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The Effect Of Imposed Time-Periodic Boundary Temperature And Electric Field On The Onset Of Rayleigh-Bénard Convection In A Micropolar Fluid

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ABSTRACT

The effect of imposed time periodic temperature of small amplitude and AC electric field on the onset of Rayleigh-Bénard convection in a micropolar fluid is investigated using linear stability analysis. A regular perturbation method is used to arrive at an expression for the correction Rayleigh number that throws light on the possibility of subcritical motions. The Venezian approach is adopted for obtaining eigen value of the problem. Three cases of oscillating temperature field are examined: (a) symmetric, so that the wall temperatures are modulated in-phase, (b) asymmetric, corresponding to out-of-phase modulation and (c) only the lower wall is modulated. It is observed that the system is most stable when the boundary temperatures are modulated out-of-phase. This problem is an example of external control of the internal convection.

1. INTRODUCTION

One of the effective mechanisms of controlling convection is through the maintenance of a non-uniform temperature gradient which is only space-dependent. However, in many practical situations non-uniform temperature gradients find their origin in transient heating or cooling at the boundaries, hence the basic temperature profile depends explicitly on position and time. This problem, called the **thermal modulation** problem, involves the solution of the energy equation under suitable time-dependent boundary conditions. These profiles can be used as an effective mechanism to control the convective flow by proper tuning of its parameters, namely, amplitude and frequency of modulation. There can be an appreciable enhancement of heat, mass, or momentum if an imposed modulation can destabilize an otherwise stable system. Similarly, if it can stabilize an otherwise unstable system, higher efficiency can be achieved in many processing techniques, particularly in solidification processes. For example, in crystal growth during solidification of metallic alloys, one can consider time-dependent temperature gradient to influence the transport process, thus to control the quality and structure of the resulting solid. The thermal modulation can be used as a mechanism to delay convection in the case of material processing applications to attain higher efficiencies and to advance it in achieving major enhancement of mass, heat and momentum transfer.

The pioneer work in this field is due to Venezian [1], who investigated the effect of time-dependent heating on the onset of thermal convection in a horizontal fluid layer heated from below. Actually, the investigation due to Venezian was motivated by the experiment of Donnelly [2], in which, he investigated the effect of rotational speed modulation on the onset of instability in fluid flow between two concentric cylinders. Many authors (Bhaduria [3, 4, 5], Pranesh and Sangeetha [6], Siddheshwar et. al. [7, 8]) have considered the effect of temperature modulation under different conditions to study its effect on the onset of convection.

In most part of the last century the engineering applications of fluid mechanics were restricted to systems in which electric and magnetic fields played no role. In recent years, the study of the interaction of electromagnetic fields with fluids started gaining attention with the promise of applications in areas like nuclear fusion, chemical engineering, medicine and high speed noiseless printing. The investigation of convective heat transfer together with the electrical and magnetic forces in non-Newtonian fluids is of practical importance. A systematic study through a proper theory is essential to understand the physics of the complex flow behavior of these fluids and also to obtain invaluable scaled up information for industrial applications.

In dielectric fluids with low values of conductivity, the electric effects will essentially govern the motion. The forces that are exerted by an electric field on free charges present in a liquid are transmitted by collision to the neutral molecules. The fluid will be set in motion, thus changing the distribution of charges that in turn modifies the electric field. There is an analogy between Rayleigh-Bénard instability and pure electroconvection. In the latter case, the destabilizing force is proportional to the mean charge gradient. If alternating electric fields of sufficiently high frequency are employed, then Kelvin or polarization body force becomes the driving force for convection.

Onset of natural convection in the presence of an external electric field has been studied by Turnbull [9-11], Turnbull and Melcher[12], Takashima and Aldridge [13], Takashima and Gosh[14], Takashima and Hambata [15], Stiles et. al. [16], Ezzat and Othman [17], Siddheshwar [18], Siddheshwar and Abraham [19, 20], Siddheshwar and Chan [21], Shivakumara et. al. [22, 23], Rudraiah et. al. [24] and Siddheshwar and Radhakrishna [25].

The theory of micropolar fluid introduced by Eringen[26] have become an important field of research especially in many industrially important fluids like paints, polymeric suspensions, colloidal fluids, and also in physiological fluids such as normal human blood and synovial fluids. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and porous media is presented by Lukaszewicz [27].

The Rayleigh-Bénard instability in a horizontal thin layer of fluid heated from below is an important particular stability problem. The theory of thermomicropolar convection heated from below was studied by many authors Datta and Sastry [28], Ahmadi [29], Bhattacharya and Jena [30], Siddheshwar and Pranesh [31 – 35] and Pranesh and Kiran [36]. The literature pertaining to temperature modulation in micropolar fluid is mainly concerned with magnetic field and a corresponding study for micropolar fluid with effect of electric field is missing despite its importance in understanding control of convection encountered in many scientific and technological problems.

Therefore, main object of this paper is to study the effect of imposed temperature modulation and electric field on the stability of convective flow in a micropolar fluid by considering free-free boundaries.

2. MATHEMATICAL FORMULATION

Consider a layer of Boussinesquian, micropolar fluid confined between two infinite horizontal surfaces separated by a distance d apart. The uniform AC electric field is directed along the z-axis. A Cartesian system is taken with origin in the lower boundary and z-axis vertically upward (see figure 1).

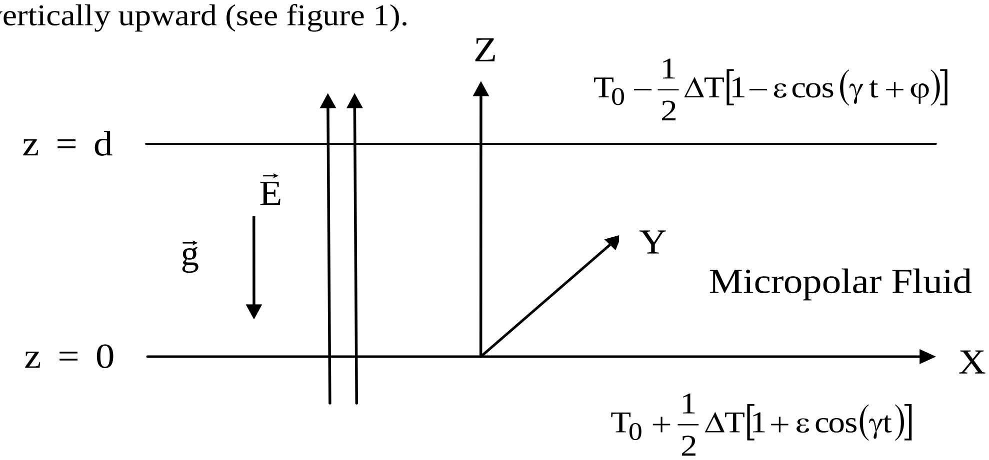

Figure 1: Physical configuration

The basic governing equations are:

Continuity equation:

$$\nabla.\vec{q} = 0, \tag{1}$$

Conservation of linear momentum:

$$\rho_o\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g\hat{k} + (2\zeta + \eta)\nabla^2\vec{q} + \zeta\nabla\times\vec{\omega} + (\vec{P}.\nabla)\vec{E}, \tag{2}$$

Conservation of angular momentum:

$$\rho_o I\left[\frac{\partial\vec{\omega}}{\partial t} + (\vec{q}.\nabla)\vec{\omega}\right] = (\lambda' + \eta')\nabla(\nabla.\vec{\omega}) + (\eta'\nabla^2\vec{\omega}) + \zeta(\nabla\times\vec{q} - 2\vec{\omega}), \tag{3}$$

Conservation of energy:

$$\frac{\partial T}{\partial t} + \left(\vec{q} - \frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}\right).\nabla T = \chi\nabla^2 T, \tag{4}$$

Equation of state:

$$\rho = \rho_o[1 - \alpha(T - T_o)], \tag{5}$$

Equation of state for dielectric constant:

$$\varepsilon_r = (1 + \chi_e) - e(T - T_0), \tag{6}$$

Faraday's law:

$$\left.\begin{aligned} &\nabla\times\vec{E} = 0 \\ &\vec{E} = -\nabla\phi, \end{aligned}\right. \tag{7}$$

Equation of polarization field:

$$\left.\begin{aligned} &\nabla.(\varepsilon_0\vec{E} + \vec{P}) = 0 \\ &\vec{P} = \varepsilon_0(\varepsilon_r - 1)\vec{E} \end{aligned}\right\}, \tag{8}$$

where, $\vec{q}$ is the velocity , ρ_0 is density of the fluid at temperature $T = T_0$, p is the pressure, ρ is the density, $\vec{g}$ is acceleration due to gravity, ζ is coupling viscosity coefficient or vortex viscosity, $\vec{P}$ is dielectric polarization, $\vec{E}$ is the electric field, λ and η are the bulk and shear spin-viscosity coefficients, $\vec{\omega}$ is the angular velocity, I is moment of inertia, λ' and η' are bulk and shear spin-viscosity coefficients, T is the temperature, χ is the thermal conductivity, β is micropolar heat conduction coefficient, α is coefficient of thermal expansion, σ is electrical conductivity, ε_r dielectric constant, $e = -\left(\frac{\partial \varepsilon_r}{\partial T}\right)_{T=T_o}$, χ_e electric susceptibility, ε_o electric permittivity of free space and ϕ electric scalar potential.

The wall temperatures are time dependent, externally imposed and are taken as

$$T(0,t) = T_0 + \frac{1}{2}\Delta T[1 + \varepsilon\cos(\gamma t)] \tag{9}$$

$$T(d,t) = T_0 - \frac{1}{2}\Delta T[1 - \varepsilon\cos(\gamma t + \varphi)] \tag{10}$$

where, ε is the amplitude of modulation, γ is the frequency of modulation and φ is the phase angle.

We consider three types of thermal modulation namely:

Case (a): Symmetric (in-phase, $\varphi = 0$)

Case (b): Asymmetric (out-of-phase, $\varphi = \pi$) and

Case (c): Only lower wall temperature is modulated while the upper wall is held at constant temperature ($\varphi = -i\infty$)

2.1 Basic State

The basic state of the fluid is quiescent and is described by:

$$\vec{q}_b = (0,0,0), p = p_b(z,t), \rho = \rho_b(z,t), \vec{E} = \vec{E}_b(z), \vec{P} = \vec{P}_b(z), T = T_b(z,t), \tag{11}$$

Substituting equation (11) into basic governing equations (1)-(8), we obtain the quiescent state solutions as:

$$\frac{\partial p_b}{\partial z} = \rho_b g + P_b \frac{\partial E_b}{\partial z}, \tag{12}$$

$$\frac{\partial T_b}{\partial z} = \chi \frac{\partial^2 T_b}{\partial z^2}, \tag{13}$$

$$\left.\begin{aligned}
&\rho_b = \rho_o\left[1-\alpha(T_b - T_o)\right],\\
&\varepsilon_r = (1+\chi_e) - e(T_b - T_0),\\
&\vec{E}_b = \left[\frac{(1+\chi_e)E_0}{(1+\chi_e)+\dfrac{e\Delta T}{h}z}\right]\hat{k},\\
&\vec{P}_b = \varepsilon_0 E_0(1+\chi_e)\left[1-\frac{1}{(1+\chi_e)+\dfrac{e\Delta T_z}{h}}\right]\hat{k},
\end{aligned}\right\}, \tag{14}$$

The solution of equation (13) that satisfies the thermal boundary conditions (9) and (10) is

$$T_b = T_0 + \frac{\Delta T}{2}\left(1-\frac{2z}{d}\right) + \varepsilon\,\mathrm{Re}\left\{\left[\alpha(\lambda)e^{\frac{\lambda z}{d}} + \alpha(-\lambda)e^{\frac{-\lambda z}{d}}\right]e^{-i\gamma t}\right\}, \tag{15}$$

where $$\lambda = (1-i)\left(\frac{\gamma d^2}{2\chi}\right)^{\frac{1}{2}}, \tag{16}$$

$$\alpha(\lambda) = \frac{\Delta T}{2}\left[\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right],$$

and Re stands for the real part.

We now superpose infinitesimal perturbations on this basic state and study the stability of the system.

2.2 Linear Stability Analysis

The stability of the basic state is analyzed by introducing the following perturbation

$$\vec{q} = \vec{q}_b + \vec{q}',\ p = p_b + p',\ \rho = \rho_b + \rho',\ T = T_b + T',\ \vec{E} = \vec{E}_b + \vec{E}',\ \vec{P} = \vec{P}_b + \vec{P}', \tag{17}$$

where the prime indicates that the quantities are infinitesimal perturbations. Let the components of perturbed polarization and electric field be $\left(P_1', P_2', P_b(z) + P_3'\right)$ and $\left(E_1', E_2', E_b(z) + E_3'\right)$.

The second equation of (8), on linearization yields

$$P_i^{'} = \varepsilon_0 \chi_e E_i^{'} \quad \text{for } i = 1,2$$

$$P_3^{'} = \varepsilon_0 \chi_e E_3^{'} - e\varepsilon_0 E_0 T^{'} \tag{18}$$

where it has been assumed that $e\Delta T << (1+\chi_e)$.

Equation (7) implies one can write $\vec{E}' = \nabla\phi'$.

Substituting equation (17) into equations (1)-(8) and using the basic state equations, we get linearized equations governing the infinitesimal perturbations in the form:

$$\nabla.\vec{q}^{'} = 0, \tag{19}$$

$$\rho_o\left[\frac{\partial\vec{q}'}{\partial t}\right] = -\nabla p - \rho' g\hat{k} + (2\zeta+\eta)\nabla^2\vec{q}' + (\zeta\nabla\times\vec{\omega}') + (\vec{P}_b.\nabla)\vec{E}^{'} + (\vec{P}^{'}.\nabla)\vec{E}_b, \tag{20}$$

$$\rho_o I\left[\frac{\partial\vec{\omega}'}{\partial t}\right] = (\lambda'+\eta')\nabla(\nabla\vec{\omega}') + (\eta'\nabla^2\vec{\omega}') + \zeta(\nabla\times\vec{q}' - 2\vec{\omega}'), \tag{21}$$

$$\frac{\partial T'}{\partial t} = \frac{\Delta T}{d}\left[\vec{q}' - \frac{\beta}{\rho_o C_v}\nabla\times\vec{\omega}'\right], \tag{22}$$

$$\rho' = -\varepsilon\rho_0 T^{'}, \tag{23}$$

$$\varepsilon' = -\varepsilon_0 e T^{'}, \tag{24}$$

$$\nabla.\left(\varepsilon_0\vec{E}' + \vec{P}'\right) = 0. \tag{25}$$

Introducing the electric potential ϕ', eliminating the pressure p in equation (20) and incorporating the quiescent state solution, we obtain the perturbed state vorticity transport equation in the form:

$$\rho_o I\left[\frac{\partial(\nabla\times\vec{\omega}')}{\partial t}\right] = (\lambda'+\eta')\nabla^2(\nabla\times\vec{\omega}') + (\eta'\nabla^2\vec{\omega}') + \zeta(\nabla^2 w - 2\nabla\times\vec{\omega}'), \tag{26}$$

Using equation (18) on equation (25), we obtain:

$$(1+\chi_e)\nabla^2\phi' - eE_0 DT^{'} = 0. \tag{27}$$

The perturbation equations (22), (25), (26) and (27) are non-dimensionalized using the following definitions:

$$(x^*, y^*, z^*) = \left(\frac{x'}{d}, \frac{y'}{d}, \frac{z'}{d}\right), t^* = \frac{t'}{d^2/\chi}, w^* = \frac{w'}{\chi/d}, T^* = \frac{T'}{\Delta T}, \phi^* = \frac{\phi'}{\dfrac{eE_0\Delta T d}{1+\chi_e}}. \tag{28}$$

to obtain the equations governing the infinitesimal perturbation (after dropping the asterisk).

$$\frac{1}{Pr}\frac{\partial}{\partial t}(\nabla^2 W) = R\nabla_1^2 T + (1+N_1)\nabla^4 W + N_1\nabla^2\Omega_z + L\nabla_1^2 T - L\frac{\partial}{\partial z}(\nabla_1^2\phi), \tag{29}$$

$$\frac{N_2}{Pr}\frac{\partial\Omega_z}{\partial t} = N_3\nabla^2\Omega_z - N_1\nabla^2 W - 2N_1\Omega_z, \tag{30}$$

$$\frac{\partial T}{\partial t} = \nabla^2 T + (W - N_5\Omega_z)\frac{\partial T_0}{\partial z}, \tag{31}$$

$$\nabla^2\phi - \frac{\partial T}{\partial z} = 0, \tag{32}$$

The non-dimensional parameters R, L, N_1, N_2, N_3, N_5 and Pr are given as

$$N_1 = \frac{\zeta}{\zeta+\eta} \qquad \text{(Coupling Parameter)},$$

$$N_2 = \frac{I}{d^2} \qquad \text{(Inertia Parameter)},$$

$$N_3 = \frac{\eta'}{(\zeta+\eta)d^2} \qquad \text{(Couple Stress Parameter)},$$

$$N_5 = \frac{\beta}{\rho_o C_v d^2} \qquad \text{(Micropolar Heat Conduction Parameter)},$$

$$R = \frac{\rho_o \alpha g \Delta T d^3}{\chi(\zeta+\eta)} \qquad \text{(Rayleigh number)},$$

$$L = \frac{\varepsilon_0 e^2 E_0^2 \Delta T^2 d^2}{(1+\chi_e)(\zeta+\eta)\chi} \qquad \text{(Electric number)},$$

$$\Pr = \frac{\mu}{\rho_0 \chi} \qquad \text{(Prandtl number).}$$

In equation (31), $\left(\frac{\partial T_0}{\partial z}\right)$ is the non-dimensional form of $\left(\frac{\partial T_b}{\partial z}\right)$, where

$$\frac{\partial T_0}{\partial z} = -1 + \varepsilon f(z), \tag{33}$$

where $f(z) = \mathrm{Re}\left\{\left[A(\lambda)e^{\lambda z} + A(-\lambda)e^{-\lambda z}\right]e^{-i\gamma t}\right\}$ (34)

and $A(\lambda) = \frac{\lambda}{2}\left[\frac{e^{-i\varphi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right],$ (35)

Equations (29) to (32) are solved subject to the conditions

$$w = \frac{\partial^2 w}{\partial z^2} = T = \frac{\partial \phi}{\partial z} = 0 \ \text{ at } z = 0, 1 \tag{36}$$

Eliminating T, Ω_z and ϕ from equations (28)-(30), we get

$$\left(\frac{N_2}{\Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1\right)\left\{\begin{aligned}&\left(\frac{\partial}{\partial t} - \nabla^2\right)\left[\nabla^4\left(\frac{1}{\Pr}\frac{\partial}{\partial t} - (1+N_1)\nabla^2\right) + N_1^2\nabla^6\right]\\ &+ L\nabla_1^4\frac{\partial T_0}{\partial z}\end{aligned}\right\}W$$
$$+ LN_1N_5\nabla^2\nabla_1^4\frac{\partial T_0}{\partial z}W = -R\nabla^2\nabla_1^2\frac{\partial T_0}{\partial z}\left(\frac{N_2}{\Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1 + N_1N_5\nabla^2\right)W \tag{37}$$

In dimensionless form, the velocity boundary conditions for solving equation (37) are obtained from equations (29) to (32) and (36) in the form:

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial^4 w}{\partial z^4} = \frac{\partial^6 w}{\partial z^6} = \frac{\partial^8 w}{\partial z^8} = 0 \qquad \text{at } z = 0, 1. \tag{38}$$

3. METHOD OF SOLUTION

We now seek the eigen-function *w* and eigen-values R of the equation (37) for the basic temperature distribution (33) that departs from the linear profile $\frac{\partial T_0}{\partial z} = -1$ by quantities of order ε. Thus, the eigen-values of the present problem differ from those of the ordinary

Bénard convection by quantities of order ε. We seek the solution of equation (37) in the form:

$$(R, w) = (R_0, w_0) + \varepsilon(R_1, w_1) + \varepsilon^2 (R_2, w_2) + \ldots\ldots\ldots\ldots \tag{39}$$

The expansion (38) is substituted into equation (36) and the coefficients of various powers of ε are equated on either side of the equation. The resulting system of equation is

$$L_1 w_0 = 0, \tag{40}$$

$$\begin{aligned} L_1 w_1 = &-L\nabla_1^4\left[\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1\right) + N_1N_5\nabla^2\right] f w_0 \\ &- \nabla^2\nabla_1^2\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1 + N_1N_5\nabla^2\right) f R_0 w_0, \end{aligned} \tag{41}$$

$$\begin{aligned} L_1 w_1 = &-L\nabla_1^4\left[\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1\right) + N_1N_5\nabla^2\right] f w_1 \\ &- \nabla^2\nabla_1^2\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1 + N_1N_5\nabla^2\right)(f R_0 w_1 - R_2 w_0), \end{aligned} \tag{42}$$

where

$$\begin{aligned} L_1 = &\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1\right)\left\{\left(\frac{\partial}{\partial t} - \nabla^2\right)\left[\nabla^4\left(\frac{1}{Pr}\frac{\partial}{\partial t} - (1+N_1)\nabla^2\right) + N_1^2\nabla^6\right] - L\nabla_1^4\right\} \\ &- LN_1N_5\nabla^2\nabla_1^4 - R_0\nabla^2\nabla_1^2\left(\frac{N_2}{Pr}\frac{\partial}{\partial t} - N_3\nabla^2 + 2N_1 + N_1N_5\nabla^2\right). \end{aligned} \tag{43}$$

3.1 Solution To The Zeroth Order Problem

The zeroth order problem is equivalent to the Rayleigh-Benard problem of Micropolar fluid with electric field in the absence of temperature modulation. The linear analysis of Rayleigh-Benard convection in micropolar fluid without electric field has been thoroughly investigated by Siddheshwar and Pranesh [31].

The stability of the system in the absence of thermal modulation is investigated by introducing vertical velocity perturbation w_0 corresponding to lowest mode of convection as:

$$w_0 = \sin(\pi z)\exp\left[i(lx + my)\right], \tag{44}$$

where l and m are horizontal wave number in x and y direction.

Substituting equation (44) into equation (40) we obtain the expression for Rayleigh number in the form

$$R_0 = \frac{\left(N_3K^2 + 2N_1\right)\left[K^8\left(1 + N_1 - N_1^2\right) - La^4\right] + LN_1N_5K^2a^4}{-K^2a^2\left(N_1N_5K^2 - N_3K^2 - 2N_1\right)}, \tag{45}$$

where $K^2 = \pi^2 + a^2$ and $a^2 = l^2 + m^2$.

In the absence of the electric field (i.e. L = 0), equation (45) reduces to

$$R_0 = \frac{\left(N_3K^2 + 2N_1\right)\left[K^8\left(1 + N_1 - N_1^2\right)\right]}{-K^2a^2\left(N_1N_5K^2 - N_3K^2 - 2N_1\right)},$$

which is the expression for Rayleigh number discussed by [30, 32]. Setting $N_1 = 0$ and keeping N_3 and N_5 arbitrary in the above expression we get

$$R_0 = \frac{K^6}{a^2},$$

which is the classical Rayleigh- Bénard result.

3.2 Solution To The First Order Problem

Equation (41) for w_1 now takes the form

$$L_1w_1 = \left(-La^4f + R_0a^2K^2f - R_1a^2K^2\right)A_1w_0, \tag{46}$$

where $A_1 = N_3K^2 + 2N_1 - N_1N_5K^2$

If the above equation is to have a solution, the right hand side must be orthogonal to the null-space of the operator L_1. This implies that the time independent part of the RHS of the equation (46) must be orthogonal to $\sin(\pi z)$. Since f varies sinusoidal with time, the only steady term on the RHS of equation (45) is $R_1a^2K^2\sin(\pi z)$, so that $R_1 = 0$. It follows that all the odd coefficients i.e. $R_1 = R_3 = ... = 0$ in equation (39).

To solve equation (46), we expand the right-hand side using Fourier series expansion and obtain w_1 by inverting the operator L_1 term by term as

$$W_1 = A_1 a^2\left(-La^2 + R_0 K^2\right)\mathrm{Re}\sum \frac{B_n(\lambda)}{L(\gamma,n)} e^{-i\gamma t}\sin(n\pi z) \tag{47}$$

where $B_n(\lambda) = A(\lambda)g_{n1}(\lambda) + A(-\lambda)g_{n1}(-\lambda)$

$$= \frac{-2n\pi^2\lambda^2\left[e^{\lambda} - e^{-\lambda} + (-1)^n\left(e^{-\lambda - i\phi} - e^{\lambda - i\phi}\right)\right]}{\left(e^{\lambda} - e^{-\lambda}\right)\left[\lambda^2 + (n+1)^2\pi^2\right]\left[\lambda^2 + (n-1)^2\pi^2\right]} \tag{48}$$

$$L_1(\gamma,n) = \left[X_1X_2 + \frac{\gamma^2 N_2}{Pr}X_3 - La^4X_2 + LN_1N_5K_n^2a^4 + R_0K_n^2a^2X^4\right]$$
$$+ i\gamma\left[-K_n^4\frac{N_2}{Pr}X_1 + X_2X_3 + \frac{LN_2a^4}{Pr} + R_0K_n^2a^2\frac{N_2}{Pr}\right],$$

$$X_1 = K_n^4\left[(1+N_1)K_n^4 - \frac{\gamma^2}{Pr} - N_1^2K_n^4\right],$$

$$X_2 = \left(N_3K_n^2 + 2N_1\right),$$

$$X_3 = K_n^4\left[-(1+N_1)K_n^2 - \frac{K_n^{\;2}}{Pr} + N_1^2K_n^2\right],$$

$$X_4 = -N_3K_n^2 - 2N_1 + N_1N_5K_n^2,$$

and

$$K_n^2 = n^2\pi^2 + a^2.$$

The equation for w_2, then becomes

$$L_1 w_2 = \left(-La^4 - R_0a^2K^2\right)fA_1w_1 + R_2a^2K^2w_0 \tag{49}$$

We shall not solve equation (49), but will use this to determine R_2. The solvability condition requires that the time-independent part of the right hand side of equation (49) must be orthogonal to $\sin(n\pi z)$, and this results in the following equation,

$$R_2 = \frac{2\left[La^2 + R_0k_n^2\right]}{k_1^2}\int_0^1 \overline{A_1 f w_1}\sin(\pi z)\,dz$$

where overbar denotes the time average.

From equation (46), we have,

$$\overline{A_1 f w_1} = \frac{\overline{A_1 L_1 w_1}}{-R_o a^2 k_1^2 - La^4}$$

simplifying we get,

$$\overline{A_1 f w_1}\sin(\pi z) = -\left(\frac{R_o a^2 k_1^2 + La^4}{2}\right)\mathrm{Re}\left\{\sum_{n=1}^{\infty}\frac{B_n(\lambda)}{L_1(\gamma,n)}\sin(n\pi z)\sum_{n=1}^{\infty}B_n^*(\lambda)\sin(n\pi z)\right\},$$

And finally,

$$R_2 = \left(\frac{-R_0 k_1^2 - La^2}{2k_1^2}\right)\sum_{n=1}^{\infty}\left[\left(La^4 + R_0 a^2 k_n^2\right)\mathrm{Re}\left[\frac{|B_n(\lambda)|^2}{L_1(\gamma,n)}\right]\right]$$

$$= \left(\frac{-R_0 k_1^2 - La^2}{2k_1^2}\right)\sum_{n=1}^{\infty}\left[\left(La^4 + R_0 a^2 k_n^2\right)\frac{|B_n(\lambda)|^2}{|L_1(\gamma,n)|^2}\left[\frac{L_1(\gamma,n) + L_1^*(\gamma,n)}{2}\right]\right]$$

$$R_{2c} = -\frac{\left(La^2 + R_0 K^2\right)}{2K^2}\sum\left(La^4 + R_0 a^2 K_n^2\right)\frac{|A_2|^2 |B_n(\lambda)|^2\left[L_2(\gamma,n) + L_2^*(\gamma,n)\right]}{2|L_2(\gamma,n)|^2} \qquad (50)$$

where

$$A_2 = N_3 K_n{}^2 + 2N_1 - N_1 N_5 K_n{}^2 - \frac{i\gamma N_2}{Pr},$$

$L_2(\gamma,n) = A_2 L(\gamma,n)$ and

$L_2{}^*(\gamma,n)$ is the conjugate of $L_2(\gamma,n)$ respectively.

4. MINIMUM RAYLEIGH NUMBER FOR CONVECTION

The value of Rayleigh number R obtained by this procedure is the eigenvalue corresponding to the eigen function w, which, though oscillating, remains bounded in time. Since R is a function of the horizontal wave number *a* and the amplitude of modulation ε, we have

$$R(a,\varepsilon) = R_0(a) + \varepsilon^2 R_2(a) + \ldots\ldots\ldots\ldots \qquad (51)$$

It was shown by [1] that the critical value of thermal Rayleigh number is computed up to $O(\varepsilon^2)$, by evaluating R_0 and R_2 at $a = a_0$. It is only when one wishes to evaluate R_4 that a_2 must be taken into account where $a = a_2$ minimizes R_2. To evaluate the critical value of R_2 (denoted by R_{2c}) one has to substitute $a = a_0$ in R_2, where a_0 is the value at which R_0 given by equation (44) is minimum.

We now evaluate R_{2c} for three cases:

Case (a): When the oscillating field is symmetric so that the wall temperatures are modulated in-phase with $\varphi = 0$. In this case, $B_n(\lambda) = b_n$ or 0, accordingly as n is even or odd.

Case (b): When the wall temperature field is antisymmetric corresponding to out-of-phase modulation with $\varphi = \pi$. In this case, $B_n(\lambda) = 0$ or b_n, accordingly as n is even or odd.

Case (c): When only the temperature of the bottom wall is modulated, the upper plate being held at constant temperature, with $\varphi = -i\infty$. In this case, $B_n(\lambda) = \dfrac{b_n}{2}$, for integer values of *n*.

where

$$b_n = \frac{-4n\pi^2\lambda^2}{[\lambda^2 + (1+n)^2\pi^2][\lambda^2 + (1-n)^2\pi^2]}.$$

The variable λ defined in equation (15), in terms of the dimensionless frequency, reduces to

$$\lambda = (1-i)\left(\frac{\gamma}{2}\right)^{\frac{1}{2}},$$

and thus

$$|b_n|^2 = \frac{16n^2\pi^4\gamma^2}{[\gamma^2 + (1+n)^4\pi^4][\gamma^2 + (1-n)^4\pi^4]}.$$

Hence from equation (50) and using the above expression of $B_n(\lambda)$, we can obtain the following expression for R_{2c} in the form

$$R_{2c} = -\frac{\left(La^2 + R_0 K^2\right)}{2K^2} \sum \left(La^4 + R_0 a^2 K_n^2\right) \frac{|A_2|^2 |B_n(\lambda)|^2 \left[L_2(\gamma, n) + L_2^*(\gamma, n)\right]}{2|L_2(\gamma, n)|^2} \qquad (52)$$

In equation (52) the summation extends over even values of *n* for case (a), odd values of *n* for case (b) and for all values of *n* for case (c).The infinite series (51) converges rapidly in all cases. The variation of R_{2c} with γ for different values of N_1 , N_3, N_5 L and Pr are depicted in figures (2)-(7).

Results and Discussions:

In this paper, an analytical study is made on the effects of temperature modulation and electric field on the onset of convection in a horizontal layer of a micropolar fluid. The expression for the critical correction Rayleigh number R_{2c} is computed as function of the frequency of the modulation γ and for different parameters. The value of R_{2c} has been calculated for the following three cases; (a) when the wall's temperature is modulated in-phase i.e., $\varphi = 0$, (b) when the wall's temperature is modulated out-of-phase, i.e., $\varphi = \pi$ and (c) when only the lower wall temperature is modulated, the upper wall is held at constant temperature, i.e., $\varphi = -i\infty$.

The analysis presented is based on the assumption that the amplitude of the modulating temperature is small compared with the imposed steady temperature difference. The validity of the results obtained here depends on the value of the modulating frequency γ. When $\gamma << 1$, the period of modulation is large and hence the disturbance grows to such an extent that it makes finite amplitude effects important. When $\gamma \to \infty$, $R_{2c} \to 0$, thus the effect of modulation becomes small. In view of this, we choose only moderate values of γ in our present study.

The results have been presented in figures (2)-(15). From the figures it is observed that the value of R_{2c} may be positive or negative. The sign of R_{2c} characterizes the stabilizing or destabilizing effect of modulation. A positive R_{2c} means the modulation effect is stabilizing while a negative R_{2c} means the modulation effect is destabilizing compared to the system in which the modulation is absent.

The effect of in-phase modulation of wall temperature on the onset of convection in a horizontal layer of micropolar fluid with electric field is shown in figures (2) – (7) for different values of coupling parameter N_1, inertial parameter N_2, coupling parameter N_3, micropolar heat conduction parameter N_5, electric Rayleigh number L and Prandtl number Pr. From these figures, we find that for low frequency γ, R_{2c} becomes more and more negative indicating that in-phase modulation for low values of γ is destabilizing and for moderate values of γ, R_{2c} becomes less and less negative indicating that in-phase modulation for moderate values of γ is stabilizing. Let γ_c be the frequency at which the R_{2c} changes from destabilizing to stabilizing, then the modulated system may be classified as destabilizing or stabilizing according as $\gamma < \gamma_c$ or $\gamma > \gamma_c$ when compared with the un-modulated system. For some particular value of γ, R_{2c} becomes zero. This is due to the fact that when the frequency of modulation is low, the effect of modulation on the temperature field is felt throughout the fluid layer. If the plates are modulated in-phase, the temperature profile consists of the steady straight line section plus a parabolic profile which oscillates in time. As the amplitude of modulation increases, the parabolic part of the profile becomes more and more significant. It is known that a parabolic profile is subject to finite amplitude instabilities so that convection occurs at lower Rayleigh number than those predicted by the linear theory. From the figures (2) – (7) for in-phase modulation the following points are noted:

Figure (2) is the plot of correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of electric Rayleigh number L in respect of in-phase modulation. The electric Rayleigh number L is the ratio of electric force to gravitational force. We see from the figure when L is greater than 1803 super critical motions occur and R_{2c} increases with an increase in L at a given frequency γ. Hence L has a stabilizing effect on the flow. When L is less than 1803 subcritical motions occurs. It is also interesting to see from the figure that for a given L (L<1803), R_{2c} first decreases with increase in γ, reaches a minimum and then increases with increase in γ and for a given L (L>1803) R_{2c} increases with increase in γ reaches the maximum and then decreases with increase in γ. This shows that for a weak dielectric fluid, the flow is destabilized for small values of γ and stabilized for large values of γ. This is due to the fact that when the frequency of modulation is low, the effect of modulation is felt throughout the fluid.

Figure (3) is the plot of correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of coupling parameter N_1 in respect of in-phase modulation. We observe that as N_1 increases, R_{2c} increases in negative direction. The increase in N_1 implies increase in the concentration of suspended particles. These suspended particles consume the greater part of the energy in forming the gyrational velocity and as a result R_{2c} becomes more and more negative.

Figure (4) is the plot of correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of inertia parameter N_2 in respect of in-phase modulation. Increase in N_2 is representative of the increase in inertia of the fluid due to the suspended particles. Thus, as is to be expected, we find that as N_2 increases R_{2c} becomes less and less negative thereby stabilizing the system. Since N_2 essentially arises with the acceleration term, it does not have any influence on R_{oc}. It influences only R_{2c}.

Figure (5) is the plot of correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of couple stress parameter N_3 in respect of in-phase modulation. The role played by the shear stress in the conservation of linear momentum is played by couple stress in the conservation of angular momentum equation. Increase in N_3 signifies decrease in gyrational velocities. Hence, as N_3 increases, we observe that R_{2c} becomes less and less negative.

Figure (6) is the plot of correction Rayleigh number R_{2c} versus frequency of modulation γ for different values of micropolar heat conduction parameter N_5 in respect of in-phase modulation. An increase in N_5 implies that the heat induced into the system also increases resulting in reduced heat transfer from bottom to top. As a result, we find from the figure that as N_5 increases R_{2c} increases in negative direction.

Figure (7) is a plot of R_{2c} versus γ, for different values of Pr for in-phase modulation. It is observed that as Pr increases R_{2c} increases in negative direction. It can be inferred from this that the effect of increasing the concentration of the suspended particle is to destabilize the system. This means that the fluids with suspended particles are more vulnerable than clean fluids to destabilization by modulation. It is appropriate to note that Pr does not affect the R_0 part of R.

The effect of out-of-phase modulation on the wall temperature on the onset of convection is shown in figures (8) and (9). It is found that R_{2c} is positive for out-of-phase whereas it is negative for in-phase. Thus N_1 and L have opposing influences for in-phase and out-of-phase modulations. The above results are due to the fact that in the case of out-of-phase modulation the temperature field has essentially a linear gradient varying in time, so that the instantaneous Rayleigh number is supercritical for half a cycle and subcritical during the other half cycle.

The above results on the effect of various parameters on R_{2c} for out-of-phase modulation do not qualitatively change in the case of temperature modulation of just the lower boundary. This is illustrated in the figures (9) and (10).

From the study following conclusion can be made:

1. The system is more stable when boundary temperature are modulated in out-of-phase.
2. In-phase temperature modulation leads to subcritical motions.
3. The results of the study throw light on an external means of controlling the internal convection with electric field, either advancing or delaying convection by temperature modulation in a micropolar fluid.
4. The suspended particles scale down the effect of temperature modulation.

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References:

[1] G. Venezian, Effect of modulation on the onset of thermal convection, J. of Fluid Mech. 35 (1969) 243-254.

[2] R. J. Donnelly, Experiments on the stability of viscous flow between rotating cylinders. III Enhancement of stability by modulation. Proc. Roy. Soc. Lond. A. 281 (1964) 130-139.

[3] B. S. Bhadauria, Lokenath Debnath, Effects of modulation on Rayleigh-Benard convection (part I). Int. J. Math. and Math. Sci. 19 (2004) 991-1001.

[4] B. S. Bhadauria, Time periodic heating of Rayleigh-Benard convection in a vertical magnetic field, Phys. Scr. 73 (2006) 296-302.

[5] B. S. Bhadauria, Fluid convection in a rotating porous layer under modulated temperature on the boundaries. Transp. in Porous Media. 67 (2007).

[6] S. Pranesh, Sangeetha George, Effect of magnetic field on the onset of Rayleigh-Benard convection in Boussinesq-Stokes suspensions with time periodic boundary temperatures, Int. J. of Appl. Math. and Mech. 6 (2010) 38-55.

[7] P.G. Siddheshwar, B.S. Bhadauria, Mishra Pankaj, Atul K. Srivastava, Study of heat transport by stationary magneto-convection in a Newtonian liquid under temperature or gravity modulation using Ginzburg-Landau model, J. Non-linear Mech. 47 (2012) 418-425.

[8] P.G. Siddheshwar, B.S. Bhadauria, Alok Srivastava, An analytical study of nonlinear double-diffusive convection in a porous medium under temperature/gravity modulation, Transp. in Porous Media, 91 (2012) 585-604.

[9] R. J. Turnbull, Electro convective instability with a stabilizing temperature gradient. I. Theory, Phys. of Fluids, 11 (1968) 2588-2596.

[10] R. J. Turnbull, Electro convective instability with a stabilizing temperature gradient. II. Experimental Results. Phys. of Fluids, 11 (1968) 2597-2603.

[11] R. J. Turnbull, Effect of dielectrophoretic forces on Benard instability, Phys. of Fluids, 12 (1969) 1809.

[12] R. J. Turnbull, J. R. Melcher, Electrohydrodynamic Rayleigh-Taylor bulk instability. Phys. of Fluids, 12 (1969) 1160.

[13] M. Takashima, K. D. Aldridge, The stability of a horizontal layer of dielectric field under the simultaneous action of a vertical dc electric field and a vertical temperature gradient. Q. J. Mech. Appl. Math., 29 (1979) 71-87.

[14] M. Takashima, A. K. Ghosh, Electrohydrodynamic instability in a viscoelastic liquid layer. J. Phys. Soc. Jpn. 47 (1979) 1717-1722.

[15] M. Takashima, H. Hamabatta, The stability of natural convection in a vertical layer of dielectric fluid in the presence of a horizontal ac electric field. J. Phys. Soc. Jpn. 53 (1984) 1728-1736.

[16] P. J. Stiles, F. Lin, P. J. Blennerhassett, Convective heat transfer through polarized dielectric liquids, Phys. of Fluids A. 5 (1993) 3273-3280.

[17] M. A. Ezzat, M. I. A. Othman, Thermal instability in a rotating micropolar fluid layer subject to an electric field, Int. J. of Eng. Sci. 38 (2000) 1851-1867.

[18] P. G. Siddheshwar, Ferrohydrodynamic and electrohydrodynamic instability in Newtonian liquids: An analogy, East West J. Math. Special Volume: Computational Mathematics and Modelling (2002) 143-146.

[19] P.G. Siddheshwar, A. Abraham, Effect of time-periodic boundary temperatures/body force on Rayleigh-Bénard convection in a ferromagnetic fluid, Acta Mech. 161 (2003) 131–150.

[20] P.G. Siddheshwar, A. Abraham, Rayleigh-Benard convection in a dielectric liquid: imposed time-periodic boundary temperatures, Chamchuri J. of Math. 1 (2009) 105-121.

[21] P. G. Siddheshwar, A. T. Y. Chan, Ferrohydrodynamic and electrohydrodynamic instability in viscoeleastic liquids: an analogy, Proc. Fourth Int. Conf. Fluid Mech. (2004)

[22] I. S. Shivakumara, M. S. Nagashree, K. Hemalatha, Electrothermoconvection instability in a heat generating dielectric fluid layer, Int. Comm. Heat Mass Transp. 34 (2007) 1041-1047.

[23] I. S. Shivakumara, Ying, Chiu-On, M. S. Nagashree, The onset of electrothermoconvection in a rotating Brinkman porous layer. Int. J. Eng. Sci. 49 (2011) 646-663.

[24] N. Rudraiah, V. Kumudini, W. Unno, Theory of non-linear magnetoconvection and its application to solar convection problems, Int. Publ. Astron. Soc. Jpn. 37 (1985) 183.

[25] P. G. Siddheshwar, D. Radhakrishna, Linear and nonlinear electroconvection under AC electric field, Comm. Nonlinear Sci. and Numerical Simul. 17 (2012) 2883-2895.

[26] A. C. Eringen, Theory of Micropolar Fluids, Int. J. Eng. Sci. 16 (1966) 1.

[27] Lukaszewicz, Micropolar fluid theory and applications, Birkhauser Boston, M.A. USA. (1999)

[28] B. Datta, V. U. K. Sastry, Thermal instability of a horizontal layer of micropolar fluid heated from below, Int. J. Eng. Sci., 14 (1976) 631.

[29] G. Ahmadi, Stability of a Micropolar fluid layer heated from below, Int. J. Eng. Sci.14 (1976) 81.

[30] S. P. Bhattacharyya, S. K. Jena, On the stability of hot layer of micropolar fluid. Int. J. Eng. Sci. 31 (1983) 1019.

[31] P. G. Siddheshwar, S. Pranesh, Magnetoconvection in a Micropolar fluid, Int. J. Eng. Sci. 36 (1998) 1173-1181.

[32] P. G. Siddheshwar, S. Pranesh, Effects of a non-uniform basic temperature gradient on Rayleigh-Bénard convection in a micropolar fluid, Int. J. Eng. Sci. 36 (1998) 1183-1196.

[33] P. G. Siddheshwar, S. Pranesh, Magnetoconvection in fluids with suspended particles under 1g and μg, Int. J. of Aerosp. Sci. and Technology, 6 (2001) 105-114.

[34] P. G. Siddheshwar, S. Pranesh, Suction-Injection effects on the onset of Rayleigh-Benard-Marangoni convection in a fluid with suspended particles, Acta Mechanica, 152 (2001) 241-252.

[35] P. G. Siddheshwar, S. Pranesh, Linear and weakly non-Linear analyses of convection in a micropolar fluid, Hydrodynamics VI-Theory and Applications – Cheng & Yow (eds) (2005) 487 – 493.

[36] S. Pranesh, R. V. Kiran, Study of Rayleigh-Bénard Magneto Convection in a micropolar fluid with Maxwell-cattaneo Law, Appl. Math.1 (2010) 467-480.