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Two Phase Thermal Boundary Layer Flow

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ABSTRACT:

Momentum integral method has been employed by using third degree profiles for velocity, temperature and particle density to study the thermal boundary layer characteristics over a flat plate. It is observed that, the particle velocity, the particle density and the temperature on the plate approaches a finite value towards the downstream. The solution is valid throughout the plate unlike previous studies available in the literature. It has been observed that, heat flows from the plate towards the fluid as Nusselt number (Nu) is positive. Irrespective of presence of heavier or lighter material particles, the particles settles down on the plate as expected and the buoyancy force stabilizes the boundary layer growth.

Key words: Two-phase flow, Boundary layer characteristics, Buoyancy force, Heat transfer

NOMENCLATURE :

$(x, y,)$	→	Space co-ordinates i.e. distance along the perpendicular to plate length
$\vec{q}(u, v)$	→	Velocity components for the fluid phase in x, y and $z-$ directions respectively
$\vec{q}_p(u_p, v_p)$	→	Velocity components for the particle phase in x, y and $z-$ directions respectively
(T, T_p)	→	Temperature of fluid and particle phase respectively
(T_w, T_∞)	→	Temperature at the wall and free-stream respectively
(υ, υ_p)	→	Kinematic coefficient of viscosity of fluid and particle phase respectively
(ρ, ρ_p)	→	Density of fluid and particle phase respectively
(ρ_s, ρ_m)	→	Material density of particle and mixture respectively
(μ, μ_p)	→	Coefficient of viscosity of fluid and particle phase respectively
(τ_p, τ_T)	→	Velocity and thermal equilibrium time
(c_p, c_s)	→	Specific heat of fluid and SPM respectively
Re	→	Fluid phase Reynolds number
Pr	→	Prandtl number
Ec	→	Eckret number
Nu	→	Nusselt number
Gr	→	Grassoff number
Fr	→	Froud number
c_f	→	Skin friction coefficient
τ_w	→	Skin friction (Shear stress for clear fluid)
p	→	Pressure of fluid phase
φ	→	Volume fraction of Suspended particulate matter (SPM)
D	→	Diameter of the particle
δ	→	Boundary layer thickness
a	→	Thermal diffusivity
κ	→	Thermal conductivity
α	→	Concentration parameter
β^*	→	Coefficient of volume

expansion

ϵ → Diffusion parameter

F → Friction parameter between the fluid and the particle $(F = 18\mu/\rho_p d^2)$

L → Reference / Characteristic length

U → Free stream velocity

A → δ^2/L^2

1. INTRODUCTION:

The boundary layer flow of a gas particulate mixture over a flat plate gives the detailed structure of the flow and estimates the surface characteristics like skin friction co-efficient, particulate velocity and density on the surface under various assumptions. Several investigators [1-9] have derived equations governing the Two Phase flow and reduce them to boundary layer type using Prandtl boundary layer approximations. Marbel's [2] solutions is valid for downstream region of the plate and the particulate velocity on the surface remains zero. Singleton [6] has studied compressible gas particulate flow over a flat plate for high and low slip flow regions by employing series solution method. Soo [7] has derived momentum integrals for the gas and particulate phases and solved the same by using linear profiles both for gas phase and particle phase and quadratic profile for particulate density. Tabakoff and Hammed [8] have used fourth degree profiles for both gas and particle velocity and particle density. Soo [3] and Tabakoff and Hammed [9] has pointed out that particle velocity decreases linearly along the plate length x and particle density increases continuously along the plate length x. Their study leads to a surface particle velocity zero and particle density to infinity at a distance along the plate length $x = 1$. No effort has been made for studying the temperature distribution inside the boundary layer. Jain & Ghosh [1] have investigated the structure and surface property of the boundary layer of a gas particulate flow over a flat plate by employing momentum integral method. They have pointed out that the third degree profile for velocity and particle density gives results which is valid to far downstream stations on the plate. With the third degree profile of particulate velocity on the surface continuously decreases from its free stream value and particulate density on surface increases rather slowly from its free stream value at the leading edge to an asymptotic value as we approach far downstream on the plate surface.

The present study is an attempt to study the temperature distribution inside the boundary layer over a flat plate which gives a better understanding of the gas particulate boundary layer flow one. In this case, the momentum integral method is adopted to study the flow and temperature distribution by using a third degree profiles.

2. MATHEMATICAL FORMULATION & SOLUTION :

The governing equations of two dimensional gas particulate flow within the boundary layer on a flat plate are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial}{\partial x}\left(\rho_p u_p\right) + \frac{\partial}{\partial y}\left(\rho_p v_p\right) = 0 \tag{2}$$

$$\rho\left(u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\,\frac{\partial u}{\partial y}\right) - \frac{\varphi}{1-\varphi}\,\frac{\rho_s}{\tau_p}\left(u - u_p\right) - \rho g \beta^*(T - T_\infty) \tag{3}$$

$$\rho_p\left(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}\right) = \varphi\frac{\partial}{\partial y}\left(\mu_s\,\frac{\partial u_p}{\partial y}\right) + \frac{\rho_p}{\tau_p}\left(u - u_p\right) + \varphi(\rho_s - \rho)g \tag{4}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\kappa\,\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\varphi}{1-\varphi}\,\frac{\rho_s c_s}{\tau_T}\left(T_p - T\right) \tag{5}$$

$$\varphi\rho_s c_s\left(u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y}\right) = \varphi\, k_p\frac{\partial^2 T_p}{\partial y^2} + \varphi\rho_s c_s\,\frac{1}{\tau_T}\,\left(T - T_p\right) \tag{6}$$

The boundary conditions are

At y = 0 : u = 0, v = 0, $u_p = a_2(x)$, $v_p = 0$, $\rho_p = a_3(x)$, T = 0, $T_p = a_4(x)$ (7)

At y = : u = U, $u_p = U$, $\rho_p = \rho_{p_\infty}$, T = T_∞, $T_p = T_\infty$ (8)

Clearly $\delta > \delta_t$ and $\delta > \delta_p$

It may be noted that, the thickness of the thermal boundary layer (δ_t), particle velocity boundary layer (δ_p), particle thermal boundary layer (δ_{p_t}) are the same as that of the velocity boundary layer(δ). Strictly speaking, they are different, in general. This assumption has its justification in that it simplifies the computational work and the results obtained are very near to the experimental results and to the exact solutions.

Now, on integration equations from (2) to (6) w. r. t. $y = 0$ (wall) to $y = \delta$, we get

$$\frac{d}{dx}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \frac{\mu}{\rho U^2}\left.\frac{\partial u}{\partial y}\right|_0 + \frac{1}{1-\varphi}\,F\int_0^\delta \frac{\rho_p}{\rho U}\left(1 - \frac{u_p}{U}\right)dy$$
$$-\frac{1}{1-\varphi}\,F\int_0^\delta \frac{\rho_p}{\rho U}\left(1 - \frac{u}{U}\right)dy + \int_0^\delta \frac{1}{U^2}g\beta(T - T_\infty)\,dy \tag{9}$$

$$\frac{\partial}{\partial x}\int_0^\delta(\rho_p u_p)(U - u_p)\,dy = \varphi\,\mu_s\left.\frac{\partial u_p}{\partial y}\right|_{y=0} - F\int_0^\delta \rho_p(u - u_p)\,dy - \left(1 - \frac{\rho}{\rho_s}\right)g\int_0^\delta \rho_p\,dy \tag{10}$$

$$\frac{\partial}{\partial x}\int_0^\delta u(T - T_\infty)\,dy = -a\left.\frac{\partial T}{\partial y}\right|_{y=0} + \frac{\mu}{\rho c_p}\int_0^\delta\left(\frac{\partial u}{\partial y}\right)^2 dy + \frac{1}{1-\varphi}\,\frac{1}{\tau_T}\frac{c_s}{\rho c_p}\int_0^\delta \rho_p\left(T_p - T\right)dy \tag{11}$$

$$-\frac{\partial}{\partial x}\int_0^{\delta}(\rho_p u_p)(T_p - T_\infty)dy = \frac{\varphi\, k_p}{c_s}\left.\frac{\partial T_p}{\partial y}\right|_{y=0} + \frac{1}{\tau_T}\int_0^{\delta}\rho_p\,(T_p - T)\,dy \tag{12}$$

By introducing the non- dimensional quantities like

$$x^* = \frac{x}{L},\ y^* = \frac{y}{\delta},\ u^* = \frac{u}{U},\ u_p^* = \frac{u_p}{U},\ T^* = \frac{T - T_\infty}{T_w - T_\infty}\ ,\ \rho_p^* = \frac{\rho_p}{\rho_{p\,0}}\ ,\ {T_p}^* = \frac{T_p - T_\infty}{T_{pw} - T_\infty} \tag{13}$$

The equations (9) to (12) reduces to

$$\frac{d}{dx^*}\left[\frac{\delta}{L}\int_0^1 u^*(1-u^*)\,dy^*\right] = \frac{\mu}{\rho U\delta}\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} + \frac{1}{1-\varphi}\,F\frac{\rho_{p\,0}}{\rho}\frac{\delta}{U}\int_0^1\rho_p^*\,(1-u_p^*)\,dy^*$$
$$-\frac{1}{1-\varphi}\,F\frac{\rho_{p\,0}}{\rho}\frac{\delta}{U}\int_0^1\rho_p^*\,(1-u^*)\,dy^* + \frac{\delta}{L}\int_0^1\frac{Gr}{Re^2}T^*dy^* \tag{14}$$

$$\frac{\partial}{\partial x^*}\delta\int_0^1(\rho_p^*\,u_p^*)\,(1-\ u_p^*)\,dy^*$$
$$= \frac{L^2}{\delta}\frac{\epsilon}{Re}\left.\frac{\partial u_p^*}{\partial y^*}\right|_{y^*=0} - \delta\frac{FL}{U}\int_0^1\rho_p^*\,(u^* - u_p^*)\,dy^* - \frac{\delta}{Fr}\left(1-\frac{\rho}{\rho_s}\right)\int_0^1\rho_p^*\,dy^* \tag{15}$$

$$\frac{\partial}{\partial x^*}\left[\delta\int_0^1\ u^*\,T^*\ dy^*\right] = -\frac{aL}{U\delta}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} + \frac{\mu}{\rho c_p}\frac{UL}{\delta(T_w - T_\infty)}\int_0^1\left(\frac{\partial u^*}{\partial y^*}\right)^2\ dy^*$$
$$+\frac{1}{1-\varphi}\frac{1}{\tau_T}\frac{c_s}{c_p}\frac{\rho_{p\,0}}{\rho}\frac{L\delta}{U}\int_0^1\rho_p^*\ (T_p^* - T^*)dy^* \tag{16}$$

$$\int_0^1\rho_p^*\,(T_p^* - T^*)dy^* = -\frac{\tau_T U}{\delta L}\left[\frac{\partial}{\partial x^*}\left\{\delta\int_0^1\rho_p^*\,u_p^*\,T_p^*dy^*\right\} - \frac{L^2}{\delta}\frac{\epsilon}{\Pr Re}\left.\frac{\partial T_p^*}{\partial y^*}\right|_{y^*=0}\right] \tag{17}$$

Subject to the boundary conditions

$$y^* = 0\ :u^* = 0,\ v^* = 0,\ u_p^* = a_2(x^*),\ v_p^* = 0,\ \rho_p^* = a_3(x^*),\ T^* = 1,\ \ {T_p}^* = a_4(x^*) \tag{18}$$

$$y^* = 1\ :u^* = u_p^* = \rho_p^* = 1,\ T^* = 0,\ \ {T_p}^* = 0 \tag{19}$$

To make the equation consistent, we use the auxiliary condition that the flux of particulate mass across any control volume is zero.

i.e. $\rho_{p0}U\delta = \int_0^{\delta}\rho_p u_p\ dy$ (20)

which gives after non – dimensionalisation

$$\frac{d}{dx^*}\int_0^1 \rho_p^* \, u_p^* \; dy^* = 0 \qquad (21)$$

Using the profiles

$$u^* = 1 - (1 - y^*)^3$$

$$u_p^* = 1 - (1 - a_2)\,(1 - y^*)^3$$

$$\rho_p^* = 1 - (1 - a_3)\,(1 - y^*)^3 \qquad (22)$$

$$T^* = (1 - y^*)^3$$

$$T_p^* = a_4(1 - y^*)^3$$

So, the two-phase boundary layer non-dimensional equations after using the third degree profiles are given by,

$$\frac{dA}{dx^*} = \frac{56\mu}{\rho UL} - \frac{2}{3}\,\frac{FL}{U}\alpha A a_2(4a_3 + 3) + \frac{14}{3}\,\frac{Gr}{Re^2}\,A \qquad (23)$$

$$\frac{da_2}{dx^*} = \frac{\frac{dA}{dx^*}(18-6a_2+12a_3-12a_2^2+16a_2a_3-28a_2^2a_3) + 2A\,(12+16a_2-28a_2^2)\,\frac{da_3}{dx^*} - 20\frac{FL}{U}A\,a_2(4a_3+3) - 1680\,\frac{\epsilon}{Re}(1-a_2) + 140\,A\,\frac{1}{Fr}\left(1-\frac{\rho}{\rho_s}\right)(3+a_3)}{2A(6+24a_2-16a_3+56a_2a_3)} \qquad (24)$$

$$\frac{da_4}{dx^*} = \frac{-\frac{3}{56}\frac{dA}{dx^*} + \frac{3}{Pr\;Re} + \frac{\mathbf{9Ec}}{\mathbf{5Re}} - \frac{1}{1-\varphi}\,\frac{\alpha\,A a_4}{105\;Pr}\left(3\frac{da_2}{dx^*}+3\frac{da_3}{dx^*}+7a_2\frac{da_3}{dx^*}+7a_3\frac{da_2}{dx^*}\right) - \frac{1}{1-\varphi}\,\frac{\alpha\,a_4}{420\;Pr}(9+6a_2+6a_3+14a_2a_3)\frac{dA}{dx^*} - \frac{1}{1-\varphi}\,\frac{2\,\alpha\,\epsilon}{(Pr)^2\,Re}\,a_4}{\frac{1}{1-\varphi}\;\frac{\alpha A}{210\;Pr}\;(9+6a_2+6a_3+14a_2a_3)} \qquad (25)$$

$$\frac{da_3}{dx^*} = -\;\frac{4a_3+3}{4a_2+3}\;\frac{da_2}{dx^*} \qquad (26)$$

3. DISCUSSION OF THE RESULTS:

Equations (23) to (26) with boundary conditions (18) and (19) are integrated numerically by Runge- Kutte 4th order scheme. The solutions are obtained for different Prandtl number (Pr), volume fraction (φ), material density of SPM (ρ_s), diameter or size of the particle (D), diffusion parameter (ε), concentration parameter (α) for uniform plate temperature. The temperature, velocity and particle density profiles are presented in figures for different values of above parameters. It is seen from fig. (1) & (2) that the carrier fluid velocity satisfied the no slip condition but the particle velocity profiles do not satisfy no slip condition at the wall and go on increasing with x i.e. towards the downstream of the plate. In fig. (1) & (4) the profiles for carrier fluid temperature display a simple shape which is found in the thermal boundary layers of pure fluid flow, but the particle temperature on the plate becomes negative towards the downstream of the plate. Fig. (3) displays the profile for the particle densities, which shows that the density of the particle on the plate go on decreasing towards the downstream. Table -2 shows that the particle density and particle velocity on the plate assumes a finite value towards the downstream station of the plate. Physically it indicates that the consideration of finite volume fraction force, arising due to stress present in the particle phase and the heat due to conduction through the particle phase in the modeling of two-phase flow may not stabilize the boundary layer growth.

From fig.(5) & (6), we conclude that irrespective of presence of heavier or lighter material particles, the particles settles down on the plate as expected and the buoyancy force stabilizes the boundary layer growth. Fig. (7) & (8) shows the presence of coarser particles decrease the magnitude of velocity and increase the magnitude of temperature of the particle phase in comparison with the presence of finer particles inside the boundary layer.

The values of Prandtl number (Pr) are taken as 0.71, 1.0 and 7.0 which physically corresponds to air, electrolyte solution and water respectively. The magnitude of the particle temperature of water is very low as compare to air and electrolyte solution. Fig. (11) shows the particle temperature increases as the number of particles per unit volume of the mixture increases, where as the magnitude of the particle velocity increases (Fig. 10).

Inclusion of Buoyancy force increase the magnitude of the particle velocity and temperature, but the temperature assumes negative value (Fig. 12 & 13). Inclusion of Buoyancy force decrease the skin friction and also heat transfer from plate fluid as can be observed from table-1.

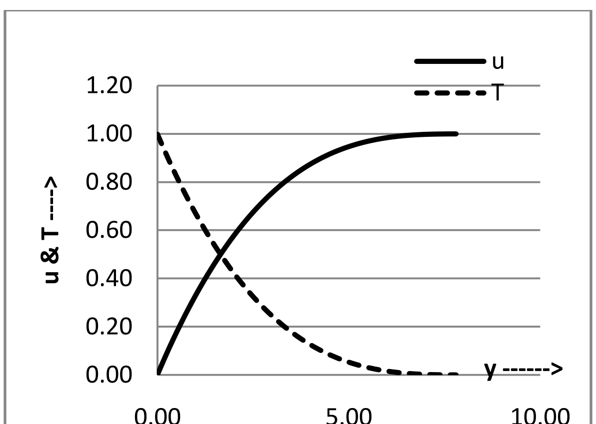

Fig. 1 : Variation of u & T with y

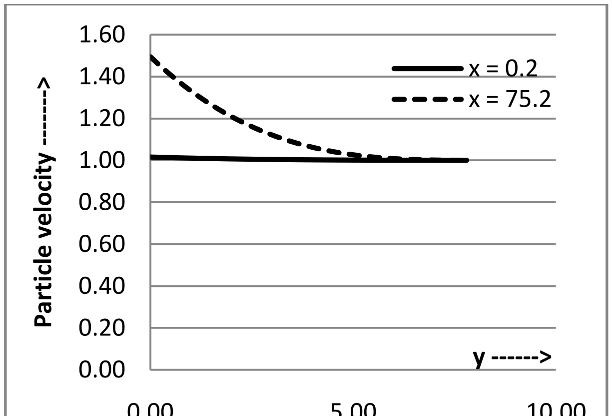

Fig.2: Variation of particle velocity with y

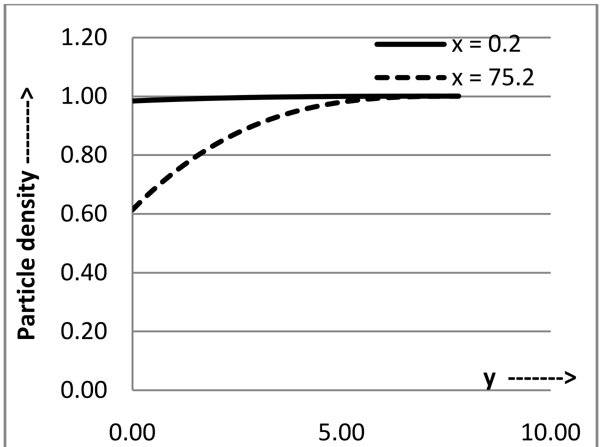

Fig.3: Variation of particle density with y

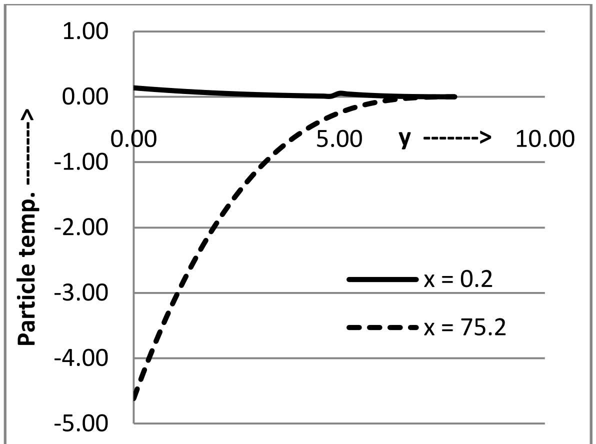

Fig.4: Variation of particle temperature with y

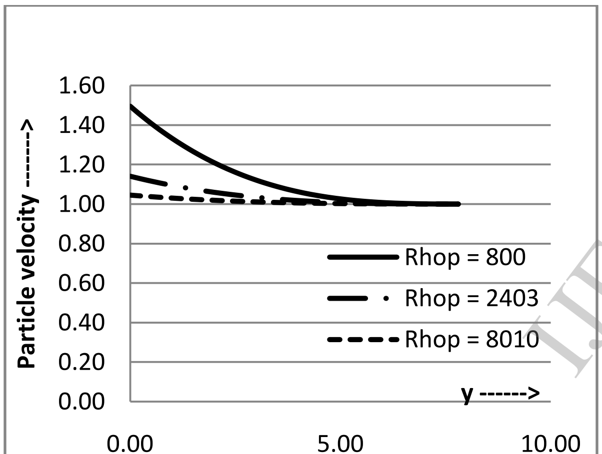

Fig.5: Variation of particle velocity with y

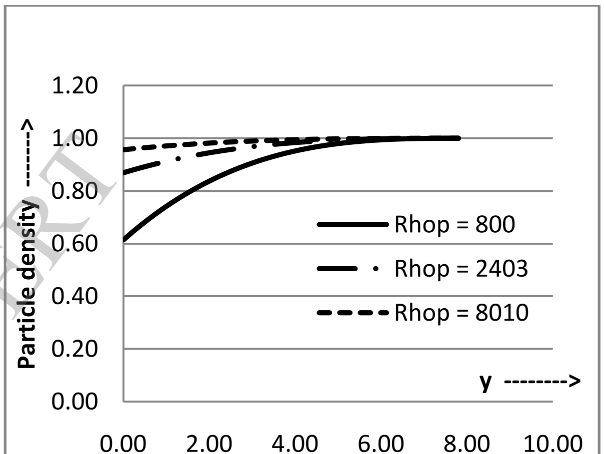

Fig.6: Variation of particle density with y

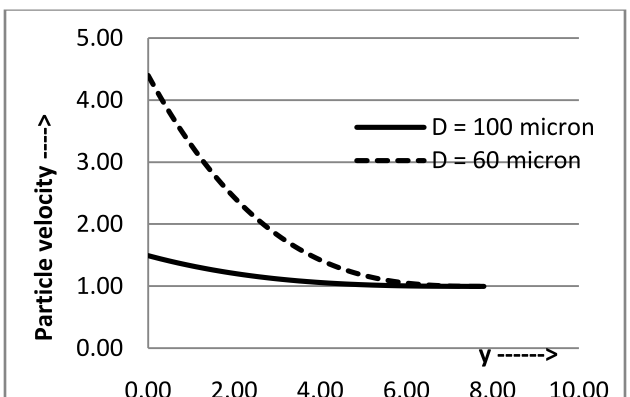

Fig.7 : Variation of particle velocity with y

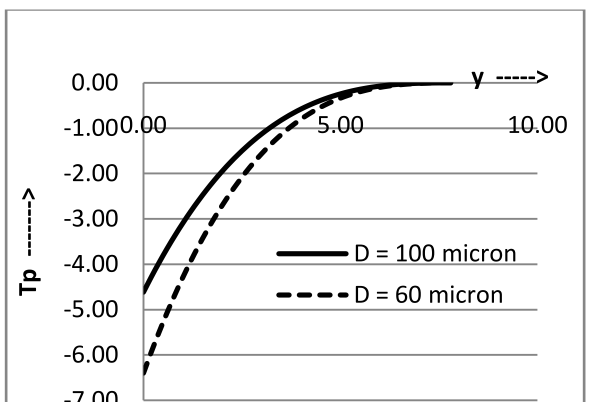

Fig.8 : Variation of particle temperature with y

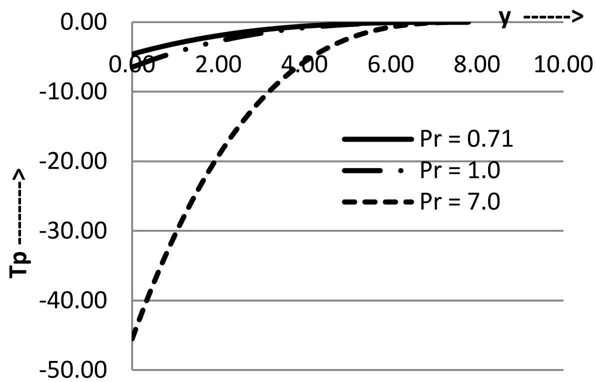

Fig. 9: Variation of particle temperature with y

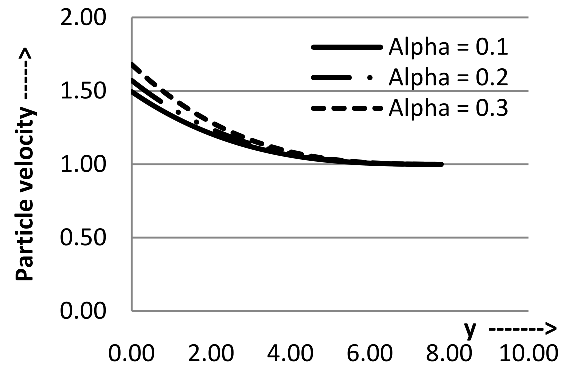

Fig. 10: Variation of particle velocity with y

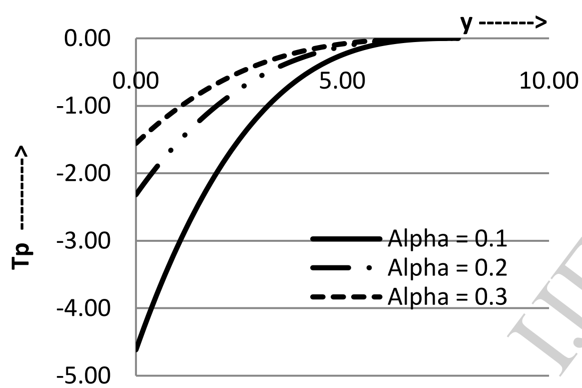

Fig. 11: Variation of particle temperature with y

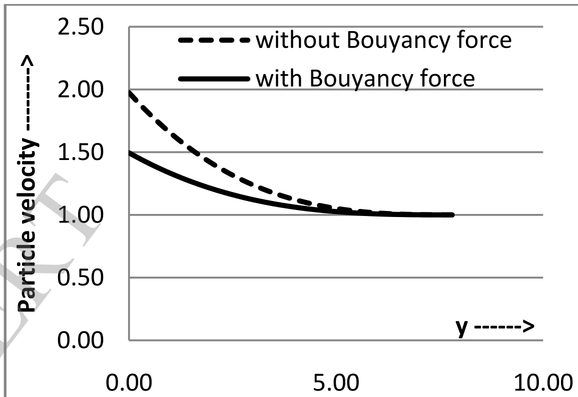

Fig. 12: Comparision of particle velocity with and without Bouyancy force

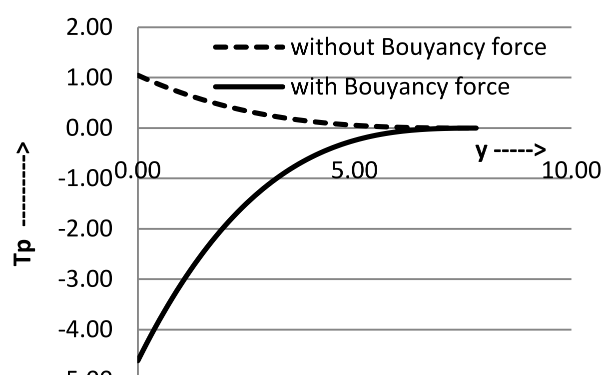

Fig. 13: Comparision of particle temperature with and without Bouyancy force

Table 1 : Comparison of Skin friction & Nusselt number with and without Buoyancy force

x	C_f Without Buoyancy force	C_f With Buoyancy force	Nu Without Buoyancy force	Nu With Buoyancy force
0.10	9.63E-01	9.80E-01	7.81E+01	7.53E+01
9.90	2.38E-04	9.68E-06	1.38E+03	5.63E+01
19.70	1.73E-04	1.59E-07	2.00E+03	1.84E+00
29.50	1.45E-04	2.60E-09	2.51E+03	4.51E-02
39.30	1.28E-04	4.26E-11	2.96E+03	9.84E-04
49.10	1.18E-04	6.98E-13	3.39E+03	2.02E-05
58.90	1.10E-04	1.14E-14	3.81E+03	3.96E-07
68.70	1.04E-04	1.88E-16	4.21E+03	7.57E-09
78.50	1.00E-04	3.07E-18	4.62E+03	1.42E-10
88.30	9.67E-05	5.04E-20	5.02E+03	2.61E-12
98.10	9.41E-05	8.26E-22	5.43E+03	4.76E-14

Table 2 : Comparison of plate values with and without Buoyancy force

	Plate values without Buoyancy force			Plate values with Buoyancy force		
x	u_p	ρ_p	T_p	u_p	ρ_p	T_p
0.10	1.02E+00	9.84E-01	1.05E+00	1.02E+00	9.84E-01	1.03E+00
9.90	2.84E+00	1.03E-01	4.38E+00	1.48E+00	6.25E-01	-4.49E+00
19.70	7.30E+00	-2.27E+00	1.19E+01	1.50E+00	6.14E-01	-4.62E+00
29.50	2.57E+00	-4.42E+00	2.28E+00	1.50E+00	6.14E-01	-4.63E+00
39.30	2.05E+00	-5.12E+00	1.19E+00	1.50E+00	6.14E-01	-4.63E+00
49.10	1.98E+00	-5.22E+00	1.05E+00	1.50E+00	6.14E-01	-4.63E+00
58.90	1.97E+00	-5.24E+00	1.03E+00	1.50E+00	6.14E-01	-4.63E+00
68.70	1.97E+00	-5.24E+00	1.03E+00	1.50E+00	6.14E-01	-4.63E+00
78.50	1.97E+00	-5.24E+00	1.03E+00	1.50E+00	6.14E-01	-4.63E+00
88.30	1.97E+00	-5.24E+00	1.03E+00	1.50E+00	6.14E-01	-4.63E+00
98.10	1.97E+00	-5.24E+00	1.03E+00	1.50E+00	6.14E-01	-4.63E+00

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