

Traffic Volume Estimate using a Third Order Lagrangian Function: A Case Study of Ibadan Metropolis

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Abstract - Reliable traffic volumes estimate is the strong footing for congestion free junctions and routes. This however is often hampered by limited financial resources, labour and time. There is therefore need to explore models that can reduce burden in all these areas. A third Lagrangian function was developed. The function was optimized using Newton's optimization technique. The model developed was applied to an intersection with 5 legs at Ibadan metropolis in Nigeria. The predicted traffic volume when one of the legs was treated as unknown was compared with the field values. Fractional difference (a criterion to determine overestimation or underestimation) of the model estimate as compared with field value was determined. Thereafter, the fraction U-turn was determined for each of the leg. The maximum model's fractional difference for both inflow and outflow were -0.21 and 0.20, respectively. The model predictions of turnings showed the incidences of U-turn were not too significant. The maximum U-turn estimates was about 8%.

Keywords: Traffic, Volume estimate, U-turn, Lagrangian function and Newton's optimization

INTRODUCTION

Efficient transport system is a pivotal/major measure of sustainable development of a nation. . Kupolati (2008) noted that road transport is the dominant form of transportation in Nigeria.

After World War II, with the tremendous increase in use of automobiles and the expansion of the highway system, there was also a surge in the study of traffic characteristics and the development of traffic flow theories. The 1950's saw theoretical developments based on a variety of approaches, such as car-following, traffic wave theory (hydrodynamic analogy) and queuing theory. Some of the seminal works of that period include the works by Reuschel (1950a; 1950b; 1950c), Wardrop (1952), Pipes (1953); Lighthill and Whitham (1955), Newell (1955), Webster (1957), Edie and Foote (1958), Chandler et al, (1958) and other papers by Herman et al (Herman 1992).

Reliable traffic volumes estimate is the strong footing for congestion free junctions and routes.

Improved vehicular traffic count and expansion of limited data to reliable figure is possible, when right predicting tool is applied to the problem of insufficient data sourcing. This problem which results in traffic congestion is common to Engineers world wide even where automated traffic count exist. Labour saving traffic count data forecasting tool modeling is the focus of this work.. Traffic is the

medium in which both land and transport find their expression in fact both produce traffic (Oguara, 2006).

The challenge of Nigerian urbanization whose component is inefficient transportation system result in urban traffic congestion is stirring a heavy drive to research solutions. Worldwide, the importance of well organized public transportation system is the over all efficient functioning of human settlement (UNCHS, 1996). In Nigeria, this fact has been shown to be very true in the rapidly growing urban centres (Adeniji 2000, 1991a &b, 1987a&b). FMT (1993) noted that transport system need to be developed to accommodate the present traffic schedule, as such development is based upon data sourcing through efficient traffic monitoring and distribution models.

Considerable work has been done by researchers in different parts of the world on how to reliably estimate traffic flow. However as noted in some Nigeria cities, most traffic surveys are sourced manually for long hours hence such data are prone to error, incompleteness, misses and unavailability. Adebisi (1987) used already existing algorithms to estimate turning traffic flow from manual count volumes at road junctions. Furness and fratar models have been developed for estimation of turning traffic volume. However, these models which can be used to predict reliably turnings at intersections require complete inflow and outflow volumes at each leg of the intersection. A lot of human, material and financial resources are needed to obtained complete inflow and outflow volumes for all legs in an intersection. This may be tiring in some developing economy. In the light of this, there is the need to develop suitable turning traffic model that may accommodate missing or incomplete information of a leg in an intersection. An optimization model from furness equation is therefore developed for use in predicting traffic volume for the city of Ibadan and subsequently turning traffic at each leg of an intersection.

Description of Study Area and Selected Intersections

The model developed is expected to be used to carry out traffic study in the city of Ibadan, Nigeria. The detailed description of the city and selected intersections in the city is hereby presented.

Study Area Description

Ibadan has a tropical wet and dry climate, with a lengthy wet season and relatively constant temperatures throughout the course of the year. Ibadan's, wet season runs from

March through October, though August sees somewhat of a lull in precipitation. This lull nearly divides the wet season into two different wet seasons. The remaining months forms the city's dry season. Like a good portion of West Africa, Ibadan experiences the harmattan between the months of November and February. Ibadan has an airport, Ibadan Airport, and was served by the Ibadan Railway Station on the main railway line from Lagos to Kano.(No longer operating). Poorly-maintained roads are particularly problematic in the rainy season.. There are not many miles of divided highways in Ibadan. The primary routes go from Ibadan to Lagos and to Benin City(see Fig. 1). Adding to the weather and terrain, roads typically have few or no speed limit signs or warning signs to alert the motorist of curves, hills, intersections or

problems with the road itself such as large potholes or eroded road beds
 In-town transportation comes in a variety of forms. Modes of transportation include, taxis, taxi-vans commonly called *danfos*, private cars that are hired out by the day with a driver, personal family cars, scooters, and walking (Wikipedia)

Description of Intersection for Study

There are varieties of intersection in the study area. However for this study a T-leg Intersection was selected. The intersection is located at near outskirts of the city. The traffic is sometimes heavy due to presence of many long vehicles that goes to the northern part of Nigeria. A sketch of the intersection is shown in Figure 1.

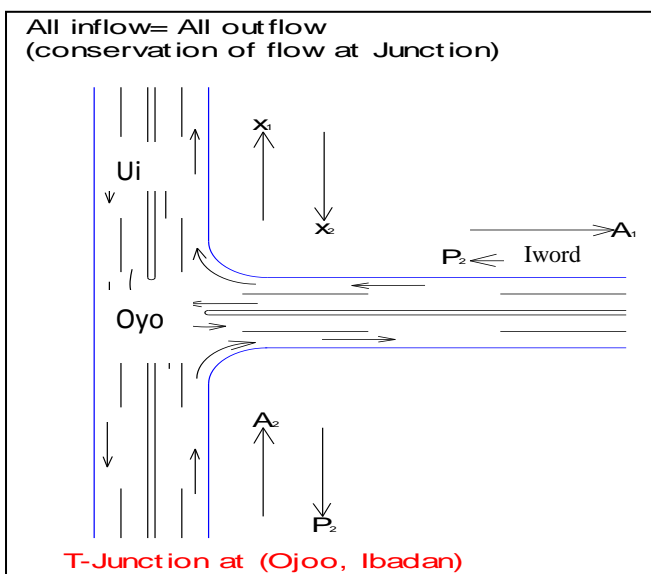


Fig. 1: A Selected T-Leg Intersection

Development of a third Order Lagrangian Model

Let t_{ij} is the turning traffic volume from leg i to leg j ,

The Furness turning traffic equation(Furness, 1965) can be expressed as

$$t_{ij}^{n+1} = \frac{P_i}{\sum_{k=1}^m r_k A_k} t_{ij}^n = R_i t_{ij}^n$$

where $R_i = \frac{P_i}{\sum_{k=1}^m r_k A_k}$ -----(5)

$$r_i = \frac{t_{ij}^{n-1}}{\sum_{j=1}^m t_{ij}^{n-1}}$$

$$\text{If } R_i = \frac{\sum_{k=1}^{2m} a_k x_k}{\sum_{k=1}^{2m} b_k x_k} \text{ -----(6)}$$

where $a_k = 1$ if $k=i$ otherwise $a_k=0$

$b_i = 0$ for $i=1, m$

$b_i = r_{i-m}$ for $i=m+1, 2m$

$A_k =$ inflow into a leg when regarded as known values

$P_k =$ outflow into a leg when regarded as known values

$x_k =$ inflow into a leg(when treated as unknown) for $k=1$ to m

$x_k =$ outflow from a leg(when treated as unknown) for $k=m+1$ to $2m$

An aggregate objective function can be written as $Q =$

$$\sum_{i=1}^m R_i$$

A third order Lagrangian function can be formulated as

$$L=Q^3 + \lambda (C_a + C_b) \text{ ----- (10)}$$

λ is Lagrangian multiplier

$$C_a = \sum_{j=1}^m x_j - A_j \text{ -----(8)}$$

$$C_b = x_m - x_{2m} + \sum_{j=1}^{m-1} x_j - \sum_{j=1}^{m-1} x_{j+m} \text{ -----(9)}$$

Equation (10) is solved to obtain unknown inflow and outflow values using Newton's Optimization technique such that

$$(X)_{new} = (X)_{old} - H^{-1} \partial L$$

Let $da_i = a_i, db_i = b_i$

$$\text{Then } aa = \sum_{i=1}^{2m} a_i, bb = \sum_{i=1}^{2m} b_i$$

By setting

$$D_i = (bb da_i - aa db_i) bb^2$$

$$G_{i,j} = 2 D_i D_j \frac{(bb^2 (da_i db_j - da_j db_i) - 2bb (da_i - da_j) db_i) db_j}{bb^4}$$

The Hessian matrix, H which is 2m x 2m matrix is expressed as

$$H_{i,j} = 6 \frac{aa}{bb} (D_i D_j + G_{i,j})$$

while ∂L_i which is a 2m column vector is given as

$$\partial L_i = 6 \frac{aa}{bb} D_i$$

With the unknown inflow and outflow values for the leg m in the junction (x_m, x_{2m}) determined, the normal Furness equations can then be used to estimate the turning traffic volume. The procedures are listed below:

- . Guess the turning volume for the various legs of the intersections
- . Develop a third order Lagrangian function
- . Optimise the Lagrangian function using Newton's optimization technique using the known inflow and output data
- . Use the Furness to estimate the turning traffic volume for all the legs.

Field Study

Traffic count was carried out at the selected intersections in the city. The inflow and outflow traffic volume was determined for all classes of vehicles. The study was carried out for seven days. Thereafter the average daily traffic (ADT) for inflow for each of the legs of the intersection was determined.

RESULTS AND DISCUSSIONS

The hybrid model developed is used to estimate turning at selected intersection in Ibadan city. The computer programme developed converges irrespective of initial trial turning traffic volumes. The programme was tested by stepping down each of the legs one after the other. Then the fractional difference of the model values as compared with the field values were determined. The turning volume estimates were used to investigate the possibility of U-turn in the selected intersection. The results are presented in Table 1-6 with the stepped down legs in highlighted form (designated as Leg 3).

For the case when Iword leg was stepped down, the fractional difference of 0.06 and 0.02 were estimated for inflow and outflow respectively. When Ui leg was stepped down, the fractional difference for the inflow and outflow were -0.21 and -0.11 respectively. When also Oyo leg was stepped down, the fractional difference for the inflow and outflow were 0.06 and 0.20 respectively. The percentage U-turn estimates as depicted in Tables 2,4 and 6 suggest little or insignificant proportion of U-turn in the selected intersection.

REFERENCES

- [1] Adebisi, O. 1987: Improving Manual Counts of Turning Traffic Volumes at road Junctions *Journal of Transportation Engineering*, Vol. 113, No. 3, ASCE
- [2] Adeniji Kunle, 1987a: Public Transportation and Basic Needs Satisfaction in Nigeria, *Research for Development* Vol. 4, No. 2, pp. 292-304.
- [3] Adeniji Kunle, 1987b: Para-transit Models in Nigeria; Problems and Prospects, *Cities International Journal*, Vol. 4 No. 4, pp. 338-347.
- [4] Adeniji Kunle, 1991a: Urban Mobility Crisis: Where do we go from here? Ogun State Public Service lecture delivered at the Gateway Hotel, Abeokuta. (1991).
- [5] Adeniji Kunle, 1991b: Urban Mobility Crisis: Ogun State Public Service Lecture, delivered at the Gateway Hotel, Abeokuta 18th April, 1991.
- [6] Adeniji Kunle, 2002: Transport Challenges in Nigeria in the next two decades, key note Address delivered at the fifth meeting of the National Council on Transport need at the ECOWAS Secretariat, Abuja. 29th -31st August, Report of Proceedings, Federal Ministry of Transport.
- [7] Chandler, R.E., Herman, R. and Montroll, E.W. 1958: *Traffic Dynamics: Studies in Car Following*, Opns. Res. 6, pp. 165-183.
- [8] Edie, L.C. and Foote, R.S. 1958. *Traffic Flow in Tunnels*, Proc. Highway Research Board, 37, pp.334-344.
- [9] FMT. 1993 National Transport Policy (Main Document). Abuja, Federal Ministry of transport pp3-92.
- [10] Furness, K.P. 1965. "Time Function Iteration". *Traffic Engineering and Control*, 7(7), London, England, 458-460.
- [11] Herman, R. 1992. *Technology, Human Interaction, and Complexity: Reflections on Vehicular Traffic Science*. Operations Research, 40(2), pp.199-212.
- [12] Kupolati, W.K. 2008. *Characterisation of used Asphalt pavement for Road Construction*. PhD Thesis Civil Engineering Department, University of Ibadan, Nigeria.
- [13] Lighthill, M.J. and G.B. Whitham, 1955. *On Kinematic Waves: II. A Theory of Traffic Flow on Long Crowded Roads*. Proceedings of the Royal Society: A229, pp.317-347, London.
- [14] Michael, P.D.; Ahmed, A; Michael, Phil. R and Howard, C. 2005. *Field Evaluation of Roundabout Turning Movement Estimation Procedure* Unpublished.
- [15] Newell, G.F. 1955. *Mathematical Models for Freely Flowing Highway Traffic*. Operations Research 3, 176-186.
- [16] Nihan, N.L. and G.A. Davis, 1989. "Application of Prediction - Error Minimization and Maximum Likelihood to Estimate Intersection O-D matrices from Traffic Counts". *Transportation Science* 23 (2):77-90.
- [17] Oguara, T. M. 2006: *Highway Engineering Geometric Design*. Malthouse Press Limited.
- [18] Pipes, L.A. 1953. *An Operational Analysis of Traffic Dynamics*. J. Appl. Physics. pp. 274-281.
- [19] Reuschel, A. 195a. *Fahrzeugbewegungen in der Kolonne*. Oesterreichisches Ingenieur-Archiv 4, No. 3/4, pp.193-215.
- [20] Reuschel, A. 1950b and 1950c. *Fahrzeug bewegungen in der Kolome bei Gleichformig Beschleunigtem oder Verzogertem Leitfahrzeug*. Zeitschrift des Oesterreichischen ingenieurund Architekten-Vereines 95, No.7/8 59-62, No. 9/10 pp.73-77.
- [21] UNCHS, 1996-*An Urban using, World, Global Report on Human Settlement*, Oxford, Pergamon Press, pp. 314-322.
- [22] Wardrop, J.G. 1952. *Some Theoretical Aspects of Road Traffic Research*. Proceedings of the Institution of Civil Engineers, Part II, 1(2), pp.325-362, U.K.
- [23] Webster, F.V. 1958. *Traffic Signal Settings*. Road Research Technical Paper No. 39. Road Research Laboratory, London, U.K.

[24] Wikipedia Free Encyclopedia. Retrieved from
 [25] Wikipedia 2007: Newton's Method in optimization.
http://en.wikipedia.org/wiki/Newton's_method_in_optimization

Table 1: Inflow and OutFlow Values for Field and Model with Iword leg Unknown

	A:in(model)	B:in(field)	(B-A)/B	D:out(model)	E:out(field)	(E-D)/E
Leg 1	8006	8006	.00	2605	2605	.00
Leg 2	2325	2325	.00	4661	4661	.00
Leg 3	2343	2482	.06	5408	5547	.02

Table 2: Turning Volume Estimate with Iword Leg Unknown

	leg 1	leg 2	leg 3	% uturn
Leg 1	149	1309	1146	.06
Leg 2	3111	146	1404	.03
Leg 3	4212	1091	105	.02

Key
 Leg 1-oyo
 Leg 2-ui
 Leg 3-iwordad

Table 3: Inflow and OutFlow Values for Field and Model with ui leg Unknown

	A:in(model)	B:in(field)	(B-A)/B	D:out(model)	E:out(field)	(E-D)/E
Leg 1	8006	8006	.00	2605	2605	.00
Leg 2	2482	2482	.00	5547	5547	.00
Leg 3	2814	2325	-.21	5150	4661	-.11

Table 4: Turning Volume Estimate with Ui Leg Unknown

	leg 1	leg 2	leg 3	% uturn
leg 1	133	1319	1153	.05
leg 2	3549	189	1809	.03
leg 3	3898	1142	110	.02

Key
 Leg 1-Oyo
 Leg 2-Iwordad
 Leg 3-Ui

Table 5: Inflow and OutFlow Values for Field and Model with Oyo leg Unknown

	A:in(model)	B:in(field)	(B-A)/B	D:out(model)	E:out(field)	(E-D)/E
Leg 1	2482	2482	.00	5547	5547	.00
Leg 2	2325	2325	.00	4661	4661	.00
Leg 3	7493	8006	.06	2092	2605	.20

Table 6: Turning Volume Estimate with Oyo Leg Unknown

	leg 1	leg 2	leg 3	% uturn
Leg 1	101	1643	3803	.02
Leg 2	1413	123	3125	.03
Leg 3	1306	627	160	.08

Key
 Leg 1-Iwordad
 Leg 2-Ui
 Leg 3-Oyo