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Ultra L-Topologies in the Lattice of L-Topologies

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Abstract

We study the principal and nonprincipal ultra L-filters on a nonempty set X,where L is a completely distributive lattice with order reversing involution. Using this notion we study the topological properties of principal and nonprincipal ultra L- topologies. If X has n elements and L is a finite pseudo complemented lattice or a Boolean lattice, there are $n(n-1)mk$ principal ultra L-topoogies, where m is the number of dual atoms and k is the number of atoms. If X is infinite, there are $|X|$ principal ultra L-topologies and $|X|$ nonprincipal ultra L-topologies.

Keywords. L - filter, Principal and Nonprincipal Ultra L-Topologies, Simple extension.
AMS Subject Classification. 54A40

1 Introduction

The Purpose of this paper is to identify the Ultra L-Topologies in the lattice of L-Topologies. For a given topology τ on X , T. P. Johnson [5] studied the properties of the lattice $S_{\tau,L}$ of L-Topologies defined by families of scott continuous functions with reference to τ on X. In [5] Johnson has proved that $S_{\tau,L}$ is complete, atomic and not complemented. Also he has showed that$S_{\tau,L}$ is neither modular nor dually atomic in general. In [3] Frolich proved that if $|X| = n$ there are $n(n-1)$ Principal ultra topologies in the lattice of topologies. In [8] A. K. Steiner studied some topological properties of the ultraspaces. In this paper we showed that if $|X| = n$ and L is a finite pseudocompleted chain or a Boolean lattice, there are $n(n-1)mk$ Principle Ultra L- Topologies, where m and k are the number of dual atoms and atoms in L respectively. If X is infinite, there are $|X|$ Principal Ultra L- Topologies and $|X|$ Nonprincipal Ultra L-topologies. Also we studied some topological properties of the Ultra L Topologies and characterise the T_1, T_2 L-topologies.

2 Preliminaries

Let X be a non empty ordinary set and $L = L(\leq, \vee, \wedge, ')$ be a completely distributive lattice with the smallest element 0 and largest element $1((0 \neq 1)$ and with an order reversing involution $a \longrightarrow a'$. We identify the constant function with value α by $\underline{\alpha}$. The fundamental definition of L-fuzzy set theory and L-fuzzy topoogy are assumed to be familiar to the reader (in the sense of Chang [2] and Goguen [4]). Here we call L-fuzzy subsets as L-subsets and L-fuzzy topology as L-topology. For a given topology τ on X, the family $S_{\tau,L}$ of all L-topologies defined by families of Scott continuous functions from (X, τ) to L forms a lattice under the natural order of set inclusion. The least upper bound of a collection of L-topologies belonging to $S_{\tau,L}$ is the L-topology generated by their union and the greatest lower bound is their intersection. The smallest and largest elements in $S_{\tau,L}$ are denoted by $0_{s,\tau}$ and $1_{s,\tau}$ respectively.

In this paper, L-filter on X are defined according to the definition given by A. K. Katsaras [6] and P. Srivastava and R. L. Gupta[7], by taking L to be the membership lattice, instead of the closed unit interval$[0, 1]$.

Definition 2.1. A non empty subset $\mathscr{U}$ of L^X is said to be an L-filter if
$i.\underline{0} \notin \mathscr{U}$
$ii.f, g \in \mathscr{U}$ implies $f \wedge g \in \mathscr{U}$ and
$iii.f \in \mathscr{U}, g \in L^X$ and $g \geq f$ implies $g \in \mathscr{U}$.

An L-filter is said to be an ultra L-filter if it is not properly contained in any other L-filter.

Definition 2.2. Let $x \in X, \lambda \in L$ An L-point x_λ is defined by $x_\lambda(y) =$

$\begin{cases} \lambda & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$ where $0 < \lambda \leq 1$

Definition 2.3. In a filter $\mathscr{U}$, if there is an L subset with finite support, then $\mathscr{U}$ is called a principal L-filter.

Example 1: $\{f \in L^X | f \geq x_\lambda$, where x_λ is an L-point$\}$.

Definition 2.4. In a filter $\mathscr{U}$, if there is no L subset with finite support, then $\mathscr{U}$ is called a nonprincipal L-filter.

Example 2: $\{f \in L^X | f > 0$ for all but finite number of points$\}$. Let f be a nonzero L-subset with finite support. Then $\mathscr{U}(f) \subset L^X$ defined by $\mathscr{U}(f) = \{g \in L^X | g \geq f\}$ is an L-filter on X, called the Principal L-filter at f.Every L-filter is contained in an ultra L-filter. From the definition it follows that on a finite set X, there are only Principal ultra L-filters.

3 Ultra L-topologies

An L-topology F on X is an ultra L-topology if the only L-topology on X strictly finer than F is the discrete L-topology.

Definition 3.1. [9] Let (X, F) be an L-topological space and suppose that $g \in L^X$ and $g \notin F$. Then the collection $F(g) = \{g_1 \vee (g_2 \wedge g) | g_1, g_2 \in F\}$ is called the simple extension of F determined by g.

Theorem 3.1. *[9] Let (X, F) be an L-topological space and suppose that $F(g)$ be the simple extension of F determined by g. Then $F(g)$ is an L-topology on X.*

Theorem 3.2. *[9] Let F and G be two L-topologies on a set X such that G is a cover of F. Then G is a simple extension of F.*

Theorem 3.3. *[3] The ultraspaces on a set E are exactly the topologies of the form $\mathfrak{S}(x, \mathscr{U}) = P(E - \{x\}) \cup \mathscr{U}$ where $x \in E$ and $\mathscr{U}$ is an ultrafilter on E not containing $\{x\}$.*

Analogously we can define ultra L-topologies in the lattice of L-topologies according to the nature of Lattices. If it contains Principal ultra L-filter, then it is called Principal ultra L-topology and if it contains nonprincipal ultrafilter, it is called nonprincipal ultra L-topology.

Theorem 3.4. *[1] A principal L-filter at* x_λ *on* X *is an ultra L-filter iff* λ *is an atom in* L*.*

Theorem 3.5. *Let* a *be a fixed point in* X *and* $\mathscr{U}$ *be an ultra L-filter not containing* $a_\alpha, 0 \neq \alpha \in L$*. Define* $\mathscr{F}_a = \{f \in L^X | f(a) = 0\}$*. Then* $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}) = \mathscr{F}_a \cup \mathscr{U}$ *is an L-topology.*

Proof. Can be easily proved. □

Theorem 3.6. *If* X *is a finite set having* n *elements and* L *is a finite pseudo complemented chain or a Boolean lattice, there are* $n(n-1)mk$ *Principal ultra L-topologies, where* m *and* k *are the number of dual atoms and atoms in* L *respectively. If* $k = m$ *there are* $n(n-1)m^2$ *ultra L-topologies.*

Illustration :
1. Let $X = \{a, b, c\}, L = \{0, \alpha, \beta, 1\}$, a pseudo complemented chain. Here α is the atom and β is the dual atom.

Figure 1:

Let $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_\alpha)) = \{f | f(a) = 0\} \cup \{f | f \geq b_\alpha\}$, $\mathfrak{S}$ does not contain the L-points a_α, a_β, a_1 Then $\mathfrak{S}(a, \mathscr{U}(b_\alpha), a_\beta) = \mathfrak{S}(a_\beta) =$ simple extension of $\mathfrak{S}$ by $a\beta = \{f \vee (g \wedge a_\beta) | f, g \in \mathfrak{S}, a_\beta \notin \mathfrak{S}\}$ is an ultra L-topology, since$\mathfrak{S}(a_1)$ is the discrete L-topology. similarly
if $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(c_\alpha))$, then$\mathfrak{S}(a_\beta)$ is an ultra L-topology.
if $\mathfrak{S} = \mathfrak{S}(b, \mathscr{U}(a_\alpha))$, then$\mathfrak{S}(b_\beta)$ "
if $\mathfrak{S} = \mathfrak{S}(b, \mathscr{U}(c_\alpha))$, then$\mathfrak{S}(b_\beta)$ "
if $\mathfrak{S} = \mathfrak{S}(c, \mathscr{U}(a_\alpha))$, then$\mathfrak{S}(c_\beta)$ "
if $\mathfrak{S} = \mathfrak{S}(c, \mathscr{U}(b_\alpha))$, then$\mathfrak{S}(c_\beta)$ "

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Number of ultra L-topologies= $6 = 3 * 2 * 1 * 1 = n(n-1)m^2$, where $n = 3, k = m = 1$
2. Let $X = \{a, b, c\}, L = [0, 1]$, then there is no ultra L-topology, since $[0, 1]$ has no dual atom and atom.
3.Let $X = \{a, b, c\}, L =$Diamond lattice $\{0, \alpha, \beta, 1\}$

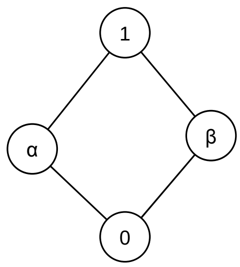

Figure 2:

Here α, β are the atoms as well as the dual atoms. Let $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_\alpha)) = \{f | f(a) = 0\} \cup \{f | f \geq b_\alpha\}$, does not contain the L-points a_α, a_β, a_1. Then the simple extension $\mathfrak{S}(a_\alpha)$ contains the fuzzy point a_α also. Let $\mathfrak{S}_1 = \mathfrak{S}(a_\alpha)$. Then the simple extension $\mathfrak{S}_1(a_\beta)$ contains all L-points and hence it is discrete. So $\mathfrak{S}(a_\alpha) = \mathfrak{S}(a, \mathscr{U}(b_\alpha), a_\alpha)$ is an ultra L-toology. Similarly the simple extension $\mathfrak{S}(a_\beta) = \mathfrak{S}(a, \mathscr{U}(b_\alpha), a_\beta)$ is an ultra L-topology. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_\beta) = \{f | f(a) = 0\} \cup \{f | f \geq b_\beta\}$, Then the simple extensions $\mathfrak{S}(a_\alpha)$ and $\mathfrak{S}(a_\beta)$ are ultra L-topologies.That is corresponding to the elements a and b there are 4 ultra L-topologies. Similarly corresponding to the elements a and c , there are 4 ultra L-topologies. So there are 8 ultra L-topologies corresponding to a. Similaly there are 8 ultra L-topologies corresponding to b and 8 ultra L- topologies corresponding to c . Hence total ultra L-topologies $= 8 + 8 + 8 = 24 = 3 * 2 * 2 * 2 = n(n-1)m^2$, where $n = 3, k = m = 2$

4.Let $X = \{a, b, c\}, L = P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}.\alpha_1 = \{a\}, \alpha_2 = \{b\}, \alpha_3 = \{c\}, \beta_1 = \{a, b\}, \beta_2 = \{a, c\}, \beta_3 = \{b, c\}$. Atoms are $\alpha_1, \alpha_2, \alpha_3$and dual atoms are $\beta_1, \beta_2, \beta_3$

Let $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 1})) = \{f | f(a) = 0\} \cup \{f | f \geq b_{\alpha 1}\}$, does not contain the L-points $a_{\alpha 1}, a_{\alpha 2}, a_{\alpha 3}, a_{\beta 1}, a_{\beta 2}, a_{\beta 3}, a_1$. Let $\mathfrak{S}_1 =$ Simple extension of $\mathfrak{S}$ by $a_{\beta 1}$ denoted by $\mathfrak{S}(a_{\beta 1})$.Then $\mathfrak{S}_1$ contains more L-subsets than $\mathfrak{S}$,but

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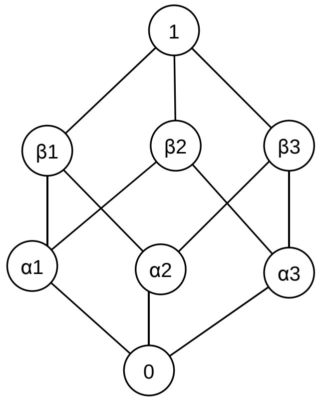

Figure 3:

not discrete L-topology. Let $\mathfrak{S}_2 = \mathfrak{S}_1(a_{\beta 2})$, Simple extension of $\mathfrak{S}_1$ by $a_{\beta 2}$. Then $\mathfrak{S}_2$ contain more L subsets than $\mathfrak{S}_1$ but not discrete L-topology. Let $\mathfrak{S}_3 = \mathfrak{S}_2(a_{\beta 3})$, Simple extension of $\mathfrak{S}_2$ by $a_{\beta 3}$, which is a discrete L-topology. Hence $\mathfrak{S}_2 = \mathfrak{S}_1(a_{\beta 2})$ is an ultra L-topology, which is the L-topology generated by $\mathfrak{S}(a_{\beta 1})$ and $\mathfrak{S}(a_{\beta 2})$. Also L-topology generated by $\mathfrak{S}(a_{\beta 1})$ and $\mathfrak{S}(a_{\beta 3})$ and L-topology generated by $\mathfrak{S}(a_{\beta 2})$ and $\mathfrak{S}(a_{\beta 3})$ are ultra L-topologies. That is if $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 1}))$, there are 3 ultra L-topologies. Similarly if $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 2}))$, there are 3 ultra L-topologies and if $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 3}))$, there are 3 ultra L-topologies. So corresponding to the elements a, b there are 9 ultra L-topologies. Similarly corresponding to the elements a, c there are 9 ultra L-topologies. Hence there are 18 ultra$L$-topologies corresponding to the element a. Similarly corresponding to each element b and c there are 18 ultra L-topologies. So total number of ultra L-topologies $= 54 = 3 * 2 * 3 * 3 = n(n-1)m^2, n = 3, k = m = 3$.

5.Let $X = \{a, b, c, d\}, L = P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{c, d, a\}, X\}$. Let $\{a\} = \alpha_1, \{b\} = \alpha_2, \{c\} = \alpha_3, \{d\} = \alpha_4, \{a, b\} = \gamma_1, \{a, c\} = \gamma_2, \{a, d\} = \gamma_3, \{b, c\} = \gamma_4, \{b, d\} = \gamma_5, \{c, d\} = \gamma_6, \{a, b, c\} = \beta_1, \{a, b, d\} = \beta_2, \{b, c, d\} = \beta_3, \{c, d, a\} = \beta_4$ If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 1}))$, there are 4 ultra L -topologies.
If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 2}))$ ”
If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 3}))$ ”

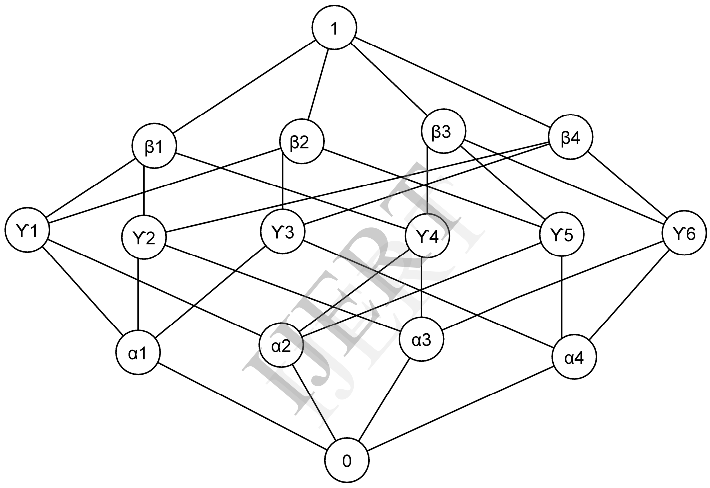

Figure 4:

If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 4}))$ "
So corresponding to the elements a, b,there are 16 ultra L-topologies. Similarly corresponding to the elements a, c,there are 16 ultra L-topologies and corresponding to the elements a, d, there are 16 ultra$L$-topologies. Hence there are 48 ultra L-topologies corresponding to the element a. Similarly corresponding to each elements b, c and d, there are 48 ultra L-topologies. So total number of ultra L-topologies $= 48 * 4 = 192 = 4 * 3 * 4 * 4 = n(n-1)m^2, n = 4, k = m = 4$. In general if $|X| = n$ and L is a finite pseudo complemented chain or a Boolean lattice, there are $n(n-1)mk$ ultra L-topologies where m and k are the number of dual atoms and number of atoms respectively. If $k = m$, it is equal to $n(n-1)m^2$.

Remark 3.1. If L is neither a finite pseudo complemented Lattice nor a Boolean Lattice, we cannot identify the ultra L-topologies in this way.

Example:
Let $X = \{a, b, c\}, L = D_{12} = \{1, 2, 3, 4, 6, 12\}$

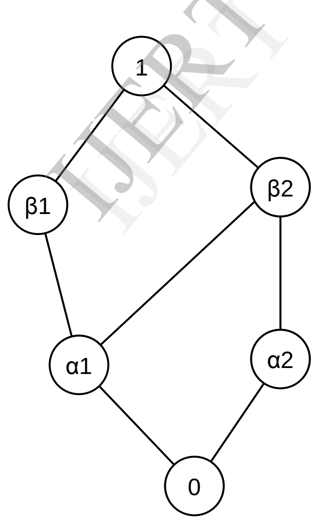

Figure 5:

Here the atoms are $\alpha_1 = 2, \alpha_2 = 3$ and dual atoms are $\beta_1 = 4, \beta_2 = 6$. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_{\alpha 1})) = \{f | f(a) = 0\} \cup \{f | f \geq b_{\alpha 1}\}$,L-topology generated by $\mathfrak{S}(a_{\beta 1})$ and $\mathfrak{S}(a_{\beta 2})$ does not contain the L-point$a_{\alpha 2}$. It is not a discrete L-topology.So we cannot say that $\mathfrak{S}(a_{\beta 1})$ is an ultra L-topology.

Theorem 3.7. *If X is infinite and L is a finite pseudo complemented chain or a Boolean lattice, there are $|X|$ principal ultra L-topologies and $|X|$ non principal ultra L-topologies.*

Illustration:
If X is countably infinite, cardinality of $X, |X| = \aleph_0$. If X is uncountable, $|X| > \aleph_0$
case 1: X is infinite and L is finite

Let $X = \{a, b, ...\}, L = \{0, \alpha, \beta, 1\}$ a pseudo complemented chain.Let$\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_\alpha)) = \{f|f(a) = 0\} \cup \{f|f \geq b_\alpha\}$. $\mathfrak{S}$ does not contain the fuzzy points a_α, a_β, a_1. Here $\mathfrak{S}(a_\beta) = \mathfrak{S}(a, \mathscr{U}(b_\alpha), a_\beta)$ is a principal ultra L-topology since $\mathfrak{S}(a_1)$ is the discrete L-topology, where $\mathfrak{S}(a_\beta)$ is the simple extension of $\mathfrak{S}$ by a_β. Similarly we can identify other L-topologies. Hence corresponding to the element a, there are $|X| - 1 = |X|$ principal ultra L-topologies. Similarly corresponding to each element $b, c, d, ...$ there are $|X|$ principal ultra Ltopologies So total number of principal ultra L-topologies $= |X||X| = |X|$. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}) = \{f|f(a) = 0\} \cup \mathscr{U}$, where $\mathscr{U}$ is a non-principal ultra filter not containing $a_\lambda, 0 \neq \lambda \in L$. Then the simple extension of $\mathfrak{S}$ by $a_\beta = \mathfrak{S}(a_\beta) = \mathfrak{S}(a, \mathscr{U}, a_\beta)$ is a nonprincipal ultra L-topology since $\mathfrak{S}(a_1)$ is discrete L-topology. So there are $|X|$ non principal ultra L topologies.
case 2: X and L are infinite

Let$X = \{a, b, c, ...\}, L = P(X)$. There are $|X|$ atoms and $|X|$ dual atoms. Number of Principal ultra L-topologies corresponding to one element$= |X||X|(|X|-1) = |X|$ Hence total number of principal ultra L-topologies$= |X||X| = |X|$. Let $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}) = \{f|f(a) = 0\} \cup \mathscr{U}$, where $\mathscr{U}$ is a nonprincipal ultra filter not containing $a_\lambda, 0 \neq \lambda \in L$. There are $|X|$ nonprincipal ultra L-filters . Corresponding to a there are $|X||X| = |X|$ nonprincipal ultra L-topologies. So total number of nonprincipal ultra L-topologies $= |X||X| = |X|$.

4 Topological Properties

PRINCIPAL ULTRA L-TOPOLOGIES

Let X be a nonempty set and L is a finite pseudo complemented chain. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_\lambda) = \{f|f(a) = 0\} \cup \{f|f \geq y_\lambda\}$, then a principal ultra L-topology $= \mathfrak{S}(a, \mathscr{U}(b_\lambda), a_\beta) = \mathfrak{S}(a_\beta)$, which is the simple extension of $\mathfrak{S}$ by $a_\beta = \{f \vee (g \wedge a_\beta), f, g, \in \mathfrak{S}, a_\beta \notin \mathfrak{S}\}$, where $a, b \in X, \lambda$ and β are the atom and dual atom in L respectively.

Let X be a nonempty set and L is a Boolean lattice. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}(b_\lambda)) = \{f|f(a) = 0\} \cup \{f|f \geq b_\lambda\}$ where $a, b \in X, \lambda$ is an atom, then a Principal ultra L-topology $= \mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j}) = L$-topology generated by any$(m-1)$, $\mathfrak{S}(a_{\beta i})$ among m, $\mathfrak{S}(a_{\beta i}), i = 1, 2, ..., m, j = 1, 2, ..., m, i \neq j$ if there are m dual atoms $\beta_1, \beta_2, ...\beta_m$, where $\mathfrak{S}(a_{\beta i}) = \mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta i})$

Theorem 4.1. *Let X be a nonempty set and L is a finite pseudo complemented chain or a Boolean Lattice. Then every principal ultra L-topology is $L - T_0$ but not $L - T_1$.*

Example:
Let X is a non empty set

Let L is a finite pseudo complemented chain and $a, b \in X, \lambda, \beta$ are atom and dual atom in L respectively. Take two distinct L points a_1, b_λ. b_λ is an open L subset contain b_λ but not a_1. Since$\mathscr{U}(b_\lambda) = \{f | f \geq b_\lambda\}$, any open set contain a_1 must contain b_λ. So $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_\beta)$ is $L - T_0$ but not $L - T_1$.

Let L is a Boolean lattice and $a, b \in X, \lambda$ is an atom and $\beta_1, \beta_2, ...$ are dual atoms in L. Take two distinct L-ponints $a_1, b_\lambda . b_\lambda$ is an open L subset contain b_λ but not a_1 . Since $\mathscr{U}(b_\lambda) = \{f | f \geq b_\lambda\}$, any open set contain a_1 must contain b_λ. So $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j})$ is $L - T_0$ but not $L - T_1$.

Definition 4.1. An L-topological space $(X, F), F \subseteq L^X$ is called door L-space if every L-subset g of X is either L-open or L-closed in F.

Example : Let $X = \{a, b\}$ and $L = \{o, .5, 1\}$.Define $f_1(a) = 0, f_1(b) = 0, f_2(a) = 0, f_2(b) = .5, f_3(a) = 0, f_3(b) = 1, f_4(a) = .5, f_4(b) = 0, f_5(a) = .5, f_5(b) = .5, f_6(a) = .5, f_6(b) = 1, f_7(a) = 1, f_7(b) = 0, f_8(a) = 1, f_8(b) = .5, f_9(a) = 1, f_9(b) = 1$. Let $F = \{f_1, f_9, f_2, f_3, f_4, f_5, f_6\}$. Then$f_7$ and f_8 are closed L-subsets. So (X, F) is a door L-space. Let $X = \{a, b, c\}, L = [0, 1]$ and the the L-topology $F = \{\underline{0}, \mu_{\{a\}}, \mu_{\{b,c\}}, \underline{1}\}$.Then (X, F) is not a door L-space since$\mu_{\{b\}}$ is neither an L open set nor an L-closed set. In a principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j})$ every L subset of X is either open or closed if L is a finite pseudocomplemented chain or a Boolean lattice . So the principal ultra L-topological space is a door L space.

Definition 4.2. An L-topological space (X, F) is said to be regular at an L-point a_λ if for every closed L subset h of X not containing a_λ, there exists disjoint open sets f, g such that $a_\lambda \in f$and $h \in g.(X, F)$ is said to be regular L-topology if it is regular at each of its L-points.

Theorem 4.2. *Let X be a non empty set and $L = P(X)$. Then the principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j})$ is not regular if $|X| \geq 3$.*

Example: let $X = \{a, b, c\}, L = P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}.\alpha_1 = \{a\}, \alpha_2 = \{b\}, \alpha_3 = \{c\}, \beta_1 = \{a, b\}, \beta_2 = \{a, c\}, \beta_3 = \{b, c\}$. Atoms are

$\alpha_1, \alpha_2, \alpha_3$ and dual atoms are $\beta_1, \beta_2, \beta_3$. Take $\lambda = \alpha_1$ in the principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta 3})$. Consider $a_{\beta 1}$. $a_{\alpha 3}$ is a closed L subset not containing $a_{\beta 1}$.Consider the open sets f, g such that $f(a) = \beta_1, f(b) = \alpha_1, f(c) = \alpha_1, g(a) = \beta_2, g(b) = 0, g(c) = 0.f$ is an open set containing $a_{\beta 1}$ and$g$ is an open set containing $a_{\alpha 3}$ but $f \wedge g \neq \underline{0}$. That is f and g are not disjoint.

Definition 4.3. An L topological space (X, F) is said to be Normal if for every two disjoint closed L subsets h and k , there exists two disjoint open L subsets f, g such that $h \in f$ and $k \in g$.

Theorem 4.3. *Let X be a nonempty set and $L = P(X)$. Then the principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j})$ is not a normal L-topology if* $|X| \geq 3$.

Example: Let $X = \{a, b, c\}, L = P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}.\alpha_1 = \{a\}, \alpha_2 = \{b\}, \alpha_3 = \{c\}, \beta_1 = \{a, b\}, \beta_2 = \{a, c\}, \beta_3 = \{b, c\}$. Atoms are $\alpha_1, \alpha_2, \alpha_3$ and dual atoms are $\beta_1, \beta_2, \beta_3$.Take $\lambda = \alpha_1$ in the principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta 3}).a_{\alpha 2}$ and $a_{\alpha 3}$ are disjoint closed L subsets. There are no disjoint open L subsets containing $a_{\alpha 2}$ and $a_{\alpha 3}$.

NON PRINCIPAL ULTRA L- TOPOLOGY

Let X be an infinite set and L is a finite pseudocomplemented chain. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U}) = \{f | f(a) = 0\} \cup \mathscr{U}$ where $\mathscr{U}$ is a non principal ultra L-filter not containing $a_\lambda, 0 \neq \lambda \in L$ .Then the non principal ultra L-topology= $\mathfrak{S}(a, \mathscr{U}, a_\beta) = \mathfrak{S}(a_\beta)$,is the simple extension of $\mathfrak{S}$ by $a_\beta = \{f \vee (g \wedge a_\beta), f, g, \in \mathfrak{S}, a_\beta \notin \mathfrak{S}\}$,where $a, b \in X, \beta$ is the dual atom in L.

Let X be an infinite set and L is a Boolean lattice. If $\mathfrak{S} = \mathfrak{S}(a, \mathscr{U})$, Then a non principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}, a_{\beta j}) = L$-topology generated by any$(m-1)$, $\mathfrak{S}(a_{\beta i})$ among m, $\mathfrak{S}(a_{\beta i}), i = 1, 2, ..., m, j = 1, 2, ..., m, i \neq j$ if there are m dual atoms $\beta_1, \beta_2, ...\beta_m$ where $\mathfrak{S}(a_{\beta i}) = \mathfrak{S}(a, \mathscr{U}, a_{\beta i})$. Here m can be assumed infinite values.

Theorem 4.4. *Every non principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}, a_{\beta j})$ is* $L - T_1$.

Proof. Let a_α, b_β be any two distinct L-points. Since $\mathscr{U}$ is a nonprincipal ultra L-filter,there exists L open sets containing each L-points but not the other. □

Theorem 4.5. *Every non principal ultra L topology $\mathfrak{S}(a, \mathscr{U}, a_{\beta j})$ is* $L - T_2$.

Proof. Let a_α, b_β be any two distinct L-points. Since U is a nonprincipal ultra L-filter, we can find disjoint open sets f and g such that $a_\alpha \in f, b_\beta \notin f$ and $b_\beta \in g, a_\alpha \notin g$ □

Theorem 4.6. *Every non principal ultra L-topology is a door L-space.*

Proof. In $\mathfrak{S}(a, \mathscr{U}, a_\beta)$ every L subset of X is either L closed or L open, if L is a finite pseudo complemented chain . If L is a Boolean Lattice,in $\mathfrak{S}(a, \mathscr{U}, a_{\beta j})$ every L subset is either L cosed or L open □

Theorem 4.7. *If X is an infinite set and L is a finite pseudo complemented chain or a diamond lattice, then the non principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}, a_\beta)$ is a regular L-topology.*

Proof. It is trival. □

Theorem 4.8. *Let X is an infinite set and $L = P(X)$.Then the non principal ultra L-topology $\mathfrak{S}(a, \mathscr{U}, a_{\beta j})$ is not a regular L-topology.*

Proof. Let $X = \{a, b, c....\}, L = P(X)$. Let $\alpha_1, \alpha_2, ...$ are atoms and $\beta_1, \beta_2, ...$ are dual atoms in L. Consider $a_{\beta 1}$. Then there exists a closed L subset $a_{\alpha i}$ for some i not containing $a_{\beta 1}$. But we cannot find disjoint open L subsets f and g such that f contains $a_{\beta 1}$ and g contains $a_{\alpha i}$. □

Theorem 4.9. *If X is infinite set and L is a finite pseudo complemented chain or a diamond lattice, the non principal ultra L topology $\mathfrak{S}(a, \mathscr{U}, a_\beta)$ is a normal L topology.*

Proof. It is trivial □

Theorem 4.10. *If X is an infinite set and $L = P(X)$, then the non principal ultra L topology $\mathfrak{S}(a, \mathscr{U}, a_{\beta j})$ is not a normal L-topology.*

Proof. Let $X = \{a, b, c, ...\}, L = P(X)$. Let $\alpha_1, \alpha_2, ...$ are atoms and $\beta_1, \beta_2, ...$ are dual atoms in L. Then there exists two closed L subsets $a_{\alpha i}$ and $a_{\alpha j}$ for some i and j . But there doesnot exists disjoint open L subsets f and g such that f contains $a_{\alpha i}$ and g contains $a_{\alpha j}$. □

Theorem 4.11. *Let X is an infinite set and L is a finite pseudo complemented chain or a Boolean lattice. An ultra L-topology F is a $T_1 - L$ topology if and only if it is a non principal ultra L-topology.*

Proof. Suppose that the ultra L-topology F is a $T_1 - L$ topology. We have to show that F is a non principal ultra L-topology. F is a principal ultra L-topology implies F is not a $T_1 - L$ topology. So we can say that F is a $T_1 - L$ topology implies F is a non principal ultra L-topology.

Next assume that F is a non principal ultra L-topology. Then by theorem 4.4 F is a $T_1 - L$ topology. □

Theorem 4.12. *An L-topology F on X is a $T_1 - L$ topology if and only if it is the infimum of non principal ultra L-toologies.*

Proof. Necessary part

Any L-topology finer than a $T_1 - L$ topology must also be a $T_1 - L$ topology. So a $T_1 - L$ topology can be the infimum of only non principal ultra L topologies.

Sufficient part

Each non principal ultra L-topology on X contains non principal ultra L- filter. So there exists distinct L points a_λ, b_γ where $a, b \in X; \lambda, \gamma \in L$and L opensets f, g such that $a_\lambda \in f, b_\gamma \notin f$ and $a_\lambda \notin, b_\gamma \in g$. This is also true in the infimum of any family of non principal ultra L-topologies since every L points are closed in non principal ultra L-filters. So infimum of any family of non principal ultra L-topologies is a $T_1 - L$ topology. □

Theorem 4.13. *Let X is an infinite set and L is a finite pseudo complemented chain or a Boolean lattice . Then an ultra L-topology is a $T_2 - L$ topology if and only if it is a non principal ultra L-topology.*

Proof. Suppose that an ultra L-topology is a $T_2 - L$ topology. This implies that the ultra L-topology is a $T_1 - L$ topology. Hence it is a non principal ultra L-topology.

Conversely suppose that the ultra L-topology is a non principal ultra L-topology. Since a non principal ultra L-topology contains a non principal ultra L-filter, for any two distinct L points in the non principal ultra L-topology there exists disjoint L open sets contains each L point but not the other. So it is a $T_2 - L$topology. □

5 Mixed L- topologies

In[8] Steiner studied the mixed topologies. Analogously we ca say that a mixed L-topology on X is not a $T_1 - L$ topology and does not have a principal representation. Thus a mixed L-topology is the intersection of a $T_1 - L$ topology and a principal L-topology.

The representation of a mixed L-topology as the infimum of a $T_1 - L$ topology and a principal L topology need not be unique.

Example:

Let $\mathscr{C} = \{\mu_A | X - A \text{ is finite}\} \cup \underline{0}$, is a $T_1 - L$ topology and δ and δ' be the principal L-topologies given by $\delta = \wedge_{a \in X-\{b,c\}} \mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j})$, $\delta' = \wedge_{a \in X-\{b\}} \mathfrak{S}(a, \mathscr{U}(b_\lambda), a_{\beta j})$, λ is an atom and β_j's are dual atoms in L. $\mathscr{C} \wedge \delta = \{\mu_A | b \in A, X - A \text{ is finite or } f = \underline{0}\} = \mathscr{C} \wedge \delta'$ is a mixed L-topology. Here $c_\lambda \in \delta$ and$c_\lambda \notin \delta'$. That is the representation of a mixed topology as the infimum of $T_1 - L$ topology and principal L-topology need not be unique.

6 Conclusion

In this paper we identified the principal and non principal ultra L-topologies and studied some topological properties. Also we introduced mixed L-topologies.

7 Future scope

Study some properties of mixed L-topologies.

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