# Underwater Passive Target Tracking from a Stationary Observer Using Modified Gain Extended Kalman Filter

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ABSTRACT: Target tracking in underwater ,for a stationary observer, observability is less compared to moving observer. Modified Gain Extended Kalman Filter (MGEKF) developed by Song and Speyer [2] was proven to be suitable algorithm for angles only passive target tracking applications in air. In this paper, this improved MGEKF algorithm is explored for underwater applications with some modifications. In underwater, the noise in the measurements is very high, turning rate of the platforms is low and speed of the platforms is also low when compared with the missiles in air. These characteristics of the platform are studied in detail and the algorithm is modified suitably for tracking applications in underwater. Monte-Carlo simulated results for one typical scenario is presented for the purpose of explanation. From the results it is observed that this algorithm is suitable for stationary observer in underwater passive target tracking using angles only measurements.

**1.INTRODUCTION:** In the ocean environment, an observer monitors noisy sonar bearings and elevations from a radiating target. The measurements are extracted from a single stationary observer and the observer processes these measurements to find out target motion parameters-Viz., range, course, bearing. elevation and speed of the target. Here the measurements are nonlinear; making the whole process nonlinear. However, the modified gain extended kalman filter (MGEKF) developed by Song and Speyer [2], was the successful contribution for angles only passive target tracking applications in air. This MGEKF algorithm was further improved by P.J. Galkowiski and M.A. Eslam [4]. In this paper, this improved MGEKF algorithm is explored for underwater applications with some modifications. In underwater, the noise in the measurements is very high, turning rate of the platforms is low and speed of the platforms is also low when compared with the missiles in air. These characteristics of the platform are studied in detail and the algorithm is modified suitably for tracking applications in underwater.

Section 2 deals with mathematical modelling of bearing and elevation measurements. Section 3 describes the implementation of the filter and section 4 is about the the results obtained in simulation.

**2.MATHEMATICAL MODELING:** Let a target be at a point P and the observer be at the origin, as shown in Fig-1. The measurement vector ,Z, is written as

$$Z = \begin{bmatrix} B_{m} \\ \phi_{m} \end{bmatrix} = \begin{bmatrix} \tan^{-1} \frac{\mathbf{r}_{x}}{\mathbf{r}_{y}} + \sigma_{B} \\ \tan^{-1} \frac{\mathbf{r}_{xy}}{\mathbf{r}_{z}} + \sigma_{\phi} \end{bmatrix}$$
(1)

Where  $\sigma_{\rm B}$  and  $\sigma_{\phi}$  are zero mean, uncorrelated normally distributed errors in the bearing (B<sub>m</sub>) and elevation ( $\phi_{\rm m}$ ) measurements respectively. Let the state vector be

$$X_{s} = \dot{x} \quad \dot{y} \quad \dot{z} \quad r_{x} \quad r_{y} \quad r_{z} \qquad (2)$$

The measurement matrix H is given by

$$H = \begin{bmatrix} 0 & 0 & 0 & \frac{\cos\hat{B}}{\hat{r}_{xy}} & -\frac{\sin\hat{B}}{\hat{r}_{xy}} & 0\\ 0 & 0 & 0 & \frac{\cos\hat{\phi}\,\sin\hat{B}}{\hat{r}} & \frac{\cos\hat{\phi}\,\cos\hat{B}}{\hat{r}} & -\frac{\sin\hat{\phi}}{\hat{r}} \end{bmatrix} (3)$$

# 2.1 HORIZONTAL PLANE AND BEARING MEASUREMENTS :-

If the range in horizontal plane is  $\sqrt{r_x^2 + r_y^2}$ , then the estimated range be  $\hat{r}_{xy} = \sqrt{r_x^2 + r_y^2}$  (4)

By adding  $r_{xy}$  and  $\hat{r}_{xy}$ , eqn. (5) is obtained .

 $\begin{aligned} r_{xy} + \hat{r}_{xy} &= r_x \sin B + r_y \cos B + \hat{r}_x \sin \hat{B} + \hat{r}_y \cos \hat{B} \quad (5) \\ \text{adding} & \text{both} & \text{sides} \\ - r_x \sin \hat{B} - \hat{r}_x \sin \hat{B} - \hat{r}_y \cos B + r_y \cos \hat{B} \text{ to the above} \\ \text{equation and after straight forward} \\ \text{simplification, the following equations are} \\ \text{obtained.} \end{aligned}$ 

$$r_{xy} + \hat{r}_{xy} = \frac{(r_x - \hat{r}_x)(\sin B - \sin \hat{B}) + (r_y - \hat{r}_y)(\cos B - \cos \hat{B})}{1 - \cos(B - \hat{B})}$$

$$r_{xy} - \hat{r}_{xy} = \frac{(r_x - \hat{r}_x)(\sin B + \sin \hat{B}) + (r_y - \hat{r}_y)(\cos B + \cos \hat{B})}{1 + \cos(B - \hat{B})}$$

using (6) and (7)  

$$2\hat{\mathbf{r}}_{xy} = (\mathbf{r}_{x} - \hat{\mathbf{r}}_{x}) \left[ \frac{\sin \mathbf{B} - \sin \hat{\mathbf{B}}}{1 - \cos(\mathbf{B} - \hat{\mathbf{B}})} - \frac{\sin \mathbf{B} + \sin \hat{\mathbf{B}}}{1 + \cos(\mathbf{B} - \hat{\mathbf{B}})} \right] + (\mathbf{r}_{y} - \hat{\mathbf{r}}_{y}) \left[ \frac{\cos \mathbf{B} - \cos \hat{\mathbf{B}}}{1 - \cos(\mathbf{B} - \hat{\mathbf{B}})} + \frac{\cos \mathbf{B} + \cos \hat{\mathbf{B}}}{1 + \cos(\mathbf{B} - \hat{\mathbf{B}})} \right]$$
(8)

Eqn.(8) can be simplified as  $\frac{\sin B - \sin \hat{B}}{1 - \cos(B - \hat{B})} - \frac{\sin B + \sin \hat{B}}{1 + \cos(B - \hat{B})} = \frac{(1 + \cos(B - \hat{B})(\sin B - \sin \hat{B}) - (1 - \cos(B - \hat{B})(\sin B + \sin \hat{B}))}{1 - \cos^2(B - \hat{B})}$ 

the coefficients of  $(r_x - \hat{r}_x)$  and  $(r_y - \hat{r}_y)$  are

simplified and the above equation is rewritten

as

$$2\hat{r}_{xy} = \frac{2\cos B(r_x - \hat{r}_x)}{\sin(B - \hat{B})} - \frac{2\sin B(r_y - \hat{r}_y)}{\sin(B - \hat{B})}$$
(9)

Again eqn. (9) is rewritten as  $\sin(B - \hat{B}) = \frac{\cos B(r_x - \hat{r}_x) - \sin B(r_y - \hat{r}_y)}{\hat{r}_{xy}} (10)$ 

## 2.2 ELEVATION ANGLE MEASUREMENT:

In the previous section, it is seen that

$$\tan^{-1} \frac{\mathbf{r}_{x}}{\mathbf{r}_{y}} = \mathbf{B} \text{ generates}$$
$$\sin(\mathbf{B} \cdot \hat{\mathbf{B}}) = \frac{\cos \mathbf{B}(\mathbf{r}_{x} - \hat{\mathbf{r}}_{x}) - \sin \mathbf{B}(\mathbf{r}_{y} - \hat{\mathbf{r}}_{y})}{\hat{\mathbf{r}}_{xy}} . \tag{11}$$

Similarly 
$$\tan^{-1} \frac{\mathbf{I}_{xy}}{\mathbf{r}_z} = \phi$$
 generates  
 $\sin(\phi \cdot \hat{\phi}) = \frac{\cos\phi(\mathbf{r}_{xy} - \hat{\mathbf{r}}_{xy}) - \sin\phi(\mathbf{r}_z - \hat{\mathbf{r}}_z)}{\mathbf{r}}$ 
(12)

After simple trigonometric manipulations, eqn.

$$r_{xy} - \hat{r}_{xy} = \frac{(r_x - \hat{r}_x)\sin\frac{(B + \hat{B})}{2} + (r_y - \hat{r}_y)\cos\frac{(B + \hat{B})}{2}}{\cos\frac{(B - \hat{B})}{2}}$$
(13)

Substituting (13) in (12)  

$$\sin(\phi - \hat{\phi}) = \frac{\cos\phi}{\hat{r}} \left[ \frac{(r_x - \hat{r}_x)\sin\frac{(B + \hat{B})}{2} + (r_y - \hat{r}_y)\cos\frac{(B + \hat{B})}{2}}{\cos\frac{(B - \hat{B})}{2}} \right] - \frac{\sin\phi}{\hat{r}} (r_z - \hat{r}_z)$$
(14)

Eqn. (10) and eqn.(14) are rewritten in matrix form as

$$\begin{bmatrix} (\mathbf{B} \cdot \hat{\mathbf{B}}) \\ (\phi \cdot \hat{\phi}) \end{bmatrix} = \begin{bmatrix} \frac{\cos \mathbf{B}}{\hat{\mathbf{r}}_{xy}} & -\frac{\sin \mathbf{B}}{\hat{\mathbf{r}}_{xy}} & 0 \\ \frac{\sin \left(\frac{\mathbf{B} + \hat{\mathbf{B}}}{2}\right) \cos \phi}{\cos \left(\frac{\mathbf{B} - \hat{\mathbf{B}}}{2}\right) \hat{\mathbf{r}}} & \frac{\cos \left(\frac{\mathbf{B} + \hat{\mathbf{B}}}{2}\right) \cos \phi}{\cos \left(\frac{\mathbf{B} - \hat{\mathbf{B}}}{2}\right) \hat{\mathbf{r}}} & \frac{-\sin \phi}{\hat{\mathbf{r}}} \end{bmatrix} \begin{bmatrix} \mathbf{r}_{x} - \hat{\mathbf{r}}_{x} \\ \mathbf{r}_{y} - \hat{\mathbf{r}}_{y} \\ \mathbf{r}_{z} - \hat{\mathbf{r}}_{z} \end{bmatrix}$$
(15)

As true bearing is not available, it is replaced by measured bearing in eqn.(15) and obtained eqn.(16) as follows.

$$\begin{bmatrix} (\mathbf{B} \cdot \hat{\mathbf{B}}) \\ (\phi \cdot \hat{\phi}) \end{bmatrix} = \cdot \begin{bmatrix} \frac{\cos \mathbf{B}_{m}}{\hat{\mathbf{r}}_{xy}} & -\frac{\sin \mathbf{B}_{m}}{\hat{\mathbf{r}}_{xy}} & 0 \\ \frac{\sin \left( \frac{\mathbf{B}_{m} + \hat{\mathbf{B}}}{2} \right) \cos \phi}{\cos \left( \frac{\mathbf{B}_{m} - \hat{\mathbf{B}}}{2} \right) \hat{\mathbf{r}}} & \frac{\cos \left( \frac{\mathbf{B}_{m} + \hat{\mathbf{B}}}{2} \right) \cos \phi}{\cos \left( \frac{\mathbf{B}_{m} - \hat{\mathbf{B}}}{2} \right) \hat{\mathbf{r}}} & \frac{-\sin \phi}{\hat{\mathbf{r}}} \end{bmatrix} \\ \begin{bmatrix} \mathbf{r}_{x} - \hat{\mathbf{r}}_{x} \\ \mathbf{r}_{y} - \hat{\mathbf{r}}_{y} \\ \mathbf{r}_{z} - \hat{\mathbf{r}}_{z} \end{bmatrix} &= g \begin{bmatrix} \mathbf{r}_{x} - \hat{\mathbf{r}}_{x} \\ \mathbf{r}_{y} - \hat{\mathbf{r}}_{y} \\ \mathbf{r}_{z} - \hat{\mathbf{r}}_{z} \end{bmatrix}$$
(16)

Considering  $\dot{x}$ ,  $\dot{y}$  and  $\dot{z}$  also, g is given by

$$g = \begin{bmatrix} 0 & 0 & 0 & \frac{\cos B_m}{\hat{r}_{xy} = \hat{r}_{s.} \sin \hat{\phi}_m} & \frac{-\sin B_m}{\hat{r}_{xy} = \hat{r}_{s.} \sin \hat{\phi}_m} & 0 \\ 0 & 0 & 0 & \frac{\cos \phi \, \sin \left(\frac{B_m + \hat{B}}{2}\right)}{\hat{r} \cos \left(\frac{B_m - \hat{B}}{2}\right)} & \frac{\cos \phi \cos \left(\frac{B_m + \hat{B}}{2}\right)}{\hat{r} \cos \left(\frac{B_m - \hat{B}}{2}\right)} & -\frac{\sin \phi}{\hat{r}} \end{bmatrix}$$

#### **3. IMPLEMENTATION OF THE ALGORTIHM :**

The above mentioned improved algorithm is implemented using MGEKF (MGEKF equations are not given in this paper due to space constraint. These equations are available in [4].)for underwater passive target tracking as follows. As only bearing and elevation measurements are available, the velocity components of the target are assumed to be each 10 m/sec which is very close to the realistic speed of the vehicles in underwater. The range of the day, say 15000 meters, is utilized in the calculation of initial position estimate of the target is as

 $X(0|0) = \begin{bmatrix} 10 & 10 & 10 & 15000 \text{sin } B_{\text{m}} & (0) \text{sin } \phi_{\text{m}}(0) \\ 15000 = \begin{bmatrix} 1 & 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 10 & 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 \end{bmatrix} =$ 

 $15000 \sin \phi_{\rm m}(0) \cos B_{\rm m}(0) \ 15000 \cos \phi_{\rm m}(0) \]^{\rm T}$ 

where  $B_m(0)$  and  $\phi_m(0)$  are initial bearing and elevation measurements.

#### 4. SIMULATION RESULTS :

elevation All raw bearings and measurements are corrupted by additive zero mean Gaussian noise with a r.m.s level of 0.3 degree. Corresponding to a tactical scenario in which the target is at the initial range of 20000 meters at initial bearing and elevation of 0.5 and 45 degrees respectively. The target is assumed to be moving at a constant course of 130 degrees. Observer is assumed to be stationary. The results have been ensemble averaged over several Monte Carlo runs. The errors in estimates are plotted in Fig.2. It is observed that this required accuracy is obtained from 800 seconds onwards and so this algorithm seems to be very much useful for underwater passive target tracking when observer is stationary, for a moving target.

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Fig.2(a). Error in range estimate versus time



Fig.1. A typical target observer geometry



Fig.2(c). Error in Course estimate verses time

Fig.2(b). Error in Speed estimate versus time