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Thermo-Elastic Analysis of Angle-Ply Laminates with Circular Cut-Out under Cylindrical Bending

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Abstract

Four-layered symmetric and anti-symmetric laminates with circular cutout and subjected to transverse uniform pressure load under cylindrical bending are analyzed using three-dimensional finite element analysis. The problem is modelled in ANSYS software. Transverse deflection and rectangular components of normal and shear stresses are evaluated and variation of these results with respect to fiber angle for both the sequences is discussed.

Keywords: FRP, FEM, Symmetry, Anti-Symmetry, cut out, Interface.

1. Introduction

The increasing use of fiber reinforced laminates in space vehicles, aircrafts, automobiles, ships and chemical vessels has necessitated the rational analysis of structures for their mechanical response. In addition, the anisotropy and non-homogeneity and larger ratio of longitudinal to transverse modulii of these new materials demand improvement in the existing analytical tools. As a result, the analysis of laminated composite structures has attracted many research workers, and has been considerably improved to achieve realistic results. In the three-dimensional elasticity solution, each layer is modelled as a three-dimensional solid. Usually, the anisotropy in laminated composite structures causes complicated responses under different loading conditions by creating complex couplings between extensions, bending, and shear deformation modes. To capture the full mechanical behaviour, it must be described by three dimensional elasticity theories.

2. Literature Review

In solving the three-dimensional elasticity equations of rectangular plates, quite a number of solution approaches have been proposed. One would find the earlier research work of Pagano, who studied the static bending of infinitely long and finite size composite laminates under sinusoidal lateral loading using an analytical method. The elasticity solutions were compared with the classical thin plate (CTP) theory solutions, and the limitations of the CTP theory were pointed out in his work.

Srinivas and Rao [1] and Srinivas et al. [2] presented a set of complete analytical analyses on bending, buckling and free vibration of plates with both isotropic and orthotropic materials. Based on the analysis of Srinivas and Rao [1] and Srinivas et al. [2], Wittrick [3] worked out a detailed analytical three-dimensional elasticity solution of simply supported plates for Eigen value problems of buckling and free vibration and for static deflections under sinusoidal lateral loading. Pagano et al. [4] has given exact solutions for the deflections and stresses of a cross-ply laminated rectangular composite without holes using elasticity theory.

Paolo et al. [5] analyzed the behaviour of an arbitrary laminated composite plate by assuming a layer wise polynomial expansion along the thickness direction for displacements. In contrast with other proposed approaches and in order to take into account the transverse normal stress distribution, out-of-plane displacements are not assumed to be constant along the thickness. Based on the proposed Kinematic assumptions the continuity of the interlaminar stress components at the interface can also be achieved. A finite element procedure is established and plate models are derived in which the stress field is obtained directly from the constitutive relations and not by the integration of the three-dimensional equilibrium equations.

Busby and Saidiwakar [6] modified the finite quasi-prismatic (FQP) element to analyze anisotropic materials. The finite quasi-prismatic element is a three-dimensional finite element which uses conventional interpolating functions in two directions and functions based on Chebyshev polynomials in the third direction. Kong and Cheung [7] proposed a displacement-based, three-dimensional finite element scheme for analyzing thick laminated plates by treating the plate as a three-dimensional inhomogeneous anisotropic elastic body.

Limited literature is available on bending of composite plates with a cutout. Hwang and Sun [8] presented a continuous mixed field iterative scheme based on a three-dimensional finite element displacement method. This method is very powerful in the determination of stress distributions for problems with either material and/or geometric discontinuities. For laminated composite materials this method is reliable in stress evaluation at locations away from the optimal Gauss points such as free edges near a notch or a hole. It is also useful in the determination of interlaminar stresses at an interface of laminated composites.

Prasad and Shuart [9] presented a closed form solution for the moment distributions around holes in symmetric laminates subjected to bending moments. Delale et al. [10] considered the stress analysis of plate made up of two bonded dissimilar isotropic materials with a central circular hole subjected to axi-symmetric bending.

Lo and Leissa [11] considered the bending of isotropic square plates with a circular hole subjected to uniform transverse load. Results were shown for simply supported and clamped boundary conditions. Shiau and George [12] developed an 18 degree-of-freedom higher-order triangular plane stress element to investigate the effect of variable fiber spacing on the stress concentration around a hole in a composite laminated plate subjected to in-plane boundary loadings.

Wen and Chyanbin [13] employed an asymptotic analysis to separate the 3D problem of laminate with hole into two plane problems. One is an interior problem, the other is a boundary layer problem. The former is treated by classical lamination theory and is solved by a special boundary element; the latter is then solved by the finite element method developed for the generalized plane deformation problems. Ramesh Kumar et al. [14] presented an approximate solution in the form of a polynomial for the normal stress distribution adjacent to a class of optimum holes in symmetrically laminated infinite composite plates under uniaxial loading. Sambasiva Rao [15] has studied the prediction of static and thermo-elastic behaviour of FRP composite cross-ply laminates with cut outs under cylindrical bending.

The present investigation is an extension of the work of Sambasiva Rao [15] for angle-ply laminates.

3. Problem Modelling

3.1 Geometrical Modelling

Figure1 shows the in-plane dimensions of laminate considered for the present analysis. The dimensions for ‘b’ and ‘l’ are taken as 2,000 mm and 4,000 mm respectively. The thickness of the laminate is taken as 200 mm and four layers of equal thickness are stacked in +θ/-θ/-θ/+θ in symmetrical arrangement and +θ/-θ/+θ/-θ in anti-symmetrical stacking sequence.

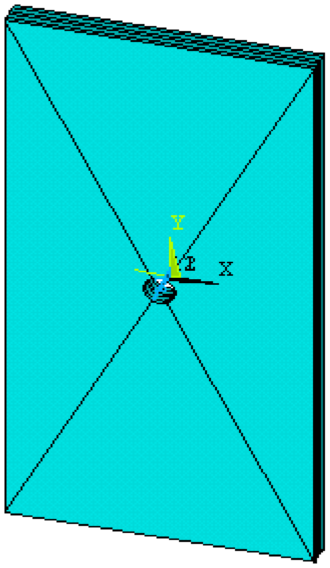

Figure 1: Composite Laminated with Circular cut out

The diameter of the hole is taken as per the ratio d/l = 0.1. Fiber angle varies from 0^0 to 90^0 with 15^0 intervals.

3.2 Finite Element Model

The finite element mesh is generated using a three dimensional brick element (SOLID 95) in ANSYS software. The element is defined by 20 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. The FE mesh on the structure is shown in figure 2.

3.3 Boundary Conditions

The longer edges of the laminate are clamped. i.e. all the degrees of freedom of nodes along this faces are constrained.

At x = +1000 mm and x = -1000 mm

$U_x = 0$; $U_y = 0$; $U_z = 0$.

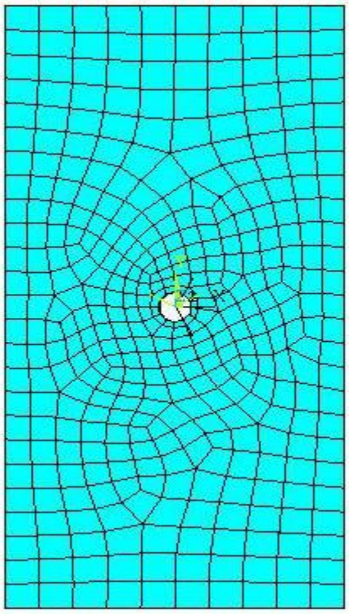

Figure 2: FE mesh on Laminate

Sambasiva Rao [15] has validated by comparing FE results with the exact elasticity results given by Pagano [4] for a cross-ply laminate under cylindrical bending subjected to transverse pressure loading with s =10 (Figure 3). Later he extended the analysis for laminates with cut outs due to pressure and thermal loads. The present analysis is extended for angle-ply laminates with cut outs under cylindrical bending subjected to transverse pressure load.

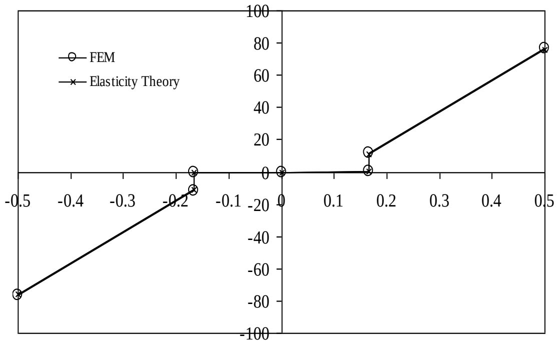

Figure 3: Validation of FE results with analytical results of Pagano [4]

3.4 Material Properties

The properties evaluated by Sambasiva Rao [15] from micrmechanical numerical approach for T300-Epoxy composite for 60% volume fraction are used for the present analysis.

E_1 = 134.48 GPa, E_2 = 9.92 GPa, E_3= 9.92 GPa

ν_{12} =0.257, ν_{23}= 0.363, ν_{31} = 0.257
G_{12}= 4.13 GPa, G_{23}= 3.75 GPa, G_{13}= 4.13 GPa

4. Discussion of Results

Variations of in-plane stresses with respect to fiber angle are shown in figures 4 to 6. σ_x increases up to 30^0 of fiber angle and there after decreases for both laminates, the stresses in symmetric angle-ply condition are more when compared to that of the anti-symmetric ply condition and maximum difference is found at $\theta=30^0$ (Figure 4).

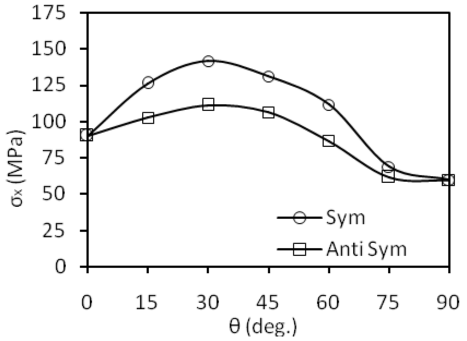

Figure 4 : Variation of σ_x with respect to fiber angle [d/l=0.1, s=10]

It is observed that in both cases σ_y increases up to 60^0 fiber angle and there after decreases, the stresses in symmetric angle-ply condition are more when compared to that of the anti-symmetric ply condition and maximum difference is found at $\theta=60^0$ (Figure 5).

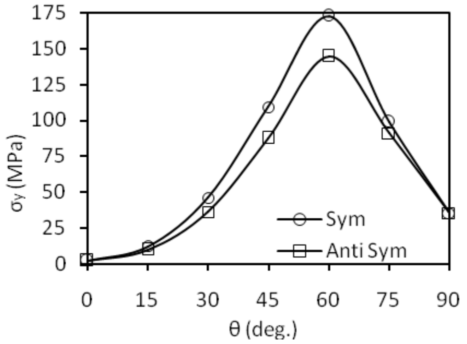

Figure 5: Variation of σ_y with respect to fiber angle [d/l=0.1 s=10]

τ_{xy} for both symmetric and anti symmetric conditions is observed to be increasing up to about 52^0 of fiber angle and there after decreases, the stresses in symmetric angle-ply condition are more when compared to that of the anti-symmetric ply condition and maximum difference is found at $\theta=52^0$ (Figure 6).

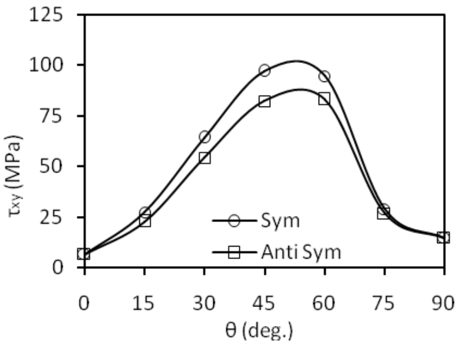

Figure 6: Variation of τ_{xy} with respect to fiber angle [d/l=0.1 s=10]

σ_z for both symmetric and anti symmetric conditions is observed to be increasing up to 75^0 of fiber angle and there after decreases, the stresses in symmetric angle-ply condition are more when compared to that of the anti-symmetric ply condition and maximum difference is found at $\theta=45^0$ (Figure 7).

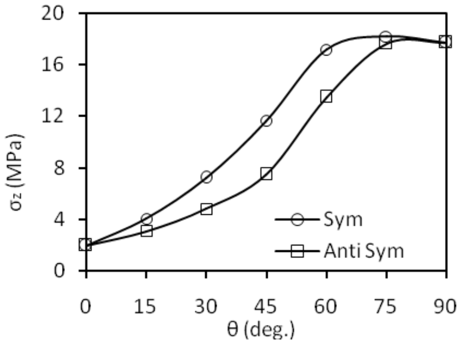

Figure 7: Variation of σ_z with respect to fiber angle [d/l=0.1 s=10]

τ_{yz} for both symmetric and anti symmetric conditions is observed to be increasing up to 75^0 of fiber angle and there after decreases, the stresses in symmetric angle-ply condition are more when compared to that of the anti-symmetric ply condition and maximum difference is found at $\theta=75^0$ (Figure 8).

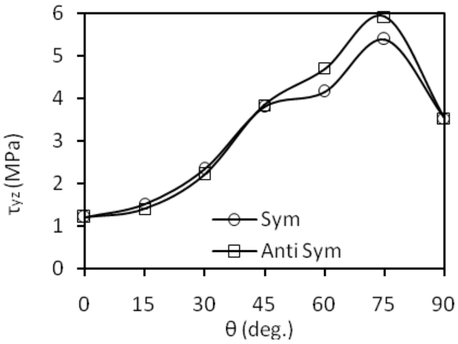

Figure 8 : Variation of τ_{yz} with respect to fiber angle [d/l=0.1 s=10]

τ_{zx} for both symmetric and anti symmetric conditions is observed to be increasing up to 45^0 of fiber angle and there after decreases, the stresses in symmetric angle-ply condition are more when compared to that of the anti-symmetric ply condition and maximum difference is found at $\theta=45^0$ (Figure 9).

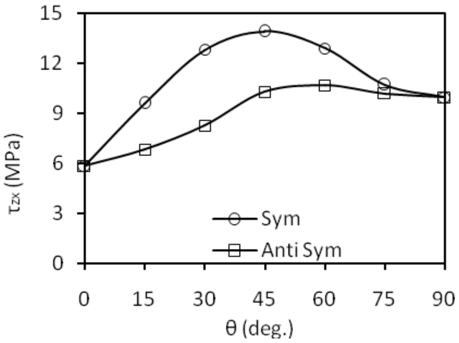

Figure 9: Variation of τ_{zx} with respect to fiber angle [d/l=0.1 s=10]

U_z for both symmetric and anti symmetric conditions is observed to be increasing up to 75^0 of fiber angle and there after decreases, the deflection in symmetric and anti-symmetric laminates are almost equal (Figure 10).

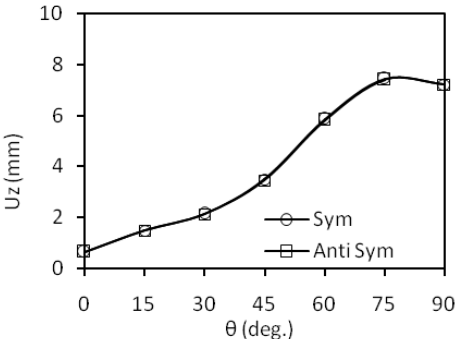

Figure10: Variation of U_z with respect to fiber angle [d/l=0.1 s=10]

In a cylindrically loaded isotropic plate, in-plane normal stress in x-direction is known to be significant. In the present case, other two in-plane stresses are also observed to be significant. This is mainly due to the interaction of layers in a laminate leading to anisotropic behavior of total laminate where any kind of load produces extension, shear and bending responses of the laminate and this tendency depends on the arrangement of fibers in laminate. The resultant stiffness in a particular direction of the laminate varies with respect to fiber angle and hence all the stresses are varying with θ.

5. Conclusions

Stress analysis of a four-layered angle-ply laminate is carried out using three-dimensional finite element method. Variation of all the six components of stresses and transverse deflection with respect to fiber angle for both symmetric and antisymmetric laminates is discussed for a rectangular laminate with cut out subjected to transverse pressure load under cylindrical bending. The following conclusions are drawn.

- All the stresses and deflection are sensitive towards change in fiber angle.
- Stresses in symmetric laminates are higher than corresponding antisymmetric laminates.
- Deflection is same for both the laminates.

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