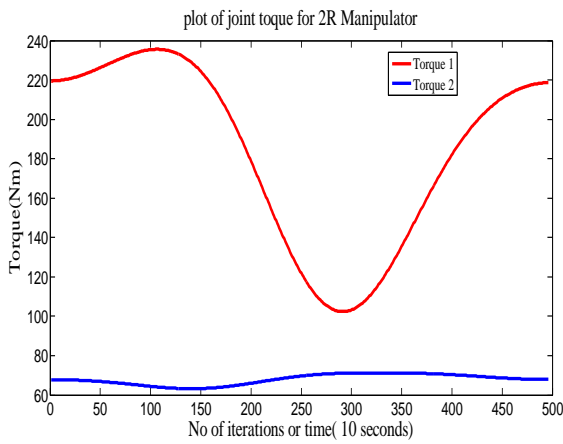


Fig.9 Computed  $\tau_1(t)$  and  $\tau_2(t)$   
plot of joint torques without gravitational force

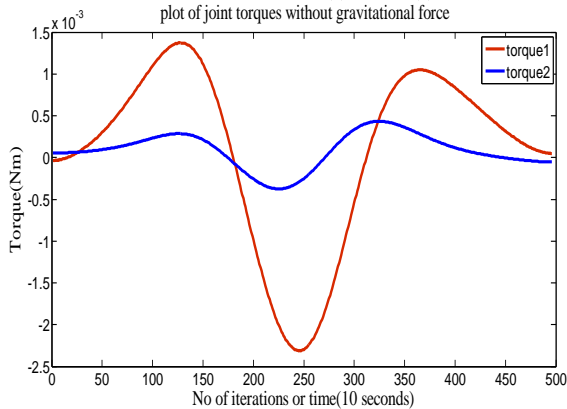


Fig.10 Torque without gravitational force

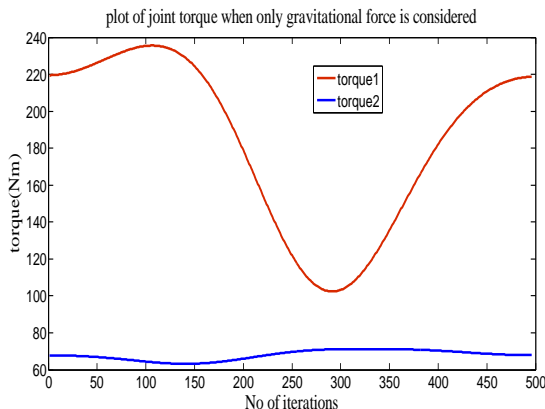


Fig.11 Effect of gravitational force on joint torques

#### IV-CONCLUSION AND SCOPE OF FUTURE WORK

##### A. Conclusion

1. In this paper the Dynamic model (EOM) is derived for the two degree of freedom serial manipulator. Two cases were considered, in one the manipulator is under the influence of gravitational force and in another gravitational force was not considered as it is freely floating in the space.
2. We have also analyzed the behavior of the torques under the influence of various factors like centrifugal force, Coriolis force and inertia.

3. The Lagrange-Euler method is used for building the Dynamic equation
4. From the case studied we found that the final angle of both the joints are same as that of initial angle
5. It was found that the initial and final velocities were zero which are as per our requirement. As the velocity was increasing linearly and then again decreasing linearly the acceleration was also varying which also showed the expected results.
6. Torque is much varying when compared in the presence and absence of gravitational force. High torque is required for the same body at particular time to move particular point in the presence of gravitational force where less is required in the absence.

These suggested that the key to solving the problem of planning and control of space robotic system lies in understanding the fundamental dynamic behavior of the systems.

##### B. Scope of future work

- 1) We hope that the results encourage the development of more effective control algorithm for free-floating space manipulator. Further work can be done on the kinematic and dynamics of space manipulators.
- 2) We can propose generalized Jacobian matrix for space manipulator, taking dynamical interactions between the arm and satellite into account, it is just an initial step for future work on space robots.
- 3) The dynamic equations obtained can be used to study dynamic behavior of free-flying and free-floating space manipulator study in detail during pre-impact and post impact operations.
- 4) A possible extension for this work would be implementation of a dynamic model for a 6 DOF space robots.

#### APPENDIX I

Derivation without the gravitational force, the Lagrangian  $L = K - P = K_1 + K_2 - P_1 - P_2$  is obtained by Substituting values of equation (15), (16), (19) and (20) in above equation

$$L = K_1 + K_2$$

$$L = \frac{1}{2} \left( \frac{1}{3} m_1 + m_2 \right) L_1^2 \dot{\theta}_1^2 + \frac{1}{6} m_2 L_2^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2) + \frac{1}{2} m_2 L_1 L_2 C_2 (\dot{\theta}_1^2 + \dot{\theta}_1 \dot{\theta}_2) \quad (A1)$$

Lagrange-Euler formulation for link1 is given by

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} = \tau_1 \quad (A2)$$

Diff (A1) w.r.t  $\theta_1$  and  $\dot{\theta}_1$

$$\frac{\partial L}{\partial \theta_1} = 0$$



$$\frac{\partial L}{\partial \dot{\theta}_1} = \left(\frac{1}{3}m_1 + m_2\right)L_1^2\dot{\theta}_1 + \frac{1}{3}m_2L_2^2(\dot{\theta}_1 + \dot{\theta}_2) + \frac{1}{2}m_2L_1L_2C_2(2\dot{\theta}_1 + \dot{\theta}_2)$$

Diff equation A3 w.r.t time

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \left[\left(\frac{1}{3}m_1 + m_2\right)L_1^2 + \frac{1}{3}m_2L_2^2 + m_2L_1L_2C_2\right]\ddot{\theta}_1 + m_2\left[\frac{1}{3}L_2^2 + \frac{1}{2}L_1L_2C_2\right] - m_2L_1L_2\dot{S}_2\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2}m_2L_1L_2S_2\dot{\theta}_2^2 \quad (A3)$$

Substituting equation A3 and A4 in A2 will give

$$\tau_1 = \left[\left(\frac{1}{3}m_1 + m_2\right)L_1^2 + \frac{1}{3}m_2L_2^2 + m_2L_1L_2C_2\right]\ddot{\theta}_1 + m_2\left[\frac{1}{3}L_2^2 + \frac{1}{2}L_1L_2C_2\right] - m_2L_1L_2\dot{S}_2\dot{\theta}_1\dot{\theta}_2 - \frac{1}{2}m_2L_1L_2S_2\dot{\theta}_2^2 \quad (A4)$$

Similarly for joint 2

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_2} - \frac{\partial L}{\partial \theta_2} = \tau_2 \quad (A5)$$

$$\frac{\partial L}{\partial \theta_2} = -\frac{1}{2}m_2L_1L_2S_2(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2) \quad (A6)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2\left[\frac{1}{3}L_2^2 + \frac{1}{2}L_1L_2C_2\right]\ddot{\theta}_1 + \frac{1}{3}m_2L_2^2\ddot{\theta}_2 - \frac{1}{2}m_2L_1L_2\dot{S}_2\dot{\theta}_1\dot{\theta}_2 \quad (A7)$$

Substituting equation (A6) and (A7) in equation in (A5)

$$\tau_2 = m_2\left[\frac{1}{3}L_2^2 + \frac{1}{2}L_1L_2C_2\right]\ddot{\theta}_1 + \frac{1}{3}m_2L_2^2\ddot{\theta}_2 + \frac{1}{2}m_2L_1L_2S_2\dot{\theta}_1^2$$

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