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Prof. Prakash B Lugade
Professor in Mathematics,
P V P I T ,Budhghoan College of Engineering,
Tal:- Miraj,Dist:- Sangli
Res.: Harihar Ganesh nagar Budhghoan
Tal:-Miraj,Dis:- Sangli (MS)-416304

TRINORMAL VECTOR OF THE WORLDLINE OF A PARTICLE

[1]P.B. and [2]D.B. Unde

Department of Mathematics,

[1]Dr. J.J. Magdum College of Engineering, Jaysingpur.Dist.Kolhapur Pin-416 101

[2]P.V.P.I.T., Budhgaon Sangli, Pin – 416 304

Maharashtra (INDIA)

ABSTRACT

In this paper the trinormal vector field R^a and bitorsion scalar B of the worldline of a particle are expressed in terms of the third and lower order intrinsic derivatives of the flow vector u^a. The applications of R^a and B are seen for relativistic continuum mechanics.

Introduction :

The expression for Trinormal R^a :

We adopt the signature of the metric tensor as (+, -, -, -). The timelike tangent field u^a to the worldline is chosen to satisfy

$$u^a u_a = 1$$

The vector field P^a is a spacelike vector orthogonal to u^a. It is called as principal normal. The spacelike unit vector fields Q^a and R^a are called respectively the binormal and the trinormal fields which are orthogonal to velocity as well as acceleration. For the tetrad denoted by (u^a, P^a, Q^a, R^a) the rheotetrad formulae (Relativistic Serret Frenet formulae) are (Synge 1960, Davis 1970)

$$u'^a = KP^a \qquad (A)$$

$$P'^a = Ku^a + TQ^a \qquad (B)$$

$$Q'^a = -TP^a + BR^a \qquad (C)$$

$$R'^a = -BQ^a \qquad \text{(D)}$$

The tetrad is invented in Relativistic continuum mechanics by Pirani (1956). The tetrad is related to velocity vector u^a, principal normal P^a, binormal Q^a and trinormal R^a. Thus tetrad is denoted by (u^a, P^a, Q^a, R^a). For the tetrad Synge (1960) has introduced first, second and third curvture K,T,B, respectively. Synge(1960) also given the relativistic serret Frenet formulae, Gursey (1957) given the explicit expression for binormal Q^a interms of derivatives of velocity vector u^a. In this paper the expression for trinormal R^a and bitorsion B in terms of derivatives of u^a are given.

The orthonormal conditions are

$$u^a u_a = -P^a P_a = -Q^a Q_a = -R^a R_a = 1$$

and $$u^a P_a = u^a Q_a = u^a R_a = P^a Q_a = P^a R_a = Q^a R_a = 0$$

An overhead dot denotes the covariant differentiation along the flow vector u^a, thus

$$P'^a = P^a{}_{;b} u^b$$

Following Gursey (1957), the explicit expression for bionormal Q^a is

$$Q^a = \frac{1}{KT}\left(\H{u}_a - \frac{K'}{K} u'^a - K^2 u^a \right) \qquad \text{(E)}$$

On covariantly differentiating the expression for trinormal Q^a along the flow vector u^a, we obtain.

$$Q'^a = \frac{1}{KT}\left(\H{u}^a - L\H{u}^a + Mu'^a - NK^2 u^a \right) \qquad \text{(F)}$$

Where

$$L = \frac{T'}{T} + \frac{2K'}{K},$$

$$M = \frac{2K'^2}{K^2} + \frac{T'K'}{TK} - \frac{K'}{K} - K^2$$

$$N = \frac{K'}{K} - \frac{T'}{T}$$

Substituting Equations (A), (F) in Eq (C) yields on rearrangement,

$$R^a = \frac{1}{KTB} \left[\dddot{u}^a - L\ddot{u}^a + (M+T^2)\, u'^a - NK^2 u^a \right] \qquad \text{(G)}$$

This is the expression for R^a interms of the third and lower order intrinsic derivatives of the flow vector u^a,

The expression for Bitorsion B :

$$K^2 = -u'^a u'_a$$

$$T^2 = \frac{1}{K^2} \left(K^4 - K'^2 - \ddot{u}^a \ddot{u}_a \right)$$

To evaluate B, we use (Eq. F) in $R^a R_a = -1$ and after simplification we obtain

$$B^2 = \frac{1}{K^2 T^2} \left[9K^2 K'^2 - (K'' + K^3 - KT^2)^2 - (2K'T + KT')^2 - \dddot{u}^a \dddot{u}_a \right] \qquad \text{(H)}$$

on exploiting the identities

$$u'^a u_a = 0$$

$$u^a \ddot{u}_a = K^2$$

$$u'^a u'_a = -K^2$$

$$\dddot{u}^a u_a = 3KK'$$

$$\dddot{u}^a \ddot{u}_a = K^2(K^2 - T^2) - k''^2$$

$$\ddot{u}^a u'_a = \frac{1}{2} (u'^a u'_a)' = -KK'$$

$$\dddot{u}^a u'_a = K^2(T^2 - K^2) - KK''$$

$$\dddot{u}^a \ddot{u}_a = 3K^3 K' - T^2 KK' - K^3 T\, T' - K'K''$$

4

Thus in equation (G) we have accomplished the expression for bitorsion scatar field in terms of product of u^a, $\acute{u}^a$, $\H{u}^a$, $\H{u}^a$ and K,T. The 'quite completed' nature of R^a and B is obvious from the equations predicted by Vishweshwara (1988).

Theorem :

Remark : $\H{u}^a$ is a linear combination of u^a,u^a, $\H{u}^a$ iff B=0

Soln : Necessary Part

$$\H{u}^a = KTBR^a + L\H{u}^a - (M+T^2)u'^{\,a} - NK^2u^a \qquad \text{(by Eq. G)}$$

For linear combination of u^a,u'^a, $\H{u}a$,

KTB = 0

As $K \neq 0$, $T \neq 0$, $B = 0$

Sufficient part : if B = 0 then

$$\H{u}^a = L\H{u}^a - (M+T^2)u'^{\,a} - NK^2u^a.$$

It is linear combination of $\H{u}^a$, u^a, u^a, of vanishing Bitorsion

Physical Significance : The path of classical gravitationally self interacting spin partical with Frenkel-weissenhoff constraints (Geonner 1967) has vanishing bitorsion. The Equation of path is linear combination of $\H{u}^a$, u'^a, u^a.

$$\H{u}^a = \left(\frac{m^2}{s^2} - u'^b u'_b\right)u'^a + \frac{8Gm}{15}\left(\frac{m^2}{s^2} - u'^b u'_b\right)(u'^i u'_i u^a + \H{u}^a) - 3u'^i \H{u}_i u^a$$

Where m is the mass of the particle, G is the gravitational constant and s is spin of the particle of mass m.

Conclusion :

In this paper trinormal vector field R^a and bitorsion B are expressed interms of linear combination of derivatives of u^a. The expression of bitorsion can be studied for non-geodesic flows in Relativistic continuum mechanics. According to the exhaustive and magnificent survey of the exact solutions of Einstein's field equations by Kramer, Hertz, Maccallum and Stephani (1980), right from the inception of general relativity in 1916, the properties of the models are confined to the study of geodesic paths. Only few models with non-geodesic paths are reported in their book. This provides the motivation for this paper which explores non-geodesic flow with torsion as well as bitorsion.

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