Unsteady Flow and Heat Transfer of Viscous Immiscible Fluids Between Two Parallel Plates

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***Abstract*—This paper studies the unsteady flow and heat transfer of two viscous immiscible fluids between two parallel plates. The partial differential equations governing the flow and heat transfer are solved analytically using two-term harmonic and non-harmonic functions in both fluid regions of the channel. Effects of physical parameters such as height ratio, viscosity ratio, conductivity ratio, Prandtl number, Eckert number, periodic frequency parameter and pressure on the velocity and temperature distributions are given and illustrated graphically**

***Keywords*—Unsteady flow, heat transfer, immiscible fluids, oscillatory pressure gradient**

NOMENCLATURE

A	real positive constant
C_P	specific heat at constant pressure
$\vec{g}$	gravitational acceleration
K	thermal conductivity
P	pressure
Ec	Eckert number
Pr	Prandtl number
T	temperature
T_w	wall temperature
t	time
u	velocity components of velocity along the plate.
U_0	average velocity

Greek letters

ρ	fluid density
μ	vis cosity of fluid
ε	coeffiecient of periodic parameter
ω	frequency parameter
ωt	periodic frequency parameter
ν	kinematic viscosity
θ	nondimensional tempeaure

Subscripts

1,2 quantities for Region-I and Region-II respectively

I. INTRODUCTION

Problems involving immiscible multi-phase flow and heat transfer and multi-component mass transfer arise in a number of scientific and engineering disciplines. Important applications include petroleum industry, geophysics and plasma physics. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. In general, multi-phase flows are driven by gravitational and viscous forces. There has been some theoretical and experimental work on stratified laminar flow of two immiscible fluids in a horizontal pipe ,[1,2]. [3] studied two-phase MHD flow and heat transfer in a parallel plate channel, with one of the fluids being electrically conducting. Two-phase MHD flow and heat transfer in an inclined channel was investigated by [4]. Later on convective magnetohydrodynamic two-fluid flow and convective flow and heat transfer in composite porous medium was analysed by [5,6]. Fully developed flow and heat transfer in horizontal channel consisting of an electrically conducting fluid layer sandwiched between two fluids layers is studied analytically by [7]

All the above studies pertain to steady flow. [8-11] have presented analytical solutions for unsteady/oscillatory two-fluid and three-fluid flow and heat transfer in a horizontal channel. However, most problems of practical interest is unsteady. Keeping in view the wide area of practical importance of unsteady multi-fluid flows as mentioned above, it is the objective of the present study to investigate unsteady flow and heat transfer of two–fluid model in a horizontal channel.

II. MATHEMATICALFORMULATION

Consider a two dimensional unsteady flow of two immiscible fluids in a horizontal parallel permeable plates, extending in the Z and X direction. The region $0 \le y \le h$ (Region-I) is filled with a viscous fluid having density ρ_1, dynamic viscosity μ_1, specific heat at constant pressure C_{P_1} thermal conductivity K_1 and the region $-h \le y \le 0$ (Region-II) is filled with a different viscous fluid having density ρ_2, dynamic viscosity μ_2, specific heat at constant pressure C_{P_2} and thermal conductivity K_2.

The flow of both regions is assumed to be fully developed and fluid properties are constant and driven by a

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common pressure gradient $\left(-\frac{\partial P_1}{\partial x}\right)$ in region-I and by pressure gradient $\left(-\frac{\partial P_2}{\partial x}\right)$ in region-II . The two plates are maintained at constant temperatures $T_{w_1}\ at\ y=h\ and\ T_{w_2}\ at\ y=-h.$

Under these assumptions and taking $\rho_1=\rho_2=\rho_0$ and $C_{p_1}=C_{p_2}=C_p$ the governing equations of motion and energy (Loharsbi and Sahai, 1988) are given by:

Region-I

$$\rho_0\frac{\partial u_1}{\partial t}=\mu_1\frac{\partial^2 u_1}{\partial y^2}-\frac{\partial p_1}{\partial x} \tag{1}$$

$$\rho_0 Cp\frac{\partial T_1}{\partial t}=K_1\frac{\partial^2 T_1}{\partial y^2}+\mu_2\left(\frac{\partial u_1}{\partial y}\right)^2 \tag{2}$$

Region-II

$$\rho_0\frac{\partial u_2}{\partial t}=\mu_2\frac{\partial^2 u_2}{\partial y^2}-\frac{\partial p_2}{\partial x} \tag{3}$$

$$\rho_0 Cp\frac{\partial T_2}{\partial t}=K_2\frac{\partial^2 T_2}{\partial y^2}+\mu_2\left(\frac{\partial u_2}{\partial y}\right)^2 \tag{4}$$

where u is the x-component of fluid velocity, v is the y-component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions which required that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at y=0.

The hydrodynamic boundary and interface conditions for the two fluids can then be written as

$$\begin{aligned} &u_1(h)=0\\ &u_2(-h)=0\\ &u_1(0)=u_2(0)\\ &\mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y}\quad at\ y=0 \end{aligned} \tag{5}$$

The thermal boundary and interface conditions on temperature for both fluids are given by

$$\begin{aligned} &T_1(h)=T_{w1}\\ &T_2(-h)=T_{w2}\\ &T_1(0)=T_2(0)\\ &K_1\frac{\partial T_1}{\partial y}=K_2\frac{\partial T_2}{\partial y}\quad at\ y=0 \end{aligned} \tag{6}$$

By use of the following non-dimensional quantities:

$$u_i=U_0u_i^*\quad y=hy^*\quad t=\frac{h^2}{\nu}t^*\quad P_i=\frac{h^2}{\mu_1U_0}\left(\frac{\partial P_i}{\partial x}\right)$$
$$\theta=\frac{T-T_{w2}}{T_{w1}-T_{w2}}\quad \Pr=\frac{\mu_1C_p}{K_1}\quad Ec=\frac{U_0^2}{C_p\left(T_{w1}--T_{w2}\right)} \tag{7}$$

and for simplicity dropping the asterisks, equations (1) to (6) becomes

Region-I

$$\frac{\partial u_1}{\partial t}=\frac{\partial^2 u_1}{\partial y^2}-P_1 \tag{8}$$

$$\frac{\partial \theta_1}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta_1}{\partial y^2}+Ec\left(\frac{\partial u_1}{\partial y}\right)^2 \tag{9}$$

Region-II

$$\frac{\partial u_2}{\partial t}=m\frac{\partial^2 u_2}{\partial y^2}-P_2 \tag{10}$$

$$\frac{\partial \theta_2}{\partial t}=\frac{b}{\Pr}\frac{\partial^2\theta_2}{\partial y^2}+Ec\,m\left(\frac{\partial u_2}{\partial y}\right)^2 \tag{11}$$

where $m=\frac{\mu_2}{\mu_1}$ is the ratio of viscosities and $b=\frac{K_2}{K_1}$ is the ratio of thermal conductivities.

The hydrodynamic and thermal boundary and interface conditions for both fluids in non-dimensional form become

$$\begin{aligned} &u_1(1)=0\\ &u_2(-1)=0\\ &u_1(0)=u_2(0)\\ &\frac{\partial u_1}{\partial y}=m\frac{\partial u_2}{\partial y}\qquad at\ y=0 \end{aligned} \tag{12}$$

$$\begin{aligned} &\theta_1(1)=1\\ &\theta_2(-1)=0\\ &\theta_1(0)=\theta_2(0)\\ &\frac{\partial \theta_1}{\partial y}=b\frac{\partial \theta_2}{\partial y}\qquad at\ y=0 \end{aligned} \tag{13}$$

III. SOLUTIONS

The governing equations (8) to (11) are solved subject to the boundary and interface conditions (12) and (13) for the velocity and temperature distributions in both regions. These equations are partial differential equations that cannot be solved in closed form. However, it can be reduced to ordinary differential equations by assuming

$$u_j(y,t)=u_{j0}(y)+\varepsilon e^{i\omega t}u_{j1}(y)+--- \tag{14}$$
$$\theta_j(y,t)=\theta_{j0}(y)+\varepsilon e^{i\omega t}\theta_{j1}(y)+--- \tag{15}$$
$$P_j(y,t)=P_{j0}(y)+\varepsilon e^{i\omega t}P_{j1}(y)+--- \tag{16}$$

where j-1,2 for region-I and region-II respectively.

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By substitution of equation (14) to (16) into equations (8) to (13), one obtains the following pairs of equations:

Region-I

Non-periodic coefficients

$$\frac{d^2u_{10}}{dy^2} = P_{10} \tag{17}$$

$$\frac{d^2\theta_{10}}{dy^2} + \Pr Ec\left(\frac{\partial u_{10}}{\partial y}\right)^2 = 0 \tag{18}$$

(19)

Periodic coefficients

$$\frac{d^2u_{11}}{dy^2} - i\omega u_{11} = P_{11} \tag{20}$$

$$\frac{d^2\theta_{11}}{dy^2} - i\omega \Pr u_{11} + 2Ec\frac{du_{10}}{dy}\frac{du_{11}}{dy} = 0 \tag{21}$$

Region -II

Non-periodic coefficients

$$\frac{d^2u_{20}}{dy^2} = \frac{P_{20}}{m} \tag{22}$$

$$\frac{d^2\theta_{20}}{dy^2} + \Pr Ec\frac{m}{b}\left(\frac{du_{20}}{dy}\right)^2 = 0 \tag{23}$$

Periodic coefficients

$$\frac{d^2u_{21}}{dy^2} + i\frac{\omega}{m}u_{21} = \frac{P_{21}}{m} \tag{24}$$

$$\frac{d^2\theta_{21}}{dy^2} - i\frac{\omega \Pr}{b}\theta_{21} + \frac{2\Pr Ec}{b}\frac{du_{20}}{dy}\frac{du_{21}}{dy} = 0 \tag{25}$$

The corresponding boundary and interface conditions become as follows:

Non-periodic coefficients

$$\begin{aligned} &u_{10}(1) = 0 \\ &u_{20}(-1) = 0 \\ &u_{10}(0) = u_{20}(0) \\ &\frac{\partial u_{10}}{\partial y} = m\frac{\partial u_{20}}{\partial y} \qquad at\ y = 0 \end{aligned} \tag{26}$$

Periodic coefficients

$$\begin{aligned} &u_{11}(1) = 0 \\ &u_{21}(-1) = 0 \\ &u_{11}(0) = u_{21}(0) \\ &\frac{\partial u_{11}}{\partial y} = m\frac{\partial u_{21}}{\partial y} \qquad at\ y = 0 \end{aligned} \tag{27}$$

Non-periodic coefficients

$$\begin{aligned} &\theta_{10}(1) = 1 \\ &\theta_{20}(-1) = 0 \\ &\theta_{10}(0) = \theta_{20}(0) \\ &\frac{\partial \theta_{10}}{\partial y} = b\frac{\partial \theta_{20}}{\partial y} \qquad at\ y = 0 \end{aligned} \tag{28}$$

Periodic coefficients

$$\begin{aligned} &\theta_{11}(1) = 0 \\ &\theta_{21}(-1) = 0 \\ &\theta_{11}(0) = \theta_{21}(0) \\ &\frac{\partial \theta_{11}}{\partial y} = b\frac{\partial \theta_{21}}{\partial y} \qquad at\ y = 0 \end{aligned} \tag{29}$$

Equations (17) to (25) along with boundary and interface conditions (26) to (29) represent a system of ordinary differential equations and conditions that can be solved in closed form. Since the solutions can be obtained directly, the expressions are not presented. The results are depicted graphically and are discussed in the next section.

IV. RESULTS AND DISCUSSION

In this section representative, flow results for oscillatory flow and heat transfer of two immiscible fluids between two parallel plates are presented and discussed for various parametric conditions. The flow governing equations cannot be solved exactly. However the closed form solutions were found considering the cosine function for frequency parameter on velocity and pressure is assumed. The solutions are depicted graphically in Figs. 1 to 7 for different values of viscosity ratio, periodic frequency parameter and pressure on the flow and thermal conductivity ratio, Prandtl number and Eckert number on temperature field. The parameters are fixed as 1 except the varying one, Pr=0.7, Ec=0.5 and ωt =45^0.

Figure 1 shows that velocity profiles are suppressed for large values of viscosity ratios. The flow profile is large in region-II compare to region-I, and the similar effect observed for different values of viscosity ratio on temperature profile as shown in Fig. 2.

Figure 3 and 4 observed that the variation of periodic frequency parameter ωt on velocity and temperature profiles respectively, as ωt increases the flow increases, the ωt increases temperature profiles is also increases in both the regions, since the solutions are approximated by function of since the solutions are approximated by exponential function of ωt .

Keeping in view the physical model of the flow of two immiscible fluids such as water and oil in petroleum industries, a study is made to know the effect of pressure on the flow as shown Fig.5. We have considered different values of pressure for two fluids separately. For positive values of pressure on upper and lower fluids, the flow is promoting. For positive values of pressure in the lower region and negative values of pressure in the upper region display the effect of

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maximum velocity in region-I. On the other hand if we take negative values of pressure in lower region and positive values of pressure in the upper region also show the maximum velocity in region-I itself but the flow direction is opposite. Assigning negative values of pressure also show the similar effect to that for positive values of pressure except in opposite direction. It is observed that controlling the pressure parameter one can also control the direction of flow, which has immense applications in flow reversal problems.

The effect of thermal conductivity ratio is depicted in Fig. 6. As the ratio increases the magnitude of suppression is large in region-I compared to region-II. This is obvious because the upper plate is maintained at a low temperature compared to region-I.

Figures 7 and 8 display the effect of Prandtl number and Eckert number respectively on temperature filed. It is seen that temperature is increases with increase in Prandtl number as well as Eckert number. Since the values of Prandtl number are very small for liquid and metals and it is very high for highly viscous fluid.

Thus one can conclude that the flow can be controlled by considering different fluids having different viscosities, periodic frequency and applying different pressures.

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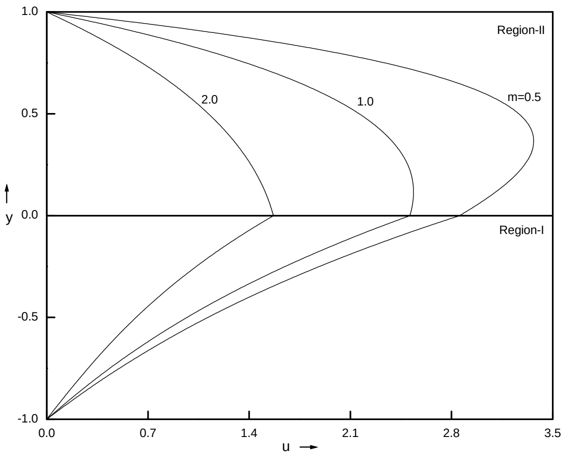

Fig.1 Velocity profiles for different values of viscosity ratio m

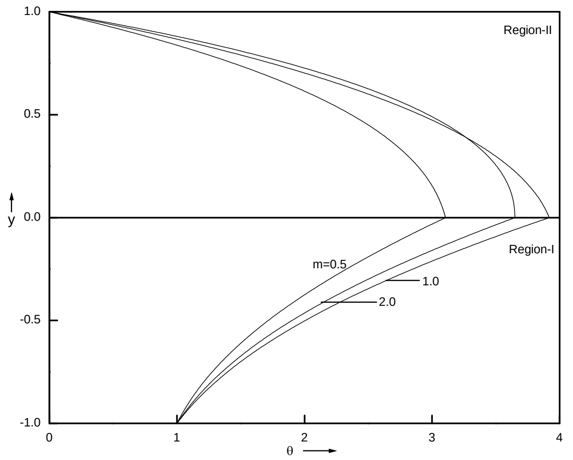

Fig.2 Temperature profiles for different values of viscosity ratio m

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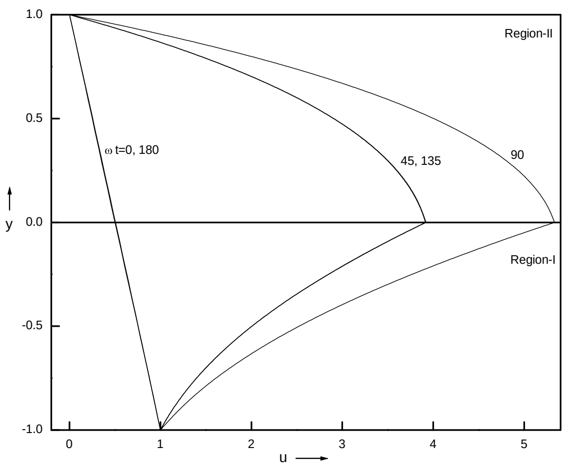

Fig.3 Velocity profiles for different values of periodic frequency parameter ωt

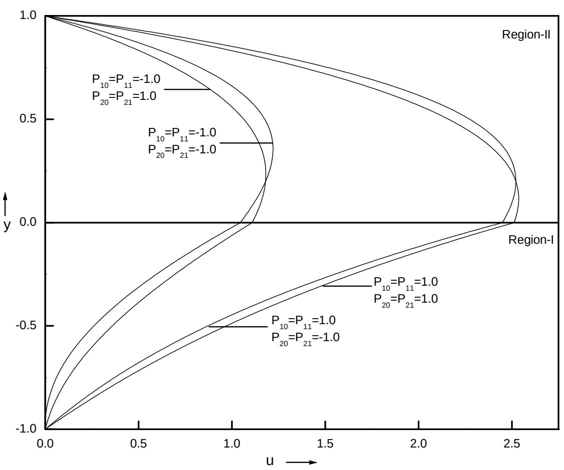

Fig.5 Velocity profiles for different values of pressure P

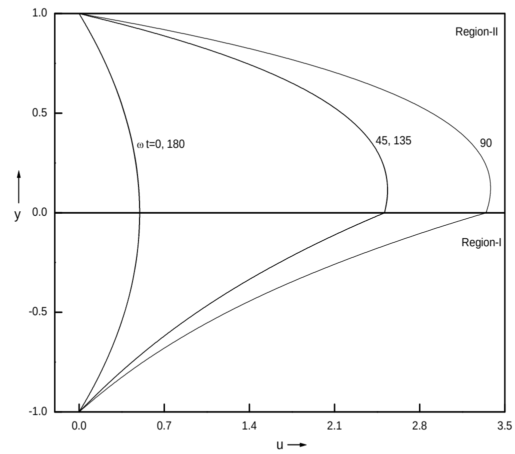

Fig.4 Temperature profiles for different values of periodic frequency parameter ωt

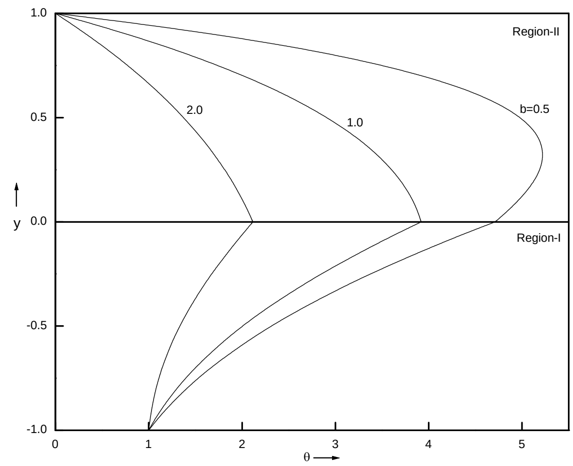

Fig.6 Temperatur profiles for different values of conductivity ratio b

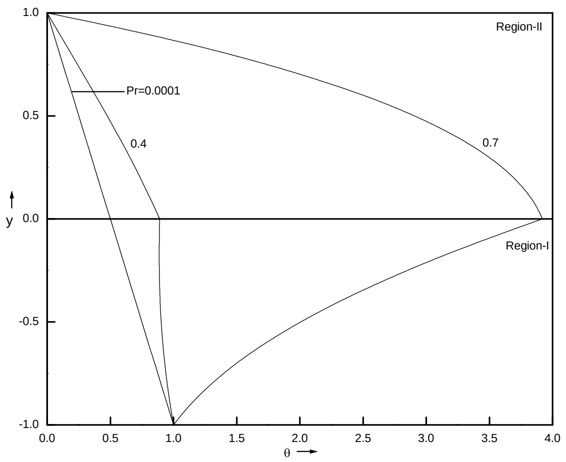

Fig.7 Temperatur profiles for different values of Prandtl number Pr

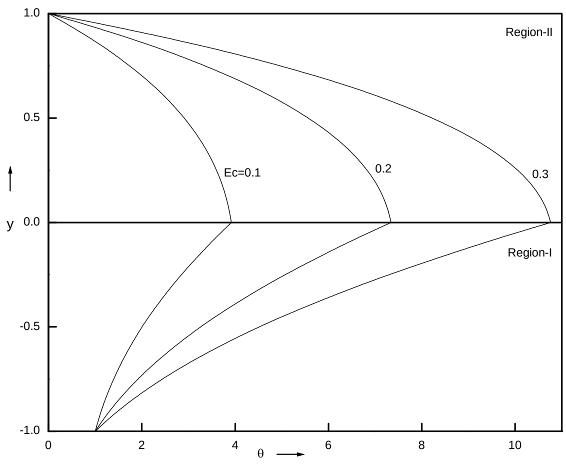

Fig.8 Temperatur profiles for different values of Eckert number Ec

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