# Underwater Bearings only Passive Target Tracking Using Pseudo Linear Estimator

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**ABSTRACT:** The passive target tracking using bearings-only measurements is studied for several underwater applications. For submarine (own ship) to submarine (target) application, Pseudo Linear Estimator and its variants are developed for various situations. The algorithm is extended to Electronic Surveillance Measures in Electronic Warfare / Intercept sonar target tracking application, where the measurements are highly aperiodic. In underwater, prior information of the target motion parameters will not be available. Therefore, Pseudo Linear Estimator is developed in such a way to work without initialization of target state vector. The pseudo measurement tracking filter manipulates the original nonlinear measurement algebraically to obtain the linear like structured measurement. The Pseudo Linear Estimator is projected in such a way that it does not require any initial estimate at all and at the same time offers all the features of extended Kalman filter based pseudo linear filter, namely sequential processing, flexibility to adopt the variance of each measurement.

## **1. INTRODUCTION**

Passive localization and tracking techniques are of interest in a variety of sonar applications. In the ocean environment, the two-dimensional bearing-only target motion analysis (TMA) is perhaps most familiar. In our scenario, a moving observer (own ship or platform at an approximately known altitude) measures noisy bearings to a target at altitude zero, and subsequently processes these measurements to obtain estimates of the position and velocity of the target. Both the own ship and the target are moving parallel in constant velocity and in the same horizontal plane, so it is a 2-D tracking problem. The underwater vehicle monitors noisy sonar bearings from a radiating target in passive listening mode and processes these measurements to find out the target motion parameters- range, course, bearing and speed of the target. As range measurement is not available and the bearing measurement is not linearly related to the target states, the whole process becomes non linear. But many times it is difficult to carry out maneuver by own ship due to tactical reasons.

Many techniques have been developed for bearing-only target tracking problem. One method of solution is the Extended Kalman filter. It is based on the linearization of the measurement equation. Another filter for the bearing-only tracking problem is the Pseudo measurement tracking filter, which is often said to be an alternative method of the EKF.

## 2.PSEUDO LINEAR ESTIMATOR

The pseudo linear estimator (PLE) using an extended Kalman filter was developed by Lindgren and Gong [1], Aidala [2] and Aidala and Nardone[3] for passive target tracking using bearings only measurements. The pseudo measurement approach has computational advantages compared to other filters with similar performance, but it has the undesirable property of generating biased estimates. It offers a biased estimate at long ranges it has an advantage that it never diverges. PLE outputs can be used as initial estimates for maximum likelihood estimator for bearings only passive target tracking.

Nardone, Lindgren and Gong presented relevant equations of pseudo linear estimator in batch processing. The solution of the gradient equation was obtained by Guassnewton iteration. PLE in batch processing is converted into sequential processing to suit real time applications like passive target tracking. Recursive SUMS are maintained throughout the algorithm. This approach avoids the computational complexity by computing only the incremental values for every new measurement.

## **Mathematical Modeling:**

Consider the source observer. The target located at the coordinates  $(x_t, y_t)$  moves with constant velocities  $(\dot{x}_t, \dot{y}_t)$  and is defined to have the state vector  $x_s$ . Similarly the observer state is defined as  $x_{ob}$  where the velocity need not be constant.



In general target state equation is

 $\overline{X_{s}}(k+1) = \boldsymbol{\Phi}(k+1, k) X_{s}(k)$ <sup>(1)</sup>

$$\overline{\mathbf{X}_{ob}}(\mathbf{k}+1) = \boldsymbol{\Phi}(\mathbf{k}+1, \mathbf{k}) \ \mathbf{X}_{ob}(\mathbf{k})$$
<sup>(2)</sup>

 $\overline{X_s}$  is a State vector.

The position components are given by

$$X_{S} = \begin{bmatrix} x_{t} \\ \dot{y_{t}} \\ x_{t} \\ y_{t} \end{bmatrix}$$
(3)

 $\Phi$  (k+1, k) is transient matrix.

It is given as

$$\Phi (\mathbf{k+1}, \mathbf{k}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t_s & 0 & 1 & 0 \\ 0 & t_s & 0 & 1 \end{bmatrix}$$
(4)

Bearing with respect to true north is given by

$$B_m(k) = B(k) + \Gamma(k)$$
<sup>(5)</sup>

$$H^T(k)$$

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$$\begin{bmatrix} 0 & 0 & \cos^2 B_m(k) & -\sin^2 B_m(k) \end{bmatrix}$$
 (6)

A(K,0) =

$$\begin{bmatrix} t \cos \mathbb{B}_m(1) & -t \sin B_m(1) & \cos \mathbb{B}_m(1) & -\sin \mathbb{B}_m(1) \\ 2t \cos B_m(2) & -2t \sin B_m(2) & \cos B_m(2) & -\sin \mathbb{B}_m(2) \\ kt \cos \mathbb{B}_m(k) & -kt \sin B_m(k) & \cos B_m(k) & -\sin \mathbb{B}_m(k) \end{bmatrix}$$

(7)

Basing on Least squares estimation

$$\widetilde{X_{S}}(^{0}/_{k}) = [A^{T}(k,0) W(k) A(k,0)]^{-1} A^{T}(k,0) W(k) Z(k)$$

 $C = A^{T}(k,0) W(k) A(k,0)$ (8)

$$D_{D=} A^{I}(k,0) W(k) Z(k)$$
(9)

$$= \begin{bmatrix} \sum \frac{i^{2}t^{2}\cos^{2}B_{m}(i)}{\sigma_{i}^{2}} & \sum -\frac{i^{2}t^{2}\cos B_{m}(i)\sin B_{m}(i)}{\sigma_{i}^{2}} \\ \sum -\frac{i^{2}t^{2}\cos B_{m}(i)\sin B_{m}(i)}{\sigma_{i}^{2}} & \sum \frac{it\sin^{2}B_{m}(i)}{\sigma_{i}^{2}} \\ \sum \frac{it\cos^{2}B_{m}(i)}{\sigma_{i}^{2}} & \sum \frac{-it\cos B_{m}(i)\sin B_{m}(i)}{\sigma_{i}^{2}} \\ \sum \frac{-it\cos B_{m}(i)\sin B_{m}(i)}{\sigma_{i}^{2}} & \sum \frac{it\sin^{2}B_{m}(i)}{\sigma_{i}^{2}} \end{bmatrix}$$

$$\frac{\sum \frac{it \cos^2 B_m(i)}{\sigma_i^2}}{\sum \frac{-it \cos B_m(i) \sin B_m(i)}{\sigma_i^2}} \sum \frac{\sum \frac{-it \cos B_m(i) \sin B_m(i)}{\sigma_i^2}}{\sum \frac{1 \sin^2 B_m(i)}{\sigma_i^2}} \sum \frac{\sum \frac{it \sin^2 B_m(i)}{\sigma_i^2}}{\sigma_i^2}}{\sum \frac{-\cos B_m(i) \sin B_m(i)}{\sigma_i^2}} \sum \frac{\sum \frac{\sin^2 B_m(i)}{\sigma_i^2}}{\sigma_i^2}}{\sum \frac{\sin^2 B_m(i)}{\sigma_i^2}}$$

(10)

(11)

$$D = \begin{bmatrix} \sum \frac{it \cos B_m(i)P(i)}{\sigma_i^2} \\ \sum -\frac{it \sin^2 B_m(i)P(i)}{\sigma_i^2} \\ \sum \frac{\cos B_m(i)P(i)}{\sigma_1^2} \\ \sum \frac{-\sin B_m(i)P(i)}{\sigma_1^2} \end{bmatrix}$$

The matrices C and are converted into sequential processing.

$$A(k, k + 1) = A(k, 0) \Phi(0, k + 1)$$

$$= \begin{bmatrix} -kt \cos B_m(1) & kt \sin B_m(1) \\ (1-k)t \cos B_m(2) & -(1-k)t \sin B_m(2) \\ (2-k)t \cos B_m(3) & -(2-k)t \sin B_m(3) \end{bmatrix}$$

 $\begin{array}{c} \cos B_m(1) & -\sin B_m(1) \\ \cos B_m(2) & -\sin B_m(2) \\ \cos B_m(3) & -\sin B_m(3) \end{array} \right|_{(12)}$ 

Sums are updated after second measurement as

$$ESUMS[1] = \frac{\cos^2 B_m(k)}{\sigma^2(k)} + previous ESUMS[1]$$

$$= \begin{bmatrix} ESUMS[3] & ESUMS[7] & ESUMS[2] & ESUMS[6] \\ ESUMS[7] & ESUMS[10] & ESUMS[6] & ESUMS[9] \\ ESUMS[2] & ESUMS[6] & ESUMS[1] & ESUMS[5] \\ ESUMS[6] & ESUMS[9] & ESUMS[5] & ESUMS[8] \end{bmatrix}$$

$$(14)$$

$$G = [GSUMS(1) & GSUMS(2) & GSUMS(3) & GSUMS(4)]$$

(15)

### SIMULATION:

The samples are corrupted by additive zero mean Gaussian noise with RMS of 1 degree. The observer is assumed to be doing a S-maneuver moving on the line of sight at a constant speed of 30 knots at a turning rate of 1deg/s. The bearing measurements are available for every second. The target is assumed to be moving at a speed of 20

knots. A number of scenarios are tested by changing the course of the target. The simulation is carried out by considering the target at an initial range of 5000 meters with zero initial bearing relative to the observer. The results in simulation are noted. The fig shows Range error, Course error and speed error. The required accuracies are obtained at 70sec, 90 sec and 1min30sec.A short scenario is considered in which target is assumed to be at an initial bearing of 40 degrees.



fig 3 Error in Course estimate

fig 6 Error in Course estimate

### LIMITATIONS:

For the estimation, Kalman filter is fine but it faces the problem of initialization. This is avoided by PLE. But PLE too faces the problem of spurious bearings, missed bearings and also the performance of PLE is limited if the own ship is not maintaining a proper maneuver. If the own ship is not maintaining a proper maneuver it's very hard to track the target. This is because the change in the bearing is very less. These all problems can be solved by using the modified PLE.

#### SUMMARY AND CONCLUSION:

The process is done in batch processing. The equations are converted into sequential mode. Recursive sums are introduced to convert batch processing into sequential processing to suit real time underwater applications such as passive target tracking. All the elements of the covariance matrix are represented recursively in terms of the measurement equation. These terms are known as recursive sums and are maintained throughout the algorithm. This approach avoids computational complexity by computing only the incremental for every bearing measurement. new These incremental values are used to update the recursive sums in the covariance matrix. Only a few recursive sums are updated on the arrival of a new bearing measurement. This method does not increase computation burden. For the estimation PLE too faces the problem of spurious bearings, missed bearings and also the performance of PLE is limited if the own ship is not maintain a proper maneuver. If the own ship is not maintaining a proper maneuver it is very hard to track the target. This is because the change in the bearing is very less. From these results it is concluded that this algorithm can be utilized for passive target tracking applications.

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