Unsteady Boundary Layer Flow of Viscous Fluid Along a Vertical Surface with Viscous Dissipation and Thermal Radiation

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***Abstract* - The present work is devoted to investigate the heat transfer analysis of a two dimensional unsteady boundary layer flow of viscous fluid at a vertical surface in the presence of viscous dissipation and thermal radiation. The modeled equations are converted into non-linear ordinary differential equations by using similarity transformations. The coupled differential equations are solved numerically by using Runge-Kutta method with shooting technique. The effects of various parameters on velocity and temperature profiles are discussed.**

Keywords - Unsteady boundary layer, viscous dissipation, thermal radiation, convective boundary conditions.

Nomenclature

a,b,c	Empirical Constants
A	Unsteady parameter
Cf	Skin friction
C	Specific heat at constant pressure
f	Dimensionless stream function
fw	Surface mass transfer parameter
$k(T)$	Thermal conductivity
kw	Thermal conductivity at the sheet
k_∞	Thermal conductivity far away from the sheet
M	Kummer's function
Nu_x	Nusselt number
Nr	Thermal radiation parameter
Pr	Prandtl number
Ec	Eckert number
q_w	Local heat flux at the sheet
q_r	Radiative heat flux
T	Fluid temperature
T_w	Given temperature at the sheet
T_∞	Constant temperature of the fluid far away from the sheet
u	Velocity in x direction
U_w	Velocity of the stretching surface
v_w	Suction/ blowing velocity

Greek Symbols

α_∞	Thermal diffusivity
ΔT	Sheet temperature
ε	Thermal conductivity parameter
υ	Kinematic viscosity
β	Thermal expansion coefficient
μ	Dynamic Viscosity
ψ	Stream function
ρ	Density
$\sigma*$	Stephan-Boltzman constant
$k*$	Mean absorption coefficient
τ_{xy}	Shear stress
θ	Dimensionless temperature variable
λ	Free convection or buoyancy parameter

I. INTRODUCTION

The study of two dimensional boundary layer flow and heat transfer of incompressible viscous fluid over a continuous stretching and heated surfaces has acquired momentum due to its various applications such as extrusion of polymer in industry ,wire and fiber coating, design of heat exchangers and chemical process. The analysis of momentum and thermal transports within the fluid on a continuously stretching surface is important for gaining of some fundamental understanding of such processes. The first among to analyze the problem on boundary layer flow over acontinuous solid surface moving with constant velocity by Sakiadis[1]. Mahmoud and Waheed [2] have studied MHD flow and heat transfer of a micro-polar fluid over a stretching surface with heat generation (absorption) and slip. Aziz [3] has studied flow and heat transfer over an unsteady stretching surface with hall effect. Qin et al. [4] have analyzed the cauchy problem for a 1D compressible viscous micro-polar fluid model: analysis of the stabilization and the regularity.Khan and Pop [5] have discussed boundary layer flow of a nano-fluid past a stretching sheet.Chamkha et al. [6] discussed similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction / injection and chemical reaction effects.

Mukhopadhyay [7] was studied the heat transfer analysis of unsteady flow over a porous stretching surface embedded in a porous medium in presence of thermal radiation .Bhattacharyya et al. [8] have analyzed slip effects an unsteady boundary layer stagnation –point flow and heat transfer towards a stretching sheet. Mukhopadhyay [9] has analyzed the effects of slip on unsteady mixed convective flow and heat transfer past a porous stretching surface.Vajravelu et al. [10] have studied unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. Hamad et al. [11] have studied magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate.

Sarkar [12] has discussed mixed convective heat transfer of nanofluids past a circular cylinder in cross flow in unsteady regime. Kousar and Liao [13] have discussed unsteady non-similarity boundary-layer flows caused by an impulsively stretching flat sheet. Rohni et al. [14] have analyzed unsteady mixed convection boundary-layer flow with suction and temperature slip effects near the stagnation point on a vertical permeable surface embedded in a porous medium. Chen [15] has studied Mixed convection unsteady stagnation-point flow towards a stretching sheet with slip.Hunsain et al. [16] have made study on heat and mass transfer analysis in unsteady boundary layer flow through porous media with variable viscosity and thermal. Raju and Varma [17] made a study on unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature.Hossain et al. [18] made a note on solution of natural convection boundary layer flow above a semi-infinite porous horizontal plate under similarity transformations with suction and blowing. Hossain et al. [19] have studied similarity solution of unsteady combined free and force convective laminar boundary layer flow about a vertical porous surface with suction and blowing. Rahman et al. [20] have analyzed thermophoresis particle deposition on unsteady two-dimensional forced convective heat and mass transfer flow along a wedge with variable viscosity and variable Prandtl number. Ishak et al. [21] have made study on boundary layer flow and heat transfer over an unsteady stretching vertical surface.

II. MATHEMATICAL FORMULATION

Consider the unsteady laminar two dimensional boundary layer flow of incompressible viscous fluid flow on a stretching permeable surface. The sheet is stretched with a velocity $U_w = \dfrac{ax}{(1-ct)}$, where a and c are constants, in the positive x direction. Here $a > 0, b > 0$ and $t < \dfrac{1}{c}$. The sheet surface temperature $T_w(x,t) = T_\infty + \dfrac{bx}{(1-ct)^2}$ varies with the distance x.

The boundary layer governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} \pm g\beta(T - T_\infty) \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(K(T)\frac{\partial T}{\partial y}\right) - \frac{\partial q_r}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Subjected to the boundary conditions

$$\begin{aligned} &u = U_w,\ \ v = v_w,\ \ T = T_w \ \ at \ \ y = 0 \\ &u \to 0, \quad T \to T_\infty, \quad as \ \ y \to \infty \end{aligned} \tag{4}$$

Where u and v are the velocity components in the x and y directions, respectively. $\upsilon = (\mu/\rho)$ is the kinematic viscosity. g is the acceleration due to gravity. β is the coefficient of thermal expansion, T is the fluid temperature, T_∞ is the ambient temperature, ρ is the density, C_p is the specific heat at constant pressure. K (T) is the variable thermal conductivity. $v_w(t) = v_0/\sqrt{1-ct}$ is the suction/injection velocity and qr is the radiative heat flux. The thermal conductivity is assumed to vary linearly with temperature

$$K(T) = K_\infty\left(1 + \frac{\varepsilon}{(T_w - T_\infty)}(T - T_\infty)\right) \tag{5}$$

The radiative heat flux con be expressed as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

We introduce new dimensionless variables:

$$\eta = \left(\frac{a}{v(1-ct)}\right)^{\frac{1}{2}} y, \quad \psi = \left(\frac{va}{(1-ct)}\right)^{\frac{1}{2}} xf(\eta),$$

$$\theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \tag{7}$$

Using Eqs (7) ,(6) and (5) in to (1), (2) and(3) reduces to

$$f''' + ff'' - f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) + \lambda\theta = 0 \tag{8}$$

$$(1 + \varepsilon\theta + Nr)\theta'' + \varepsilon\theta'^2 + \Pr f\theta' - \Pr f'\theta + \Pr Ecf''^2 - \Pr A\left(\frac{1}{2}\eta\theta' + 2\theta\right) = 0 \tag{9}$$

Where $\Pr = \dfrac{\upsilon}{\alpha_\infty}$ is the Prandtl number, $\alpha_\infty = \dfrac{k_\infty}{\rho C_p}$, $f_w = -\dfrac{v_0}{\sqrt{\upsilon a}}$, f_w is the surface mass transfer parameter, $Nr = \dfrac{16\sigma^* T_\infty^3}{3K_\infty k^*}$ is the thermal radiation parameter, $\lambda = \dfrac{g\beta b}{a^2}$ free convection parameter, $A = \dfrac{C}{a}$ unsteady parameter, and $Ec = \dfrac{U_w^{\,2}}{(T_w - T_\infty)Cp}$ is the local Eckert number. The transformed boundary conditions are

$$f'(\eta) = 1, \ \ f(\eta) = f_w, \ \ \theta(\eta) = 1, \quad \text{at} \ \ \eta = 0$$

$$f'(\eta)\to 0 \quad \theta(\eta)\to 0 \qquad \text{as} \quad \eta\to\infty \qquad (10)$$

The quantities Skin-Friction Coefficient (rate of shear stress) and Nusselt number are (rate of heat transfer).The local skin friction coefficient and Nusselt number is defined as

$$C_f = \frac{\tau_w}{\rho U_w^2/2}, \qquad Nu_x = \frac{x q_w}{K_\infty (T_w - T_\infty)} \qquad (11)$$

Where the skin friction and heat transfer from the sheet are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad q_w = -K_\infty\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

TABLE 1: COMPARISON OF VALUES OF $-\theta'(0)$ FOR VARIOUS VALUES OF A, λ *and* Pr

A	λ	Pr	**Present**	**Vajravelu et al[10]**	**Ishak et al[21]**
0	0	0.01	0.019723	0.019723	0.0197
0	0	0.72	0.8088341	0.808836	0.8086
0	0	1	1.000008	1000000	100000
0	0	3	1.923679	1.923687	1.9237
0	0	10	3.720671	3.720788	3.7207
0	0	100	12.294081	12.30039	12.2941
1	0	1	1.681993	1.681921	1.6820
1	1	1	1.703913	1.703910	1.7039
0	1	1	1.087275	1.087206	1.0873
0	2	1	1.142336	1.142298	1.1423
0	3	1	1.185289	1.185197	1.1853

III. RESULTS AND DISCUSSION

The nonlinear ordinary differential Eqs. (8) - (9), subject to the boundary conditions Eqs.(10) are solved numerically using shooting technique. The obtained results show the effects of the various non-dimensional governing parameters on the velocity and temperature fields. Comparison of values of $-\theta'(0)$ for various values of A, λ and Pr in respective figures and tables.

Figs.1-5 displays the effects of thermal buoyancy parameter on velocity and temperature profiles. It is evident from the figures that increase in buoyancy parameter enhances the velocity profiles but decreases the temperature profiles.

Figs. 6 and 7 shows the effect of Prandtl number on temperature profiles. From these figures, the temperature profiles are decreasing with increasing value of Prandtl number. Because, an increase in Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increase in thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values Pr. Hence in the case of smaller Prandtl number as the thermal boundary layer is thicker and the rate of heat transfer is reduced.

Figs.8-10 shows effect of thermal conductivity parameter ε on temperature profiles of the flow. It is evident from the figures that an increase in thermal conductivity parameter enhances the temperature profiles. Figs. 11-13 depict the influence of thermal radiation parameter on temperature profiles. It is clear from the figures that an increase in the thermal radiation parameter enhances the temperature profiles. The influence of viscous dissipation parameter (Ec) on temperature profiles is shown in figs 14-16. It observed from the figure that increasing in the Eckert number enhances the temperature profiles of the flow. This is due to the fact that an increase in the dissipation causes to improve the thermal conductivity of the flow. This helps to enhance the thermal boundary layer thickness.

Figs. 17 and 18 depict the influence of the skin friction parameter on velocity and temperature profiles. It is evident from the figures that an increase in skin friction parameter enhances the velocity and temperature profiles.

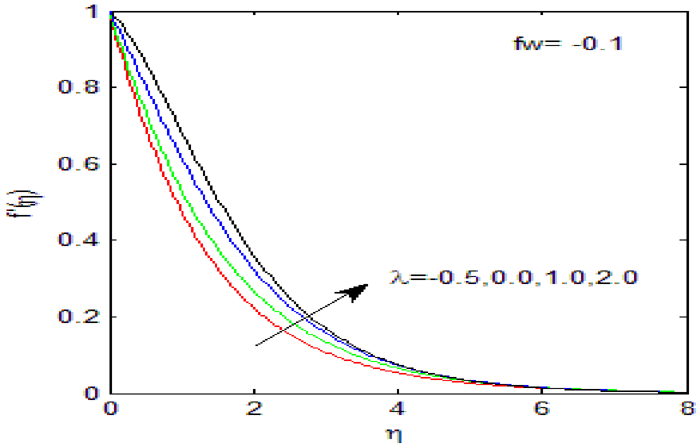

Fig.1 Velocity profile for different values of λ *with* $\Pr = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

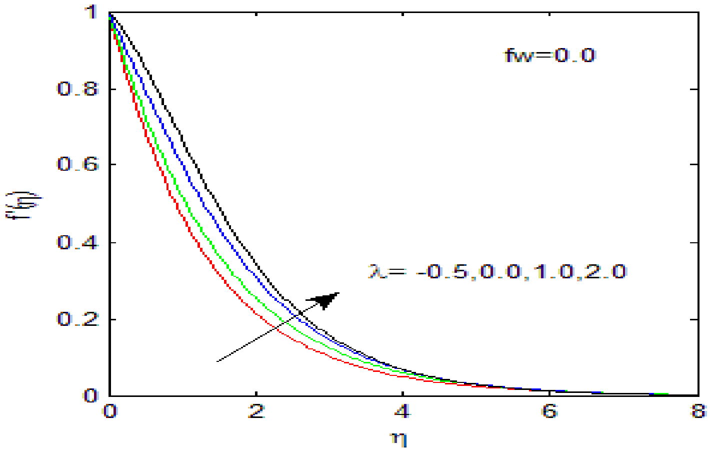

Fig.2 Velocity profile for different values of λ *with* $\Pr = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

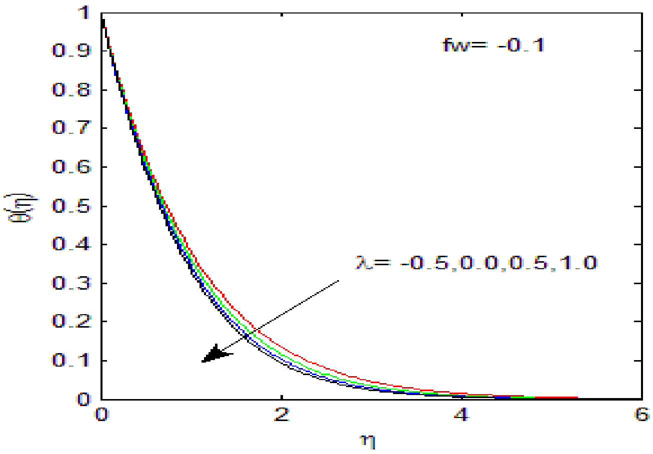

Fig.3 Temperature profile for different values of λ *with* $\Pr = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

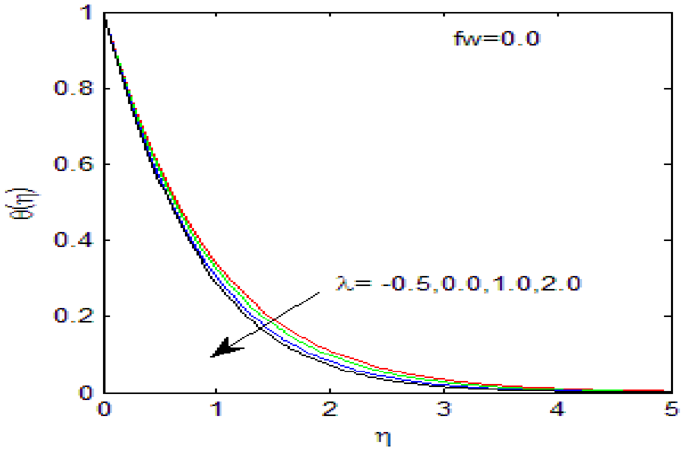

Fig.4 Temperature profile for different values of λ *with* $\Pr = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

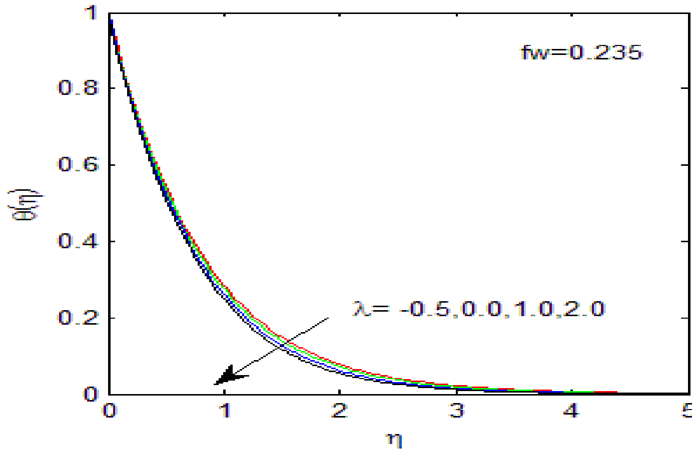

Fig.5 Temperature profile for different values of λ *with* $\Pr = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

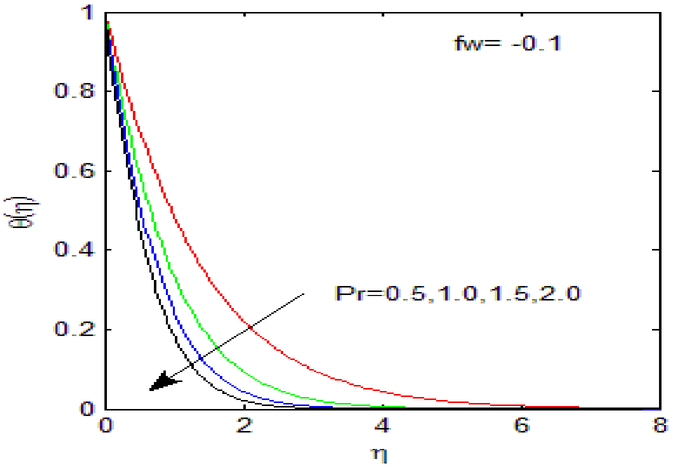

Fig.6 Temperature profile for different values of $\Pr$ *with* $\lambda = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

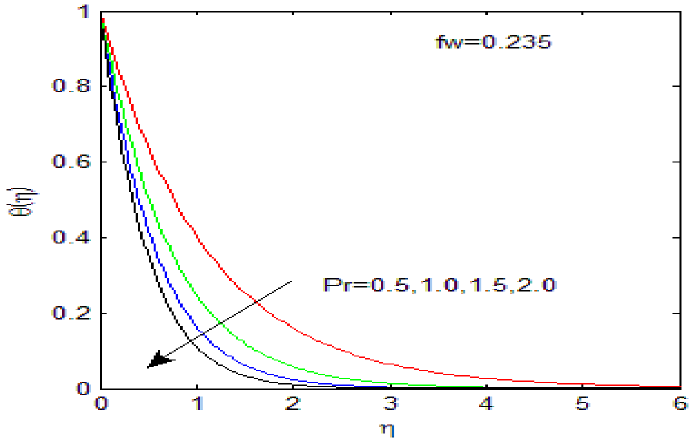

Fig.7 Temperature profile for different values of $\Pr$ *with* $\lambda = 1, \varepsilon = 0.1, Nr = 0.1, Ec = 0.1$

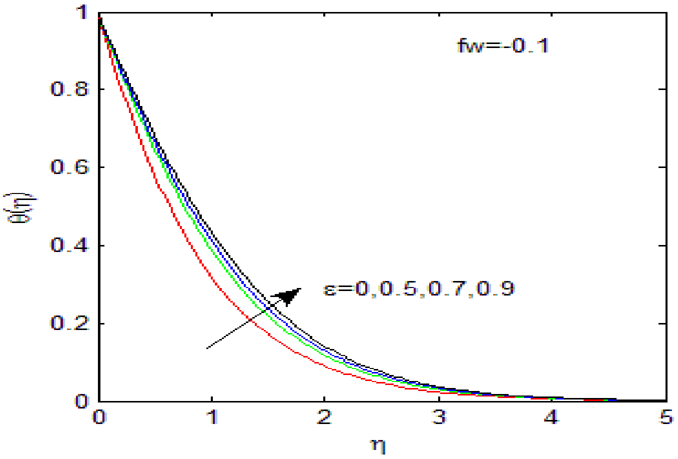

Fig.8 Temperature profile for different values of ε *with* $\lambda = 1, \Pr = 1, Nr = 0.1, Ec = 0.1$

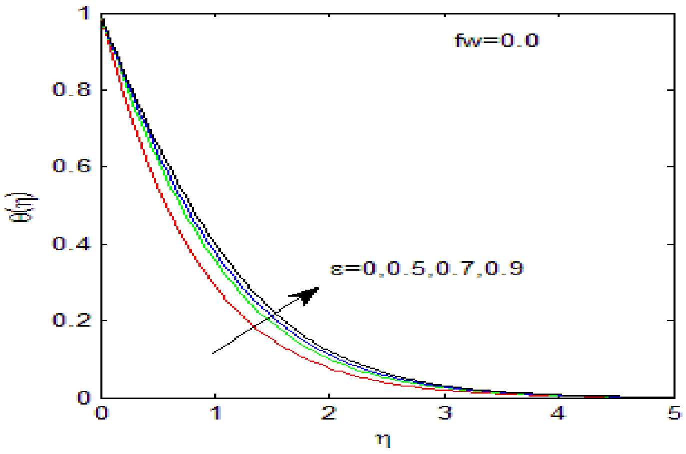

Fig.9 Temperature profile for different values of ε *with* $\lambda = 1, \Pr = 1, Nr = 0.1, Ec = 0.1$

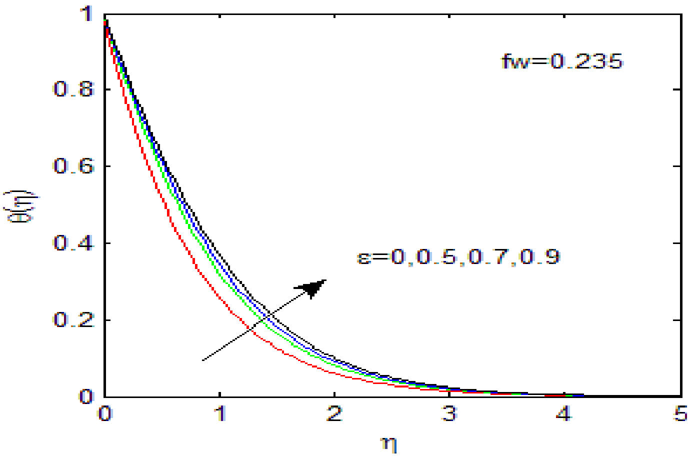

Fig.10 Temperature profile for different values of ε *with* $\lambda = 1, \Pr = 1, Nr = 0.1, Ec = 0.1$

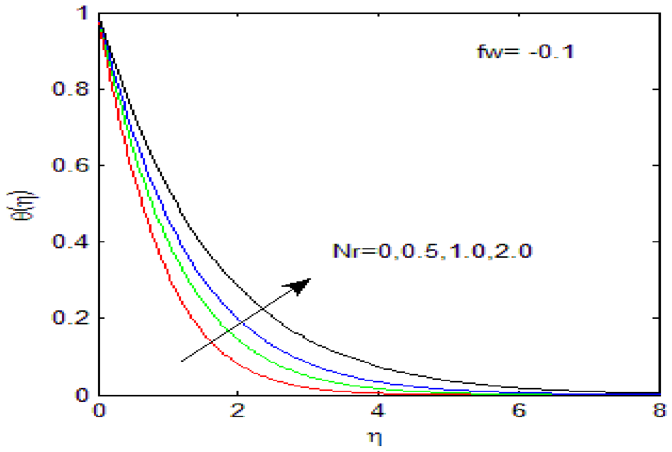

Fig.11 Temperature profile for different values of Nr *with* $\lambda = 1, \Pr = 1, \varepsilon = 0.1, Ec = 0.1$

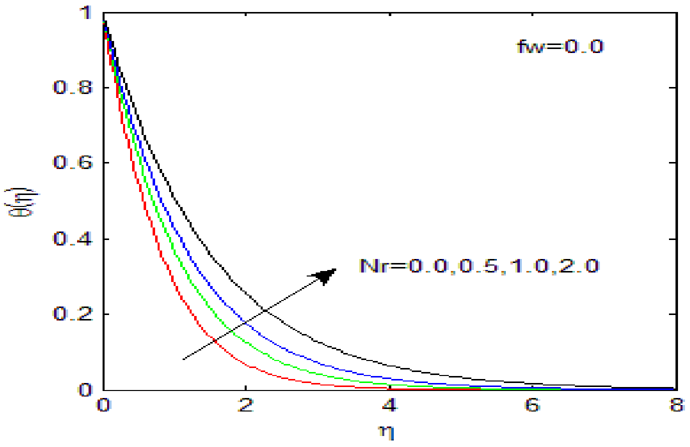

Fig.12 Temperature profile for different values of Nr *with* $\lambda = 1, \Pr = 1, \varepsilon = 0.1, Ec = 0.1$

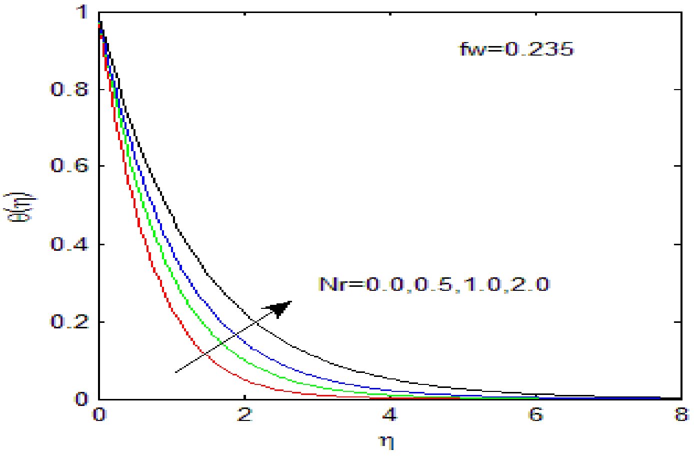

Fig.13 Temperature profile for different values of Nr *with* $\lambda = 1, \Pr = 1, \varepsilon = 0.1, Ec = 0.1$

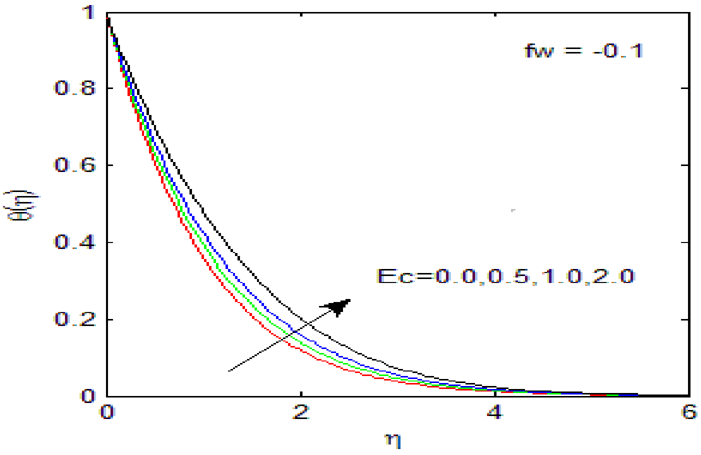

Fig.14 Temperature profile for different values of Ec *with* $\lambda = 1, \Pr = 1, \varepsilon = 0.1, Nr = 0.1$

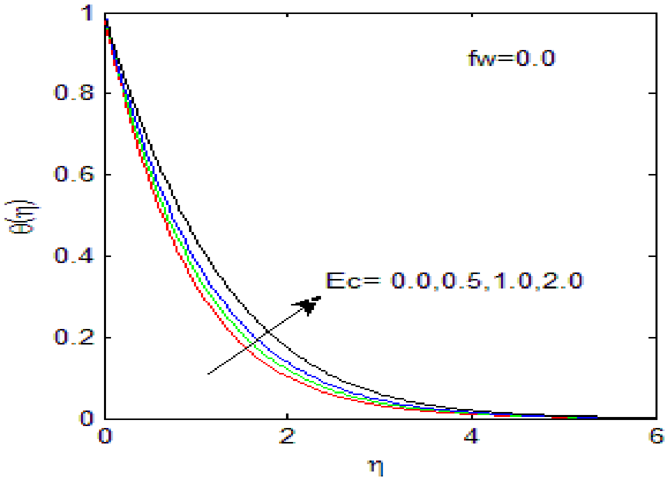

Fig.15 Temperature profile for different values of Ec *with* $\lambda = 1, \Pr = 1, \varepsilon = 0.1, Nr = 0.1$

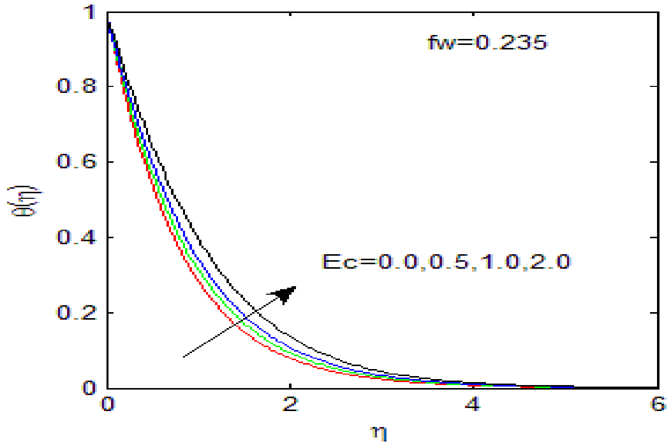

Fig.16 Temperature profile for different values of Ec *with* $\lambda = 1, \Pr = 1, \varepsilon = 0.1, Nr = 0.1$

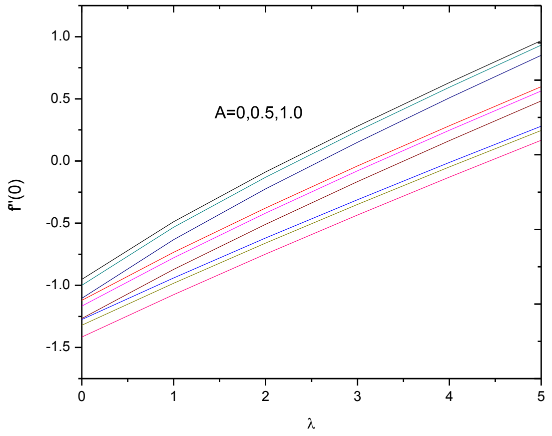

Fig.17 Variation of Skin friction $f''(0)$ *vs* λ *for different values of* A *and* fw

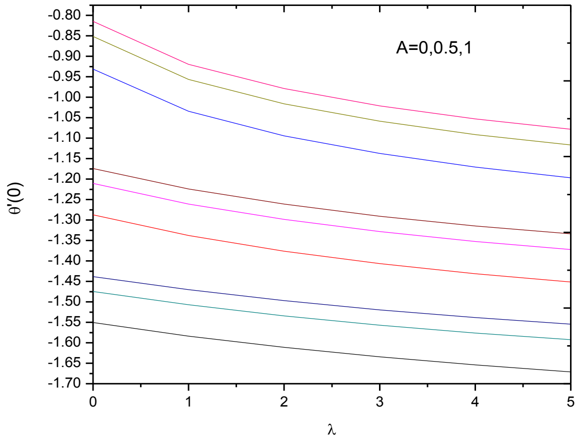

Fig.18 Variation of heat flux $\theta'(0)$ *vs* λ *for different values of* A *and* fw

IV. CONCLUSIONS

This study presented the heat transfer analysis of a two dimensional unsteady convective boundary layer flow of a viscous fluid at a vertical stretching in the presence of viscous dissipation and thermal radiation. Similarity transformations are used to convert the governing partial differential equations into nonlinear ordinary and coupled differential equations and are solved numerically by using Runge-Kutta method with shooting technique. The effects of various parameters on velocity and temperature fields are discussed and presented through graphs. Also, the Skin friction, local Nusselt and Prandtl numbers are analyzed and presented through tables. The conclusions of the present study are made as follows:

- An increase in the thermal radiation parameter decreases the heat transfer rate.
- An increase in the buoyancy parameter enhances the velocity profile and decreases temperature profile.
- An increase in Prandtl number decreases the heat transfer rate.
- An increase in skin friction parameter enhances the velocity.

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