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Thermal Stresses in a Finite Solid Cylinder

(Magnetothermodynamics Stresess in A Solid Cylinder)

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***Abstract*— This paper is concerned an analytical method for determining the thermal stresses in a finite solid cylinder under a sudden temperature change to a constant temperature with the help of integral transform technique.**

Keywords*— Hankel transform, Laplace transform, magneto thermo stresses, solid cylinder, thermo elasticity.

INTRODUCTION

Kaliski and Nowacki [1] considered the half-space problem of magneto thermo elastic waves produced by a thermal shock in a perfectly conducting medium. The problem of magneto thermo elasticity related to an infinite cylindrical region was solved by Dhaliwal and Singh [2]. Noda et al. [3] give a combined formulation of the two theories of generalized thermo elasticity to discuss the problem of an infinite solid with a cylindrical of spherical hole.

In this paper, an attempt has been made to determine the magneto thermo stresses of finite solid cylinder with help of Hankel Transform and Laplace transform technique

NOMENCLATURE

$T(v,t)$ - Temperature charge (absolute temperature minus reference temperature).

$\tilde{U}$ - Displacement vector

u - Radial displacement

$\sigma\gamma, \sigma\theta$ - Radial stress and circumferential stress.

ρ, t, a - Density, time and radius of solid cylinder.

α - Coefficient of linear thermal expansion.

λ, G - Lame constants

E, v - Young's modulus and Poisson's ratio

μ - Magnetic permeability

$\vec{H}$ - Magnetic intensity vector $_{o,o,}\ H_2$

$\vec{c}$ - Perturbation of electric field vector.

$\vec{H}$ - Perturbation of magnetic field vector (o, o, hz)

$C_1 = (\lambda + lG)/p$ - Elastic wave speed

$C_z = \sqrt{\mu H_z^t / p}$ - Magnetic interference wave speed

$C_L = \sqrt{c_1^t + c_t^t}$ - Magneto thermo elastic wave speed.

Non dimensional quantities

$$R = \gamma / a,\ \tau = (tc_L)/a,\ \sigma_\sigma^* = \sigma\gamma /(\alpha ET(\gamma,t))$$

$$\sigma_\theta^* = \sigma_\theta /(\alpha ET(\gamma,t)),\quad h_z^* = h_z /(\alpha T(\gamma,t)H_z)$$

I -STATEMENT OF THE PROBLEM

Consider a long finite solid cylinder of radius with perfect conductivity placed initially in an axial magnetic field $\vec{H}(0,0,H_z)$.

Let this cylinder be subjected to a rapid change in temperature $T(\gamma,t)$ produced by the absorption of an electromagnetic pulse or $\gamma - \gamma ay$ pulse radiant energy. Assuming that the magnetic permeability, μ of the solid cylinder equals the magnetic permeability of the medium around it and omitting. Displacement maxwell equations for a perfectly conducting elastic body are given by

$$\vec{J} = Curl\,\vec{h},\quad -\mu\frac{\partial \vec{h}}{\partial t} = Curl\,\vec{e},\quad \vec{e} = -\mu\left(\frac{\partial \bar{U}}{\partial t}\times\vec{H}\right) \tag{1}$$

An initial magnetic field vector $\overline{H}(0,0,H_z)$ in cylindrical polar co-ordinate (r,θ,z) .are

$$\vec{U} = \left(u(r,t),0,0\right),\quad \bar{e} = \mu\left(0, H_z\frac{\partial u}{\partial t},0\right) \tag{2}$$

$$\vec{h} = (0,0,h_z)\quad \vec{J} = (0,\frac{\partial h_z}{\partial r}0,) \tag{3}$$

Let magneto elastic dynamic equation of the solid cylinder becomes

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}(\sigma_\gamma - \sigma_\theta) + f_r = p\frac{\partial^2 v}{\partial t^2}, \tag{4}$$

Where fr is defined as

$$f_r = \mu(\vec{j}\times\vec{H}) = \mu H_z^{\ 2}\frac{\partial^2}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \tag{5}$$

The radial stress and the circumferential stress of a solid cylinder subjected to a thermal shock load are $T(r,t)$, are

$$\sigma_r(r,t) = (\lambda+2G)\frac{\partial u}{\partial r} + \frac{\lambda}{r}u - \frac{E\alpha}{1-2v}T(r,t) \tag{6}$$

II. SOLUTION OF THE PROBLEM

Substituting Eqs (5) to (6) into (4) the basic displacement equations of magneto thermo elastic motion is expressed as

$$\left[\frac{1}{r}\frac{\partial u(r,t)}{\partial r} - \frac{1}{r^2}\frac{\partial u_s(r,0)}{\partial t}\right] = \frac{1}{C_L^2}\left[\frac{\partial^2 u(r,t)}{\partial t^2} + \frac{E\alpha}{(1-2\nu)B}\frac{\partial^2 T(r,t)}{\partial t}\right], 0 \le r \le a, t \ge 0 \tag{7}$$

Where $B = \lambda + 2\sigma + \mu H_z^2$

Omitting the Maxwell tensor on the surface of the solid cylinder the corresponding boundary conditions are

$$u(0,t) = 0 \tag{8}$$

$$\sigma_r(a,t) = \left[(\lambda+2G)\frac{\partial u}{r} + \frac{\lambda}{r}u - \frac{E\alpha}{(1-2v)}T(r,t)\right]_{r=0} = 0 \tag{9}$$

The initial conditions are

$$u(r,0) = 0, \quad \frac{\partial u(r,0)}{\partial t} = 0 \tag{10}$$

Assume that general solution to the basic equation (7) to (10) may be expressed in the form

$$u(r,t) = u_s(r,t) + u_d(r,t) \tag{11}$$

Where $u_s(r,t)$ and $u_d(r,t)$ are the static solution and dynamic solutions to Eq. (7) to (10).
Solving Eq. (11) we have,

$$u_s(r,t) = \frac{E\alpha}{(1-2v)\beta_r}\int_0^r r.T(r,t)dr + B_1 + \frac{B_2}{r} \tag{12}$$

Where unknown constants B_1 and B_2 in Eq. (12) may be determined by

$$B_1 = \frac{E\alpha}{Ba^2}\int_0^a rT(r,t)dr - \frac{(1-v)E - B(1+v)(1-2v)}{B(1-2v)} \tag{13}$$

$$B_2 = 0 \tag{14}$$

From equations (7),(10) and (12) and using the boundary condition (8),(9) we get,

$$\frac{\partial^2 u_d(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u_d(r,t)}{\partial r} - \frac{1}{r^2}u_d(r,t) = \frac{1}{C_L^2}\left[\frac{\partial^2 u_d(r,t)}{\partial t^2} + \frac{\partial^2 u_s(r,t)}{\partial t^2}\right] \tag{15}$$

$$u_d(0,t) = 0 \tag{16}$$

$$(\lambda+2G)\frac{\partial u_d(a,t)}{\partial r} + \frac{1}{r}u_d(r,t) = 0 \tag{17}$$

$$u_d(r,0) = -u_s(r,0) = 0$$

$$\left[\frac{\partial u_d(r,0)}{\partial t} + \frac{\partial u_s(r,0)}{\partial t}\right] = u_d(0,t) = u_s(0,t) = V_0 \tag{18}$$

Where $u_s(r,t)$ is the known static solution shown in Eq. (12).
The solution of the homogeneous formula of Eq. (14) assuming $u_s(r,t) = 0$ is given by

$$Ud_0(r,t) = f(r)\exp(iwt) \tag{19}$$

Where $f(r)$ and w are the characteristic function and natural frequency respectively.
Substituting Eq. (19) into the Eq. (15) and utilizing Eqs. (16) and (17)
We have

$$\frac{d^2 f(r)}{dr^2} + \frac{1}{r^2}\frac{df(r)}{dr} + \left(k^2 - \frac{1}{r^2}\right)f(r) = 0; \quad 0 \le r \le a \tag{20}$$

$$f(0) = 0 \tag{21}$$

$$(\lambda+2G)\frac{df(a)}{dr} + \frac{\lambda}{a}f(a) = 0 \tag{22}$$

The generalized solution of Eq. (24) is given by

$$f(r) = A_1J_1(kr) + A_2Y_1(kr) \tag{23}$$

Substituting Eq. (23) into Eqs. (21) the corresponding characteristic function Eq. (23) reduces to.

$$f(r) = A_nJ_n(k_nr) \tag{24}$$

This should satisfy the characteristic equation

$$k_nJ_1^1(k_na) + \frac{\lambda}{a(\lambda+2\sigma)}J_1(k_na) = 0 \tag{25}$$

Where $J_m(k_nr)$ is an m[th] order Bessel (14) function of the first kind, $k_n(n = 1,2,3......)$ are the positive roots of eigen equation (25).
And

$$w_n = k_nc_L \tag{26}$$

By means of the normalization property of eigen function, the constant An in Eq. (24) is determined as

$$A_n = \frac{\int_0^a r f_n(r) A_n J_1(k_n r) dr}{\int_0^a r J_1^2(k_n r) dr} \tag{27}$$

Define a finite Hankel transform of $f(r)$ as

$$\bar{f}(k_n) = Hankel[F(r)] \int_0^a r f_n(r) A_n J_1(k_n r) dr \tag{28}$$

Then the inverse of Eq^n (28) is given by

$$f(r) = \sum_{k_n} \frac{\bar{f}(k_n)}{f(k_n)} J_1(k_n r) \tag{29}$$

Where

$$F(k_n) = \int_0^a r J_1^2(r)(k_n r) dr = \frac{a^2}{2}\left\{\left[J_1^1(k_n a)\right]^2 + \left[1 - \left(\frac{1}{k_n a}\right)^2\right]\left[J_1(k_n a)\right]^2\right\} \tag{30}$$

Using Eq. (28) and applying a finite Hankel transform to Eq. (15) we have

$$\frac{aJ_1(k_n a)}{(\lambda + 2\sigma)}\left[(\lambda + 2\sigma)u_d(a) + \frac{\lambda}{a}u_d(a)\right] - k_n^2 \bar{u}_d(k_n, t) = \frac{1}{C_L^2}\left[\frac{\bar{u}_d(k_n, t)}{dt^2} + \frac{\bar{u}_s(k_n, t)}{dt^2}\right] \tag{31}$$

Where

$\bar{u}_s(k_n, t) = Hankel[u_s(r, t)]$

The first term on the left hand side of Eq. (31) should be the homogeneous boundary condition (17) simplifies to

$$k_n^2 \bar{u}_d(k_n, t) = \frac{1}{C_L^2}\left[\frac{\bar{u}_d(k_n, t)}{dt^2} + \frac{\bar{u}_s(k_n, t)}{dt^2}\right] \tag{32}$$

Applying Laplace transforms to Eq. (32), one obtains

$$\bar{u}_d^*(k_n, p) = \bar{u}_s^*(k_n, p) + \frac{k_n^2 C_L^2}{k_n^2 C_L^2 + p} U_s + \frac{p^2}{k_n^2 C_L^2 + p} \bar{U}_0 + \frac{1}{k_n^2 C_L^2 + p} V_0^2 \tag{33}$$

Where p is the Laplace transform parameter. Taking the inverse Laplace transform of Eq. (33) we have

$$\bar{u}_d(k_n, t) = -\bar{u}_s(k_n, t) + k_n C_L \int_0^t \bar{u}_s \sin[k_n C_L(t - \tau)] dt + \bar{u}_0 \cos(k_n C_L t) + \frac{V_0}{k_n C_L}\sin(k_n C_L t) \tag{34}$$

Using Eqs (29) and (30) and applying finite inverse Hankel transform to Eq. (34), the elasto dynamic solution $u_d(r, t)$ of Eq. (15) to (18) may be expressed as

$$u_d(r, t) = \sum_{k_n} \frac{\bar{U}_d(k_n, t)}{F(k_n)} JL_1(k_n, r) \tag{35}$$

Substituting Eq. (16) and (39) into Eq. (12) the general solution of the basic equation (8) becomes

$$u(r, t) = \frac{E\alpha}{B(1 - 2v)} \int_0^r r, T(r, t) dr + B_1 r + \sum_{kn} \frac{\bar{u}d(k_n t)}{F(k_n)} J_1(knr) \tag{36}$$

Equations (35) and (36), are the corresponding magneto thermodynamic stress.

By using Eq. (36) and the basic solution for magneto thermo elastic motion equation (11) reduces to

$$u(r, t) = (1 + v)\alpha T_0 + (1 + v)\alpha T_0 \sum_{kn} \frac{ust(k_n)}{F(k_n)} J_1(k_n r)\cos(w_n st) \tag{37}$$

Where

$$u_{st}(k_n) = Hankel[r] = \frac{a^2}{k_n} J_0(k_n a) - \frac{2a}{k_n^2} J_1(k_n a) \tag{38}$$

From Eqs. (6) and (7) and utilizing the following properties of the Bessel functions.

$$k_n J_1^1(k_n r) = \frac{\partial J_1(k_{nr})}{\partial r} = k_n J_0(k_{nr}) - \frac{1}{r} J_1(k_n r) \tag{39}$$

$$\frac{1}{r} J_1(k_n r) = \frac{k_n}{2}[J_0(k_n r) + \frac{k_n}{2} J_2(k_n r)] \tag{40}$$

We get the magneto thermo stresses,

$$\sigma_r^*(r, t) = \sum_{kn}\left[\frac{k_n}{2(2v - 1)} J_0(k_n r) + \frac{k_n}{2} J_2(k_n r)\right]\frac{\bar{u}_{st}(k_n)}{F(k_n)}\cos(w_n t) \tag{41}$$

$$\sigma_\theta^*(r, t) = \sum_{kn}\left[\frac{k_n}{2(2v - 1)} J_0(k_n r) - \frac{k_n}{2} J_2(k_n r)\right]\frac{\bar{u}_{st}(k_n)}{F(k_n)}\cos(w_n t) \tag{42}$$

Where $\sigma_i^*(r, t) = \sigma_i(r, t)/(\alpha T_0 E)$ is normalized.

Eq. (41) and (42) are the magneto thermo stress is only dependent on the dynamic term in the basic solution (7) . Using the proportion of Bessel function;

$J_0(0) = 1$ and $J_2(0) = 0$

The magneto thermo stress response at the center (R = 0) of solid cylinder reduces to;

$$\sigma_r^*(0, t) = \sum_{kn}\left[\frac{k_n}{2(2v - 1)}\right]\frac{u_{st}(k_n)}{F(k_n)}\cos(w_n t) \tag{43}$$

$$\sigma_\theta^*(0, t) = \sum_{kn}\left[\frac{k_n}{2(2v - 1)}\right]\frac{u_{st}(k_n)}{F(k_n)}\cos(w_n t) \tag{44}$$

Equations (43) to (44) are the magneto thermo stress response at the center (R = 0) of a finite solid cylinder.

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III. CONCLUSION

In this paper, we have investigated the magneto thermo stresses in a finite solid cylinder with the help of the finite Hankel transform and Laplace transform techniques. The expressions that are obtained can be applied to the design of useful structures or machines in engineering application .

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