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Thermal Analysis and Regression for Temperature Rise in a Cuboid Shaped Electrically Conductive Pin or Terminal

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***Abstract*- The metals present in current carrying components such as connector pins, receptacles, bus-bars etc., play a crucial role in effective functioning of electronic and electrical devices today. In the components where current flow is involved, temperature rise is observed due to joule heating phenomena. The temperature rise has to be minimum such that undesirable effects due the same are avoided. Excess temperature rise may lead to uneven bending of components, acceleration of fatigue failure, uneven thermal expansion and in extreme cases burn out. So, an efficient design is necessary to avoid such catastrophes. Having an analytical formula for temperature rise obtained due to joule heating phenomena for such standard components will aid the design engineers in optimizing the design, reduce the time taken in the process and ensuring that temperature rise is minimum. This study presents such analytical formulation for a standard cuboid shaped pin to obtain a formula for temperature rise considering its dimensions and current flowing through it.**

***Keywords*- Connectors, terminals, pins, natural convection, Regression and Analytical formulation.**

INTRODUCTION

Whenever there is flow of current in an electrical conductor (such as pins, terminals or bus-bars), it heats up and as a result rise in temperature is observed. The electrical conductor gets heated up due to joule heating effect [1]. Rise in temperature of electrical conductors due to joule heating causes many undesirable problems like uneven bending of components, distortion of material properties, acceleration of damage due to fatigue, increase in electrical resistance, drop in the efficiency of electrical energy carrying capacity and in some cases the component might also get burnt out. Due to these reasons, it becomes very important to design components such that the temperature rise observed is less. So, selection of proper components from the available standard or designing a new system considering joule heating effect is very important in order to avoid undesirable effects. Hence, having analytical formula for certain standard conductors would be help full in designing components, considering the negative effects of joule heating. One such correlation exists for simple horizontal (Churchill and Chu) and vertical plates (McAdams, William H.) with constant surface temperature [2,3], where a convection heat transfer coefficient is determined in terms of non-dimensional numbers for natural convection. In natural convection systems, external flow devices like fans, blowers etc., are not used. In these systems air flows due to the effect of buoyancy caused by decrease in density with increase in temperature [4]. The velocity of air flow is very small in these systems. However, these correlations are valid only when there is heat transfer from a surface due to its high temperature. It does not consider the effect of radiation on temperature rise which has a higher significance in natural convection system. This correlation is for a general case and does not consider heat generation terms like joule heating effect.

Since there are limitations in using existing correlations as mentioned before, other alternative is to perform numerical simulations for number of cases by varying input parameters and then performing data analysis to obtain a formula. In this method, for each mesh cell, simultaneous equations involving continuity, momentum, energy (Navier- stokes equations [5]), and current flow are solved. Theoretically, this technique doesn't need measurement information. However, the method is time-consuming to compute and requires a model that has been validated [6]. Another method is to perform experiments on a physical system and then use the data obtained from the same for correlation and analytical formulation. This method is expensive and requires to build physical setup and may involve measurement errors. Ryfa, et al had conducted such study on a naturally ventilated and hermetic switch gear bus-bar system and found a formula for convection heat transfer coefficient [7]. The law of conservation of energy was used for analytical formulation, i.e., heat generated due to joule heating is equal to sum of heat lost due to convection and radiation. In order to accommodate error involved in measuring each term in experimental investigation, data reconciliation method was used wherein an uncertainty in measurement was fixed for each term in the formula and regression equation was obtained [8]. In this section we have seen how to arrive at an analytical formula for convection heat transfer rate using different methods. In the upcoming sections, an application where one of these methods are applied in order to arrive at an analytical formula for a standard shape. In this formula, temperature rise due to flow of current is obtained directly instead of convection heat transfer coefficient.

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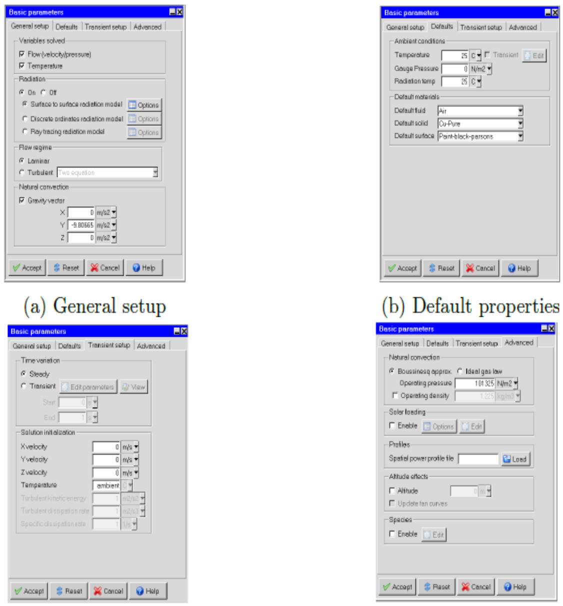
THERMAL ANALYSIS

Before proceeding into regression and analytical formulation, temperature rise data (due to joule heating effect) has to be available for any given standard geometry and current input. This data can either be obtained through experiments performed on the considered sample or by performing thermal analysis on a commercial software. Since performing experiments is an expensive and time-consuming process, thermal analysis method was selected to obtain the rise in temperature for a given current input, But, before performing thermal analysis a standard shape with dimensions needs to be defined. Hence, a bar with rectangular cross-section (cuboid shape) was selected. The length of the sample to be used for simulation were 20, 30 and 50mm respectively. Similarly, breadth of 5,10 and 15mm were considered and thickness was varied between 0.5, 1 and 1.5mm respectively. For each individual sample size, current inputs of 20A, 40A, 60A and 80A are given and thermal simulations are to be performed for a total of 108 different cases in order to obtain an analytical formula for rise in the temperature for known current. the material used for all simulations is pure copper.

Now that sample size and current inputs are decided, thermal analysis are to be performed for each case. The software package used to perform thermal analysis and obtain temperature rise (using joule heating phenomena) is Ansys Icepak. The first step is to define the basic set up of the problem, where certain prerequisites for solving are mentioned. There is a separate tab wherein these prerequisites can be added. The prerequisites defined are type of flow (laminar), direction and magnitude of gravity, default fluid, default solid, model used to solve for equation of state (ideal gas law or bousinessq approximation) [9, 10] and type of analysis (steady state in this case). Surface to surface radiation model was used to consider radiation for solving the problem and default surface was also defined in the advanced tab inside basic parameters section. Figure 1 displays the different tabs in basic parameters section. After defining the basic parameters, geometric modelling has to be done. A block is defined resembling a cuboid since the sample considered is of similar shape. Current input is defined on one side of the block and zero voltage is applied on another. This makes the current flow from one side to another. A computational domain is to be defined in order to setup the boundaries where the problem is to be solved. The boundaries of cabinet in each direction are opened to mimic natural convection real-time. The size of cabinet is defined such that its size is thrice that of block in two directions (where gravity is zero) and at least 100mm above the block (opposite to the direction where gravity is present) and 5mm below (where gravity is acting). The model depicting block and cabinet is shown in fig 2. One of the most important steps after modelling the block and cabinet is meshing. To discretize the domain into numerous smaller volumes of finite size, meshing is done. The primary purpose of discretization is to solve the navier-stokes equations by numerical method. Any given domain or hexahedral option.



(a) General setup (b) Default properties (c) Transient setup (d) Advanced tab

Fig 1: Defining basic parameters

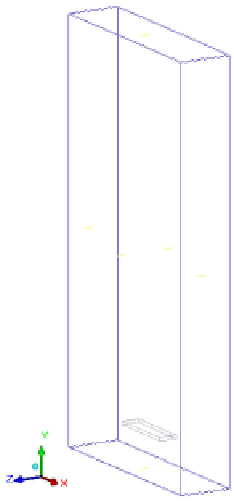


Fig 2: Model depicting block and cabinet

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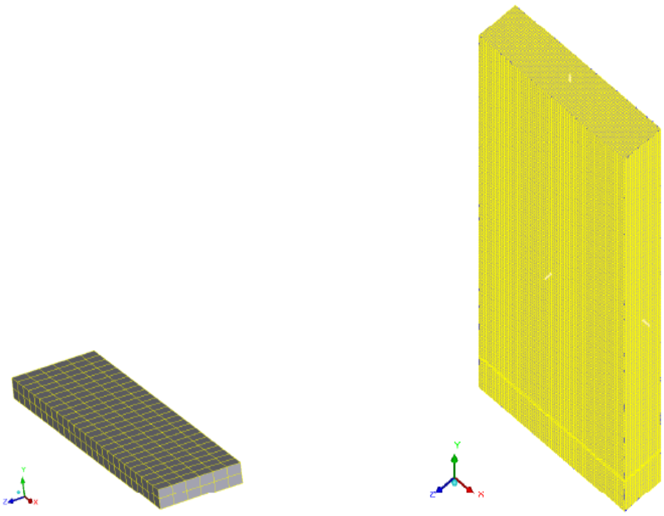
Mesher-hd divides the domain into smaller volumes with the help of tetrahedral and hexahedral elements whereas hexahedral option uses only hexahedral elements. Since the model and cabinet both resemble a hexahedron (cuboid with six faces) hexahedral mesh option is used to discretize the domain. The convention used in order to define the size of mesh is that minimum twenty elements should exist on any side of cabinet and three elements on any given edge. The meshed model of cabinet and block is shown in fig 3. Figure 4 depicts the window to define the mesh size and type.

Before proceeding into solution, the quality of mesh has to be confirmed. Three parameters are primarily checked which are face allignment, skewness and volume. Face allignment is the angle between the vector connecting two



(a) Meshing of block (b) Meshing of cabinet

Fig 3: Mesh created for block and cabinet

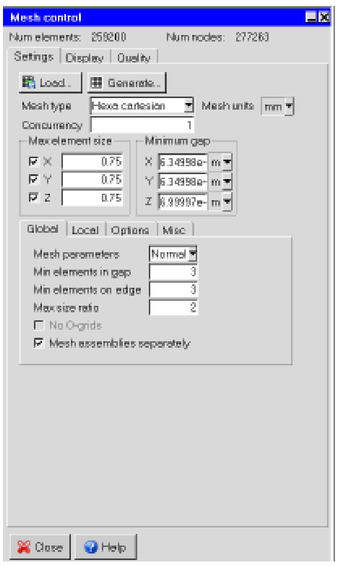
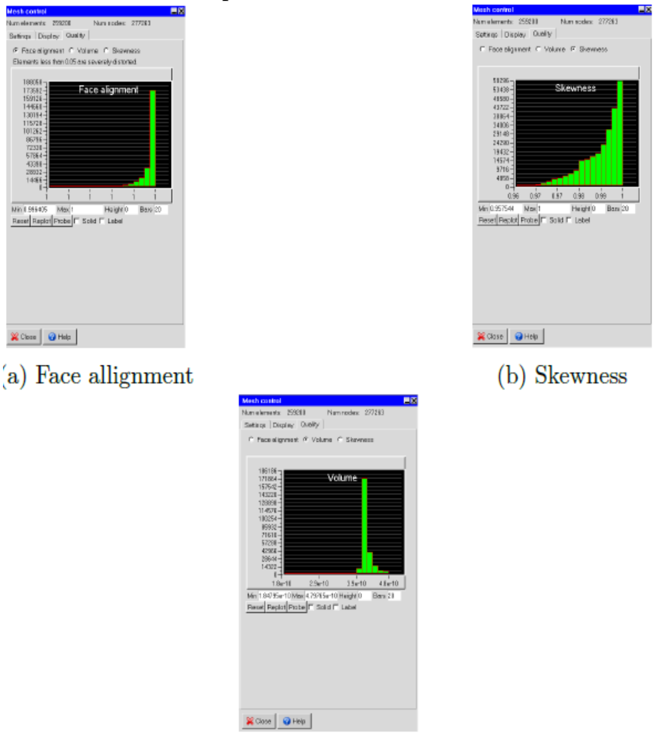


Fig 4: Window pane to input mesh size and type

neighbouring cell centers and the normal of the shared face between those centers. Skewness is the measure of deviation of the cell from an ideal one. Volume depicts the volume of individual cell. Ideally, the value of face allignment and skewness should be one, however due to geometry of model it is very difficult to obtain ideal value. Hence a face allignment value of 0.2 or above and skewness value equal to or above 0.05 is considered to be a mesh of good quality. The ratio of maximum volume to minimum should be of the order 10^7 or less for a mesh to be considered of good quality. Figure 5 displays the mesh quality obtained for one of the cases considered for simulation which depicts that the quality of mesh is very good and suitable to proceed for solution. In order to proceed with solution, residuals and



(a) Face allignment (b) Skewness

(c) Volume

Fig 5: Window pane to input mesh size and type

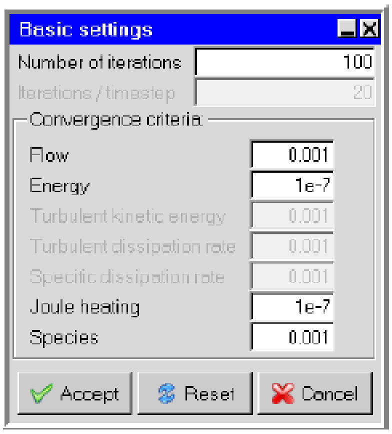


Fig 6: Window pane to input mesh size and type

solution termination criteria are to be defined before the model is put to solve.

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Residuals are to be monitored for each equation of Navier-stokes which includes three momentum equations (in x, y and z directions), Continuity equation, energy equation and one equation of state. It is very important that residuals obtained for each equation is less than mentioned value displayed in fig 7 for solution to be accurate. Temperature is also monitored throughout the process of obtaining solution. Figure 8 displays the temperature monitor point of block. Temperature monitor point results indicate that there is no sudden rise or fall in its value, hence the solution is considered to be accurate. The first three findings (For 20A current) from 108 thermal simulations are shown in table 1.

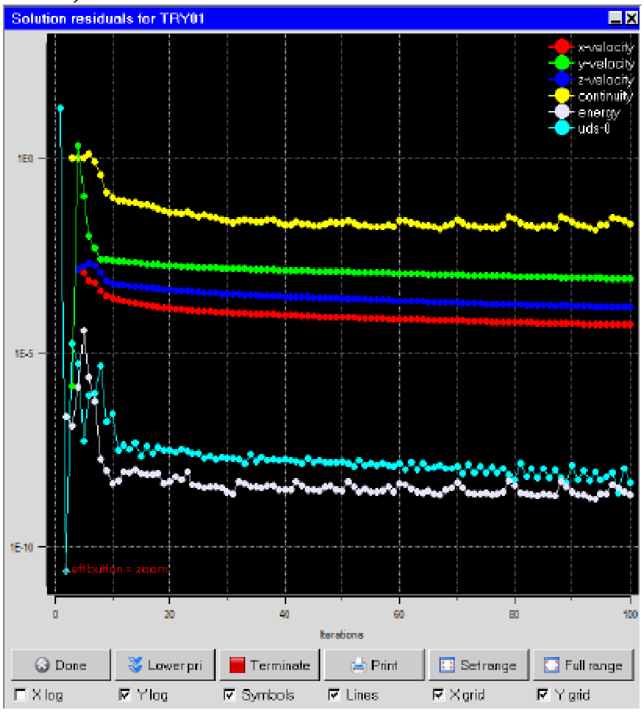

Fig 7: Monitoring residuals

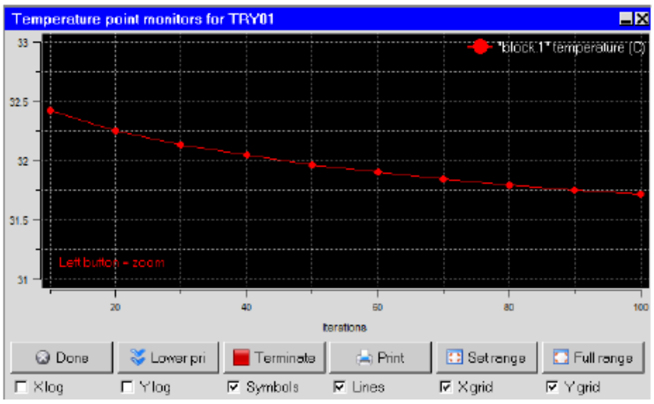

Fig 7: Temperature monitor points

Table 1: Results obtained from thermal simulations

Convection heat transfer [W]	Radiation Heat Transfer [W]	Total heat generation [W]	Maximum heat transfer coefficient(W/K-m2)	Minimum heat transfer coefficient(W/K-m2)	Mean heat transfer coefficient(W/K-m2)	Final temperature (C)	Temperature rise(C)
0.0387785	0.0191617	0.0579402	33.7009	12.0439	19.0592	38.5114	13.5114
0.01844097	0.00976843	0.0282094	27.3124	11.3353	17.5682	31.4229	6.4229
0.01188447	0.00675463	0.0186391	23.8209	11.0287	16.5916	29.0851	4.0851

The results recorded from simulation results are radiation heat transfer, total heat transfer, heat transfer coefficient and final temperature. Convection heat transfer is obtained by subtracting radiation heat transfer from total heat transfer value. Temperature rise is found by subtracting final temperature from ambient. Similar results were recorded for all 108 cases. The results obtained can be further used for data analysis and regression for finding an analytical formula.

REGRESSION AND ANALYTICAL FORMULATION

After finding results from simulation, data is recorded in excel. This data is to be used for analytical formulation. At first, resistance, heat generation and voltage drop were calculated using the value of current, resistivity and dimensions of the sample. Equations 1 to 3 depict the formula to calculate electrical resistance and heat generated due to joule heating effect.

$$R = \frac{\rho \cdot l}{A} \quad (1)$$

$$\dot{Q}_{generated} = i^2 \cdot R \quad (2)$$

$$V = i \cdot R \quad (3)$$

where, R is electrical resistance, ρ is resistivity of the material, l is length, A is cross-section area, $\dot{Q}_{generated}$ is the heat generated due to joule heating effect (without considering rise in temperature), i is current input and V is voltage drop. Table 2 shows the theoretically calculated values for three cases. These values are calculated in order to compare with results obtained from simulation. It can also be used for data analysis which helps in analytical formulation. By comparing heat generated values obtained from simulation and theoretical calculation it is clear that there is close correlation between the two. In order to obtain a regression equation for temperature rise for known dimensions of the sample and current input, energy conservation equation is used which is displayed in eqn 4.

$$i^2 \cdot R \cdot (1 + C \cdot \Delta T) = \dot{Q}_{Convection} + \dot{Q}_{radiation} \quad (4)$$

where, C is temperature coefficient of resistance, ΔT is rise in temperature, $\dot{Q}_{Convection}$ is heat transfer due to convection and $\dot{Q}_{radiation}$ is the heat transfer due to radiation. Equation 4 takes into account the increase of electrical resistance with temperature by using the value of temperature coefficient of resistance for the material. It is due to this factor that value of heat generated obtained from simulation is slightly higher than theoretically calculated value obtained from eqn 2. Typically, the value of

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temperature coefficient of resistance is of the order 10^{-3}. The theoretical radiation heat transfer is described by steffan-boltzmann law which is described by eqn 5.

Table 2: Theoretically calculated values for three sample cases

Case	Length[mm]	Width[mm]	Thickness[mm]	Current[A]	Resistivity[Ω.m]	Resistance[Ω]	Voltage Drop[V]	Heat generation[W]
1	20	5	0.5	20	1.72E-08	1.38E-04	2.75E-03	5.50E-02
2	20	5	1	20	1.72E-08	6.88E-05	1.38E-03	2.75E-02
3	20	5	1.5	20	1.72E-08	4.59E-05	9.17E-04	1.83E-02

$$\dot{Q}_{radiation} = \sigma \cdot \epsilon \cdot A \cdot (T^4 - T^4_{ambient}) \quad (5)$$

where, σ is the value of steffan-boltzmann constant, A is the surface area, ϵ is the surface's emissivity, T is the temperature of body obtained from simulation and $T^4_{ambient}$ is the surrounding air's temperature in degrees celcius. In order to verify steffan boltzmann's law the final temperature value was substituted in eqn 5 and that value was compared against radiation heat transfer value obtained from simulation. Figure 8 depicts the relation between the two values. It is clear from the graph that stefan-boltzmann law accurately describes the radiation heat transfer obtained from simulation, hence it can be used to obtain analytical formula for rise in temperature. After obtaining an analytical formula for radiation heat transfer, similar attempt has to be made for convection heat transfer as well. The existing correlations are available only for standard cases and there is no formula which directly relates to independent input variables (in this case the dimensions of the sample and current input) to convection heat transfer rate. The existing correlations indicate that convection heat transfer phenomena are more complex and depends on non-dimensional numbers raised to constant power thereby indicating that it varies according to power law. So, to estimate the convection heat transfer rate it was reasonable to assume that it would depend on input variables (dimensions and current) according to power law. Hence logarithm of convection heat transfer rate, length, breadth, thickness and current were calculated for all 108 cases. The logarithm values for three cases out of 108 cases are displayed in table 3. A regression analysis was performed for the logarithm of inputs and output in microsoft excel. For regression, logarithm of current, length, breadth and thickness were considered as independent variables and logarithm of convection heat transfer rate as dependent variable and multiple linear regression was selected for the same. Figure 9 depicts the statistics summary and f-statistic obtained after performing regression analysis in excel. The error in a regression is expressed as the value of R square. It is essentially calculated by comparing the total of squared regression errors to the mean error. The regression is quite accurate and the equation derived from that is capable of adequately describing the data if the value of R square is closer to one. As a result, it is evident from fig 9a that regression performed in this case is accurate. Adjusted R square is usually less than value of R square since the former also takes into consideration the number of variables used to explain the regression. The sample data's standard deviation is used to calculate standard error. The f-statistic obtained from the analysis of variance is displayed in fig 9b. It is the difference between estimated value when all coefficients are zero and value obtained when all coefficients are taken into account. Therefore, a greater f-statistic value suggests that independent variables are more significant. The value of f-statistic determined by regression shows that it has a very high value when compared to significance F. The chosen independent variables are therefore important for value prediction. The t-statistic and estimation of coefficients for each individual independent variable are additional significant regression statistic. The sample size is taken into account while calculating T-statistic, which is similar to the z-statistic for samples. T-statistic indicates whether the value of estimation has been obtained by chance. Estimation of errors and other statistics could be reduced since the sample size is limited in comparison to full population.

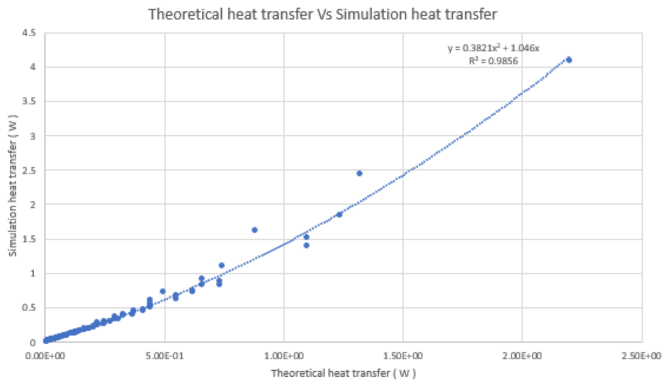

Fig 8: Graph depicting relation between theoretically calculated radiation heat transfer and value obtained from simulation

Table 3: Logarithmic values of inputs and output of the simulation

Ln(CHT)	Ln(Current)	Ln(Thickness)	Ln(Length)	Ln(Breadth)
-3.24988931	2.995732274	-0.693147181	2.995732274	1.609437912
-3.993180459	2.995732274	0	2.995732274	1.609437912
-4.432522773	2.995732274	0.405465108	2.995732274	1.609437912

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999602434
R Square	0.999205026
Adjusted R Square	0.999174154
Standard Error	0.040364612
Observations	108

(a) Regression statistic

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	4	210.9311206	52.73278	32365.26	1.2246E-158
Residual	103	0.167818097	0.001629		
Total	107	211.0989387			

(b) F-statistic

Fig 9: Summary of statistic obtained from regression analysis

The t-statistic and coefficient values obtained are shown in tabular form in the fig 10. It is evident that all of the independent variable's coefficients and intercept fall within upper and lower 95 %. The t-statistic is used to get the p-value (probability value) for each coefficient and an intercept. P-value represents the likelihood that a specific coefficient or intercept will become zero. The coefficients acquired through analysis are a good approximation for analytical formulation because it is obvious from the fig 10 that p-value is very small for all coefficients and intercept. The statistic obtained from analysis shows that the regression analysis is accurate and final formula obtained is shown in eqn 6.

$$i^2 \cdot R \cdot (1 + C \cdot \Delta T) = 1.3155 \times 10^{-5} \times i^{2.144} \times t^{-1.077} \times l^{0.974} \times w^{-1.276} + \sigma \cdot \epsilon \cdot A(T^4 - T_{ambient}^4) \quad (6)$$

where, t is thickness, l and w are length and width respectively. But in eqn 6 maximum temperature obtained is on both sides of the equation hence it needs to be modified so that temperature is on one side and its value can be obtained. For that purpose, a graph is plotted between heat generated with and without considering rise in temperature as shown in fig 11 and a regression equation is obtained for the same. Hence the final equation becomes as shown in eqs 7 and 8.

$$i^2 \cdot R_{adjusted} = 1.3155 \times 10^{-5} \times i^{2.144} \times t^{-1.077} \times l^{0.974} \times w^{-1.276} + \sigma \cdot \epsilon \cdot A\,(T^4 - T_{ambient}^4) \quad (7)$$

$$i^2 \cdot R_{adjusted} = 0.382 \cdot (i^2 \cdot R_{theoretical})^2 + 1.046 \cdot i^2 \cdot R_{theoretical} \quad (8)$$

where, $i^2 \cdot R_{adjusted}$ is the heat transfer value obtained from eqn 8 and $i^2 \cdot R_{theoretical}$ is calculated value of heat generation without considering rise in temperature. So, a formula is obtained successfully for rise in temperature in a bar of rectangular cross section (shape of a cuboid) when a known value of current flows through it. This is a unique formula which directly correlates the rise in temperature with the current

flow and dimensions of the sample. Similar studies were conducted previously however, those were focussed on finding an appropriate analytical formula for convection heat transfer rate rather than rise in temperature.

CONCLUSION

After performing thermal analysis for 108 different cases and analyzing data obtained from the results, an accurate formula is obtained for final temperature given the value of length, breadth, thickness and current input. As it can be seen from the formula that convection heat transfer rate increases with increase in length and current but it decreases with length and width. However, it is important to note that with increase in current, heat generation also increases and same is the case with length as resistance increases with length. This indicates that convection heat transfer is more complex phenomena and final temperature can be obtained from radiation term in the equation. The process of obtaining an analytical formula involved taking logarithm of inputs and out and perform multiple linear regression, this method could be used in future cases where variation according to power law is involved.

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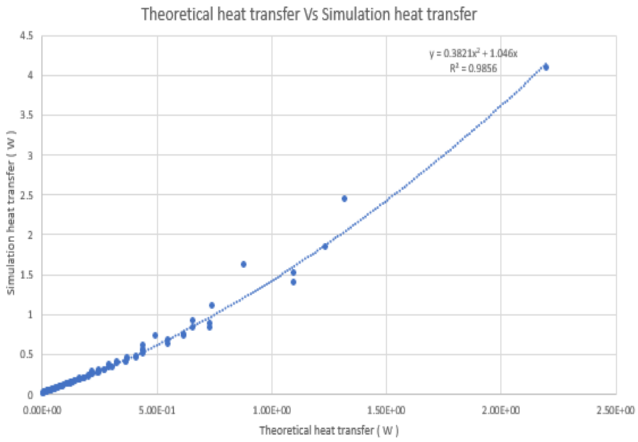

Fig 11: Comparing heat generated value with and without considering temperature rise

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-11.23876905	0.049394783	-227.529	5.45E-141	-11.33673195	-11.140806	-11.33673195	-11.14080615
In(current)	2.144029911	0.00746041	287.3877	2.02E-151	2.129233947	2.15882587	2.129233947	2.158825875
In(Thickness)	-1.077032501	0.008562738	-125.781	1.42E-114	-1.094014672	-1.0600503	-1.094014672	-1.060050329
Ln(Length)	0.973612954	0.010360395	93.9745	1.2E-101	0.953065554	0.99416035	0.953065554	0.994160354
In(width)	-1.276313861	0.008562738	-149.054	3.97E-122	-1.293296032	-1.2593317	-1.293296032	-1.259331689

Fig 10: t-statistic value obtained from regression

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