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Tuning and Analysis of Fractional Order Controllers for Hard Disk Drive Servo System

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Abstract- **Hard-Disk-Drive Servo systems are mechatronic systems that demands for high precision control of output of the system. The head positioning servo system is to maintain the read/write head at a desired track (track following). The loop gain variations caused by different hardware components is a major problem that affects the tracking control performance of HDD's. To achieve consistent tracking performance, the problem demands for a controller that is robust to loop gain variations. In this paper, a fractional order proportional integral (FO-PI) and fractional order proportional integral derivative (FO-PID) controllers are proposed and designed. Simulation results shows that the proposed controllers can ensure robustness against loop gain variations and the overall tracking control performance is improved than traditional integer order controller.**

Key Words: Hard- disk drive, fractional order proportional integral (FO-PI) control, fractional order proportional integral derivative control (FO-PID), proportional integral derivative controller (PID), robustness, isodamping

I. INTRODUCTION

Hard Disk Drives (HDD's) are widely used data storage medium in the modern era of digital technology [1], [2]. The servo mechanism in HDD systems plays an important role in meeting the high performance requirements of HDD's [3]. The servo mechanism includes a Voice Coil Motor (VCM) actuator which is used to position a read/write head assembly model for recording and retrieving data. Track following, track seeking and track settling are the control modes of the servo mechanism. Track seeking means the heads to be positioned from one track to a desired track. Track Following needs the heads to be maintained as close as possible to desired track and follow it. In track settling the heads are allowed to settle within the desired track. Track following requires that the position error, i.e., the relative difference between actual head position and desired should be minimum. For high performances, the HDD system should have consistent track following performances under the influence of gain variations that can be caused by external disturbances, changes in external temperatures [4].

Many integer order controllers/compensators [5], [6] have been designed in order to solve the problem. Even though some of the requirements are satisfied, these controllers are not found robust against loop gain variations. The systems designed with integer order controllers may exhibit variations in time domain and frequency domain requirements with gain variations. An isodamping property is not exhibited with designed integer order controller. Such a performance cannot be accepted in mechatronic system like HDD's were high precision control are necessary. Several control approaches such as Linear Quadratic Gaussian control, robust control, H_∞ control have been proposed for the track following performances. [7-9]. In [10], [11] the effects of loop gain variations are suppressed using automatic gain control methods. More control methods should be there that should come up with high accuracy and simplicity in control.

In the mean while fractional order controllers (FO controllers) have got a wide spread attraction in control engineering and technology [12], [13]. In the past the FO controllers were not used because of unfamiliarity with the use of FO parameters, and the absence of necessary computational power. But the fractional order differential equations could well define the dynamic processes and so the computational progresses can be made easier. With the introduction of fractional order parameters more accuracy in control can be obtained [14].

The main contributions of this brief include: Fractional order proportional integral and fractional order proportional integral derivative controllers are proposed and designed for the HDD- VCM actuator model. Certain tuning constraints are imposed to ensure that the gain cross over and phase margin specifications are met and robustness against gain variations are achieved. The FO controller performances are compared with an integer order PID controller. All the controllers are addressed under same design specifications. From the simulations it is clear that the FO controllers outperform the PID controllers. This brief provides an illustration of benefits of applying FO-PI and FO-PID controllers in the real HDD industry.

The outline of rest of this brief is organized as follows. Mathematical modelling of VCM actuator is described in section II. Section III explains the basic concepts of fractional calculus. In section IV initial design procedures for the FO controllers are presented. Some tuning constraints are imposed in order to ensure that the gain cross over frequency, phase margin and robust specifications are met. Then FO-PI controller design procedures are given. For the FO-PID controllers, a new tuning rule based on Ziegler- Nichols and

Aström- Hägglund methods are proposed. In section V the implementation details of the FO controllers on the HDD actuator model are given. In section VI the simulation results that are carried out to compare and evaluate the tracking performances of FO controllers with integer order PID controllers are presented. Conclusions are given in section VII.

II. HDD ACTUATOR MODELLING

In this section we present the dynamic model of HDD with the Voice Coil Motor (VCM) actuator [1]. The fig. 1 shows the circuit model of VCM actuator.

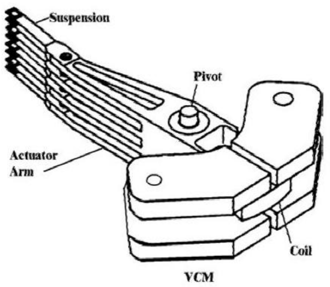


Fig.1 Voice Coil Motor (VCM) Actuator

The actuator is attached to the coil of VCM and is pivoted. When current is passed through the coil of motor, an electromagnetic force is experienced. This causes a torque on the rotary actuator. The torque of the actuator is given by:

$$\tau = k_t I_c \tag{1}$$

where, I_c is the current through the coil.

The equation of motion of rotary actuator is:

$$\tau = J\ddot{\theta} \tag{2}$$

where, J is the moment of inertia of moving parts, and $\ddot{\theta}$ being the angular displacements.

From (1) and (2) the mechanical dynamics of the actuator is

$$\tau = \frac{J\ddot{\theta}}{k_t} \tag{3}$$

For the movement of the actuator, it needs to be powered by an amplifier. Current amplifiers are commonly used due to its high impedance. The simplified transfer function of amplifier is given by:

$$\frac{I(s)}{u(s)} = \frac{k}{s\tau_a+1} \tag{4}$$

where, τ_a is the torque constant an k is the amplifier gain. I(s) and u(s) are the current input and outputs to the amplifier.

Thus the transfer function of VCM with current driver is:

$$\frac{\theta(s)}{u(s)} = \frac{k_t k}{J}\frac{1}{s^2} \tag{5}$$

TABLE I

PLANT MODEL PARAMETERS

Parameter	Description	Value
J	Moment of Inertia	0.2
k_t	Torque constant	20
k_y	Position measurement gain	10000
ω_n	Resonant Frequency	80
a	Coupling coefficient	0.0032
b	Coupling coefficient	0.25

where, $\theta(s)$ is the head position, which is given in radians. It can be transformed into data tracks and the modified transfer function is:

$$\frac{y(s)}{u(s)} = \frac{k_a k_y}{s^2} \tag{6}$$

Since the actuator structures are not perfectly rigid and contain several flexible modes that can cause resonances and vibrations. So the effects of such modes cannot be neglected. More accurate model is obtained by considering the resonance effects. The overall accurate model is given as:

$$P(s) = \frac{k_a k_y}{s^2}\frac{(a\omega_n s + b\omega_n{}^2)}{s^2+2\zeta\omega_n s+\omega_n{}^2} \tag{7}$$

where, k_a is the acceleration constant, k_y is the position measurement gain. ω_n is the resonant frequency. a and b are the resonant coupling coefficients. The parameters for the plant model are given in Table I.

Then the model described in (7) can be written as

$$P(s) = \frac{1.287e8s+8.085e8}{s^4+502.7s^3+2.527e5s^2} \tag{8}$$

III. BASIC CONCEPTS OF FRACTIONAL CALCULUS

The fractional order calculus is an area that deals with derivatives and integrals from non integer orders. The basic continuous integro-differential operator is defined as [15-18]

$$ {}_aD_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & ,\Re(\alpha) > 0 \\ 1 & ,\Re(\alpha) = 0 \\ \int_a^t d\tau^{-\alpha} & ,\Re(\alpha) < 0 \end{cases} \tag{9}$$

where α is the order of the operation; generally $\alpha \in R$ and a is a constant corresponds to initial conditions.

The most common Caputo definition for differentiation is

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \quad (10)$$

where n is an integer and n-1< α<n

Similarly Caputo's fractional order integration is given as

$$ {}_aD_t^{\alpha+1-n} f(t) = \frac{1}{\Gamma(n-\alpha-1)} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha+1-n+1}} d\tau \quad (11)$$

IV. FRACTIONAL ORDER CONTROLLER DESIGN

In this section the general form of FO controllers and the design strategies are presenting. Generally the FO controllers are modeled as:

$$ C_1(s) = K_p\left(1 + K_i s^{-\lambda}\right) \quad (12)$$

$$ C_2(s) = K_p + K_i s^{-\lambda} + K_d s^{\mu} \quad (13)$$

where, (12) and (13) represents the general form of an FO-PI and FO-PID controller.

For the plant model described in equation (8) the FO-PI /FO-PID controllers can be designed as per the three design specifications given below. For the controller of the form

The design specifications to met are:

1) Gain cross over frequency specifications.

$$ \left|\left(C(j\omega_{gc})P(j\omega_{gc})\right)\right| = 1 . \quad (14)$$

2) Phase margin specifications:

$$ < \left(C(j\omega_{gc})P(j\omega_{gc}\right) = \varphi_m - \pi \quad (15)$$

where φ_m is the desire phase margin and ω_{gc} is the gain cross over frequency.

3) Robustness to gain variations

$$ \frac{d<\left(C(j\omega_{cg})P(j\omega_{cg})\right)}{d\omega} = 0 \quad (16)$$

i.e., the derivative of the phase of the open-loop system with respect to the frequency is forced to be zero at the gain crossover frequency so that the closed-loop system is robust to gain variations, and therefore the time responses of the systems are almost invariant.

A. Fractional order PI controller

The tuning strategies for the FO-PI controllers are as follows:

1. Given ω_{gc}, the gain crossover frequency
2. Given ϕ_m, the desired phase margin.
3. Plot curve 1 corresponding to K_i with respect to λ, according to specification 2
4. Plot curve 2 corresponding to K_i with respect to λ, according to specification 3
5. Obtain λ and K_i from the intersection point between Curves 1 and 2.
6. Calculate K_p from form specification 1

By these tuning rules and specifications, FO-PI controller parameters can be obtained.

B. Fractional order PID controller

In this section the tuning strategies for an FO-PID controller is presented. For the FO-PID controller of the form in (13) there are five parameters to obtain. A tuning rule based on Ziegler- Nichols [19] and Aström- Hägglund [20] method is adopted here.

The rules are summarized as follows:

1. Obtain K_p and K_i from Ziegler- Nichols tuning rule.
2. Obtain K_d from Aströ m- Hägglund Method.
3. Solve for λ and μ using (14) - (16)

V. FRACTIONAL ORDER CONTROLLER IMPLEMENTATION ON HDD MODEL

A. Implementation of FO-PI controller

The magnitude and phase of the plant model is given as:

$$ |P(j\omega)| = \frac{3199.999\sqrt{1+(0.159154943\,\omega)^2}}{\omega^2\sqrt{(1-3.95785836\,e{-}6\omega^2)^2+(1.9894368\,e{-}3\omega)^2}} \quad (17)$$

$$ < P(j\omega) = -\pi - \tan^{-1}\left(\frac{1.989436789\,e{-}3\omega}{(1-3.957858736\,e{-}6\omega^2)}\right) + \tan^{-1}(0.159154943\omega) \quad (18)$$

For the FO-PI controller of the form (12) the magnitude and phase are:

$$ |C_1(j\omega)| = K_p\sqrt{\left(1 + K_i\omega^{-\lambda}(\cos(\lambda\pi/2))\right)^2 + \left(K_i\omega^{-\lambda}\sin(\lambda\pi/2)\right)^2} \quad (19)$$

$$ < C_1(j\omega) = -\tan^{-1}\left(\frac{K_i\omega^{-\lambda}\sin(\lambda\pi/2)}{1+K_i\omega^{-\lambda}\cos(\lambda\pi/2)}\right) \quad (20)$$

Then from specification 2

$$ \frac{K_i\omega_{gc}^{-\lambda}\sin(\lambda\pi/2)}{1+K_i\omega_{gc}^{-\lambda}\cos(\lambda\pi/2)} = \tan\left(< P(j\omega_{gc}) - \varphi_m + \pi\right) \quad (21)$$

From 21, we obtain,

$$ K_i = \frac{c}{a-bc} \quad (22)$$

where,

$$ c = \tan\left(< P(j\omega_{gc}) - \varphi_m + \pi\right) \quad (23)$$

$$ a = \omega_{gc}^{-\lambda}\sin(\lambda\pi/2) \quad (24)$$

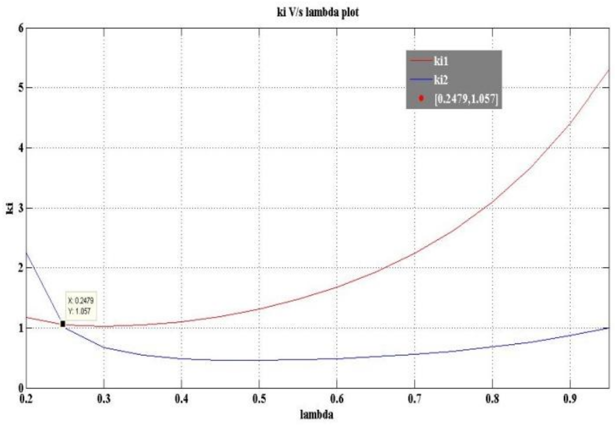


Fig. 2 K_i V/s λ plot

$$b = \omega_{gc}{}^{-\lambda}\cos\left(\lambda\pi/2\right) \tag{25}$$

From specification 3

$$\frac{d}{d\omega_{gc}}\left(-\tan^{-1}\left(\frac{K_i\omega_{gc}{}^{-\lambda}\sin\lambda\pi/2}{1+K_i\omega_{gc}{}^{-\lambda}\cos\lambda\pi/2}\right)\right) = -\frac{d}{d\omega_{gc}} < P\left(j\omega_{gc}\right)\right) \tag{26}$$

From this K_i can be evaluated as

$$K_i = \frac{-(df-e)\pm\sqrt{(df-e)^2-4gd^2}}{2dg} \tag{27}$$

where,

$$d = -\frac{d}{d\omega_{gc}}\left(< P\left(j\omega_{gc}\right)\right) \tag{28}$$

$$e = \lambda\omega_{gc}{}^{-\lambda-1}\sin\left(\lambda\pi/2\right) \tag{29}$$

$$f = 2\omega_{gc}{}^{-\lambda}\cos\left(\lambda\pi/2\right);\ g = \left(\omega_{gc}{}^{-\lambda}\right)^2 \tag{30}$$

Then value of K_p is obtained from:

$$K_p = \frac{1}{|P(j\omega_{gc})|\sqrt{\left(1+K_i\omega_{gc}{}^{-\lambda}\left(\cos\left(\lambda\pi/2\right)\right)\right)^2+\left(\sin\left(\lambda\pi/2\right)\right)^2}} \tag{31}$$

The curves between λ and K_i are shown in Fig.2. From the intersections, the optimum values of λ and K_i are obtained as 0.2479 and 1.057. Then, the designed FO-PI controller is of the form

$$C_1(s) = 0.085(1 + 1.057s^{-0.2479}) \tag{32}$$

B. Implementation of FO-PID controller

For the FO-PID controller of the form in (13).The magnitude and phase of the controller are given by (33) and (34).

$$\left|C_2\left(j\omega_{gc}\right)\right| = \sqrt{A_1{}^2 + B_1{}^2} \tag{33}$$

$$< C_2\left(j\omega_{gc}\right) = \tan^{-1}\frac{B_1}{A_1} \tag{34}$$

where,

$$A_1 = K_p + K_i\omega_{gc}{}^{-\lambda}\cos{}^{\lambda\pi}/_2 + K_d\omega_{gc}{}^{\mu}\cos{}^{\mu\pi}/_2 \tag{35}$$

$$B_1 = -K_i\omega_{gc}{}^{-\lambda}\sin{}^{\lambda\pi}/_2 + K_d\omega_{gc}{}^{\mu}\sin{}^{\mu\pi}/_2 \tag{36}$$

From specification (1)

$$\sqrt{A_1{}^2 + B_1{}^2}.\left|P(j\omega_{gc}\right| = 1 \tag{37}$$

From specification (2)

$$\tan^{-1}\frac{B_1}{A_1} + < P\left(j\omega_{gc}\right) = \varphi_m - \pi \tag{38}$$

From specification 3

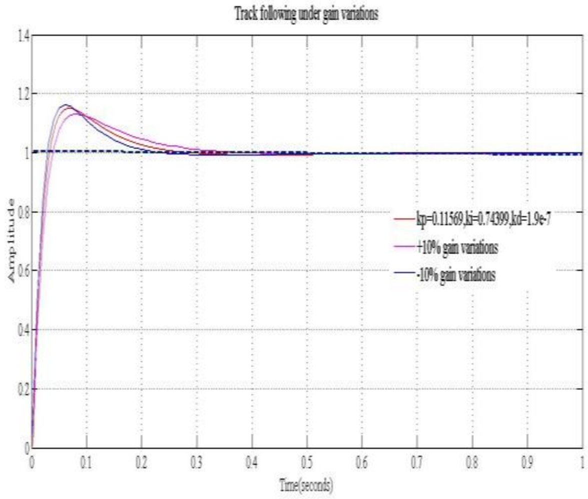
$$\frac{d}{d\omega_{gc}}\left(\tan^{-1}\frac{B_1}{A_1}\right) = -\frac{d}{d\omega_{gc}} < P\left(j\omega_{gc}\right) \tag{39}$$

The values of K_p and K_i obtained from Ziegler- Nichols tuning rule are 0.58477 and 94.133. Then from Aström-Hägglund method K_d is obtained as 0.0019. The values of λ and μ are solved using 'fsolve' command in MATLAB and are obtained as 1.2913 and 1.3556 respectively. Then the FO-PID controller has the form:

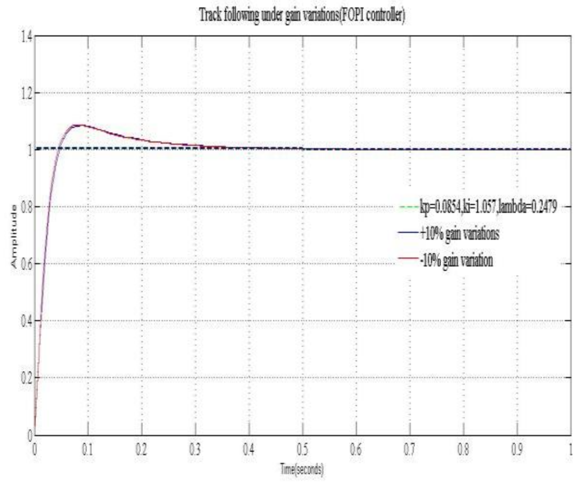
$$C_2(s) = 0.58477 + 94.13s^{-1.2913} + 0.01896s^{1.3556} \tag{40}$$

VI. SIMULATION RESULTS

This section presents the validation of the various performances with FO controllers. We have also designed an integer order controller for a fair comparison with FO controllers. The performances with each controller under gain variations are evaluated. Fig. 5 shows track following performances of the FO controllers and an integer order PID controller under gain variations. The response characteristics can be analyzed from Tables II, III and IV



(a)



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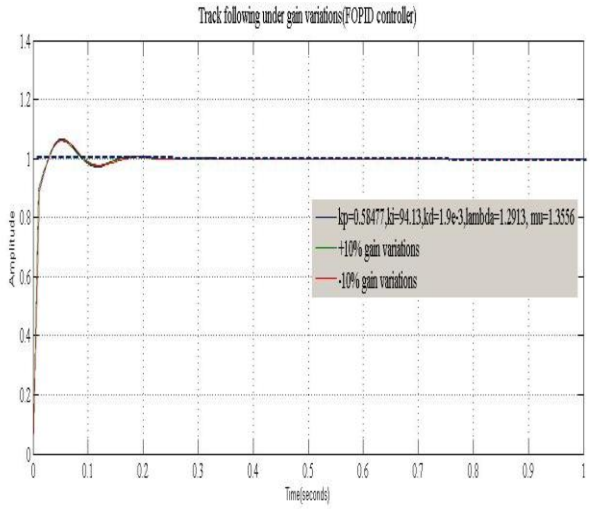


(c)

Fig 5. Track following performances under gain variations (a) PID controller (b) FO-PI controller (c) FO-PID controller

TABLE II

TRACKING PERFORMANCE CHARACTERISTICS WITH PID CONTROLLER

PID	90% gain	100% gain	110% gain
Overshoot%	16.2467	15.1036	13.3772
Settling time(seconds)	0.1823	0.2158	0.2705

TABLE III

TRACKING PERFORMANCE CHARACTERISTICS WITH FOPI CONTROLLER

FOPI	90% gain	100% gain	110% gain
Overshoot%	8.7003	8.4464	8.3916
Settling time(seconds)	0.2587	0.2597	0.2609

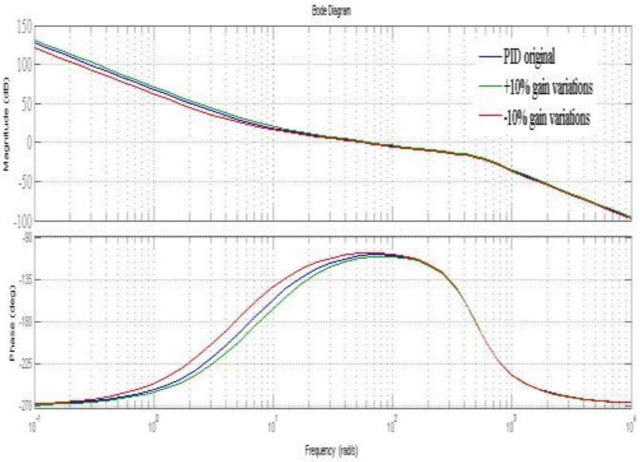
TABLE IV

TRACKING PERFORMANCE CHARACTERISTICS FOPID CONTROLLER

FOPID	90% gain	100% gain	110% gain
Overshoot%	6.8495	6.5760	6.0150
Settling time(seconds)	0.1355	0.1332	0.1303

From the tables we can infer that the settling time and overshoots are almost similar under gain variations for FO controllers. Isodamping property is exhibited with these controllers. While for the integer order PID controller the tracking performances are not so robust under gain variations. With the FO controllers overshoots are reduced to 44%-56 % .Among the FO controllers FO-PID controllers shows better track following performances when compared to FOPI controller. With the FO-PID controllers there is 38% reduction in settling time.

Now the Open loop Bode plot with gain variations of each controller is shown in fig.6. The Track following error plot with each controller are shown in fig.7. From figure 6 we can infer that the FO controllers maintain the flat phase feature which shows that they are more robust to gain variations. The phase margin and gain cross over frequencies are almost similar with gain variations. While with PID controllers, these may vary.Fig.7 shows the track following error with each of the controllers. The error is regulated in minimum time with FO controllers while it is 0.7736 in the case of PID controllers.



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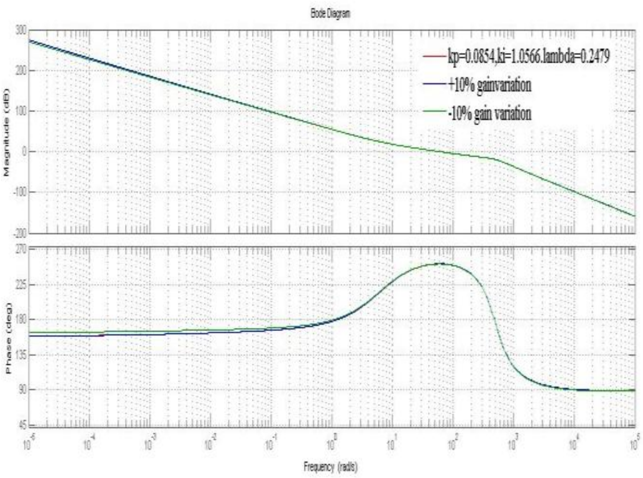

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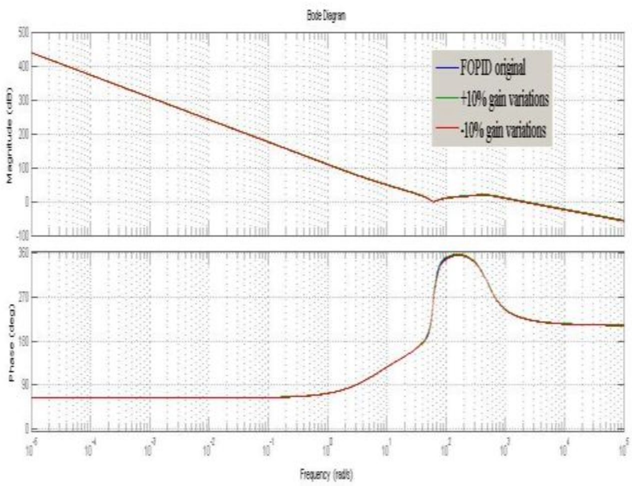
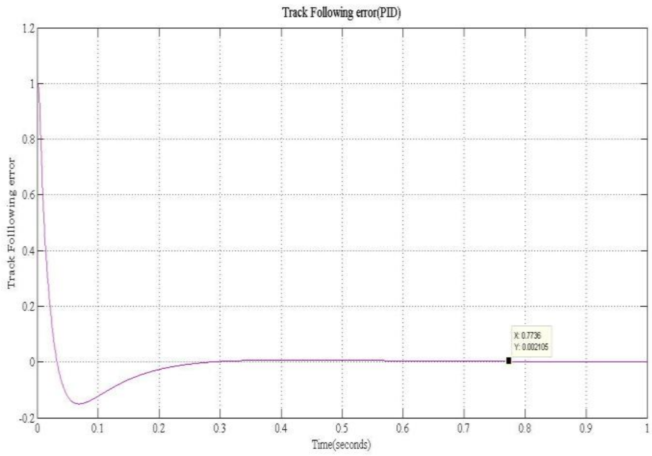
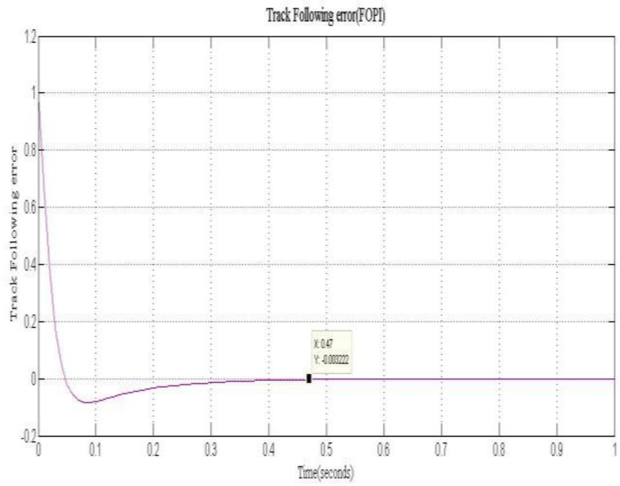


(c)

Fig. 6 Open loop Bode plot comparison with gain variations (a) PID controller (b) FO-PI controller (c) FO-PID controller



(a)



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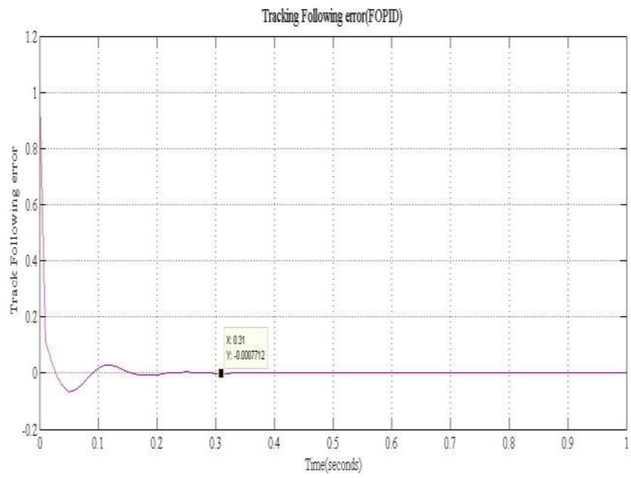

(c)

Fig 7. Track Following error (a) PID controller (b) FO-PI controller (c) FO-PID controller

VII. CONCLUSION

In this brief two fractional order controllers (FO-PI and FO-PID) controllers are proposed and designed for an HDD model so as to achieve the gain crossover frequency, phase margin and robustness requirements. FO controller implementation details on the HDD model were clearly presented. The track following performance and open loop Bode response under gain variations are evaluated with the FO controllers. The performances are compared with an integer order PID controller. From the simulation results it is clear that the FO controllers are more robust under gain variations. The track following performances is consistent with FO controllers.

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