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Using Adomian Decomposition Method for Solving Non-linear Initial Value Problem

Irfan Ul Haq, Sandeep Kumar Tiwari, Naveed Ul Haq
School of Studies in Mathematics Vikram University, Ujjain (M.P) 456010, India.

ABSTRACT
Adomian decomposition method (ADM) play a significant role for solving non-linear differential equation as an infinite series converges to the exact solution. In this paper, the Adomian decomposition method is used to solve initial value problems. The numerical results obtained are accurate and approximately close to the exact solution.
Keywords: **Adomian Decomposition Method; Non-linear; Differential Equations; Initial value problems**

INTRODUCTION

There are many numerical schemes for solving different mathematical models eitherit is linear or nonlinear. In literature, various mathematical models have been developed for solving ordinary differential equations, partial differential equations and integral equations. Some classical numerical methods are finite difference method (FDM), finite element method (FEM), Homotopy perturbation method (HPM), Variational iteration method (VIM) and wavelet methods. In this chapter, we are discussing about Adomian decomposition method. The Adomian Decomposition Method (ADM) is applied for solving a several classes of differential, algebraic, difference, integral and integro-differential equations as well.

The Adomain Decomposition Method

Consider differential equation

$$Lu + Ru + Nu = (x) \qquad (1)$$

Where N represents non-linear factor, L represents the highest order derivative which is supposed to be invertible and R represents a linear differential factor, whose orderis less than L.From equation (1), we get

$$Lu = (x) - Ru - Nu \qquad (2)$$

As L is invertible, therefore L^{-1} exists. Multiply Equation (3.2) with L^{-1}, we obtain

$$L^{-1}Lu = L^{-1}(x) - L^{-1}Ru - L^{-1}Nu \qquad (3)$$

After simplification, from (3), we get

$$u = C + Dx + L^{-1}(x) - L^{-1}Ru - L^{-1}Nu \qquad (4)$$

where C and D are constants of integration and can be obtained from the initial or boundary conditions. Adomian method approximate the solution of Equation (1) inthe form of infinite series

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \qquad (5)$$

and decomposing the non-linear operator N as

$$N(u) = \sum_{n=0}^{\infty} A_n \qquad (6)$$

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where A_n represents the Adomian polynomials as discussed in [2,3] and are givenby

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[N(\sum_{n=0}^{\infty}\lambda^i u_i]_{\lambda=0} \quad , \qquad n = 0,1,2,3, \ldots.$$

Putting (3) and (6) into (4) we obtain

$$\sum_{n=0}^{\infty} u_n = C + Dx + L^{-1}G(x) - L^{-1}R\left(\sum_{n=0}^{\infty} u_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right)$$

The recursive relationship is found to be

$$u = G(x)$$

$$u_{n+1} = -L^{-1} - L^{-1}A_n$$

Using the above recursive relationship, we can make solution of u as

$$u = \lim_{n\to\infty} \phi_n(u)$$

Where

$$\phi_n(u) = \sum_{i=0}^{n} u_i$$

Non- linear
A nonlinear system of equation is a set of equation where one or more terms have a variable of degree two higher and there is a product of variable in one of the equation.

Ordinary Differential equation
A differential equation involving derivatives with respect to a single independent variable is called an ordinary differential equation.

Numerical problem
Now for nonlinear initial value problems

$$u''(x) + \frac{2}{3}u'(x) + u^2 = 6 + x^6 \qquad (7)$$

$$u_0 = 0, \quad u'(0) = 0$$

Now equation (7) in operator form

$$Lu = 6 + x^6 - u^6 \qquad (8)$$

Apply L^{-1} on both sides of (8) we find where $L^{-1}(.) = x^{-1}\int_0^x \int_0^x x(.)dxdx$

$$u = L^{-1}(6 + x^6) - L^{-1}u^3$$

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Therefore,

$$u = x^2 + \frac{x^8}{72} - L^{-1}y^3$$

By Adomain decomposition method [6]

We divide $x^2 + \frac{x^8}{72}$ into two points and we using the polynomial series for the nonlinear term, we obtain the recursive relationship

$$u_0 = x^2$$

$$u_{k+1} = \frac{x^8}{72} - L^{-1}(A_k) \tag{9}$$

$$u_0 = x^2$$

$$u_1 = \frac{x^8}{72} - \frac{1}{x}\int_0^x \int_0^x x(x^2)^3 dxdx = 0$$

$$u_{k+1} = 0, k \geq 0.$$

In view of (9) the exact solution is given by

$$u = x^2$$

Example 1

Consider the equation

$$u' - u = xcosx - xsinx, \quad u(0) = 0 \tag{10}$$

Solving as the previous problem, apply L^{-1} to both sides gives

$$u(x) = xsinx - xcosx - xsinx + L^{-1}u(x)$$

The recursive relationship

$$u_0 = xsinx - xcosx - sinx$$

$$u_{n+1} = L^{-1}u_n, \ u \geq 0 \tag{11}$$

so

$$u_1 = L^{-1}u_0 = -xcosx + sinx + xsinx + 2(cosx - 1)$$

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Similar evaluation techniques can be used to other components.it is easily observed that the terms such as $cosx$ and $(-sinx)$ appear in u_0 and u_1 with opposite signs. Canceling these terms from u_0 gives the exact solution $u(x) = sinx$ that can be justified through substitution. It is noteworthy that additional noise terms that are present in other components disappear at the limit.

Example 2

Consider the equation

$$u''(x) + xu'(x) = u - e^{5x}, \quad u(0) = 0\,, u'(0) = 0 \qquad (12)$$

Equation (12) becomes

$$Lu = e^{5x} - u'(x) \qquad (13)$$

Apply L^{-1}on both sides of (13) we get

$$u = L^{-1}(e^{5x}) - L^{-1}u'(x)$$

By Adomian decomposition method we have

$$x^2 + \frac{x^2}{7^2} \dots \dots$$

$$u_0 = x^2$$

$$u_{k+1} = \frac{x^2}{7^2} - L^{-1}(A_k) \qquad (14)$$

$$u_1 = \frac{x^2}{7^2} - \frac{1}{x}\int_0^x \int_0^x e^{2x}\, e^{2x} e^x dxdx$$

$$u_{k+1} = 0, \quad k = 0$$

$$u = e^{2x}$$

Example 3

Consider the equation

$$u''(x) + u'(x) + u^2 = sinx, \quad u(0) = 0, \quad u'(0) = 0 \qquad (15)$$

Equation (15) becomes

$$Lu = sinx - u^2 \qquad (16)$$

Apply L^{-1}on both sides of (16) we get

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$$u = u^{-1}(sinx) - L^{-1}u^2$$

$$u = x^2 + \frac{x^2}{7^2} - L^{-1}u^2$$

By Adomian decomposition method we have

$$u = x^2$$

$$u_{k+1} = \frac{x^2}{7^2} - L^{-1}(A_k) \qquad (17)$$

$$u_1 = \frac{x^2}{7^2} - \frac{1}{x}\int_0^x x\,x'dxdx = 0$$

$$u_{k+1} = 0, \quad k = 0$$

$$y = k$$

Example 4

Consider the nonlinear differential equation of the form

$$u'(x) = u^2 + 1\,, \qquad u(0) = 1 \qquad (18)$$

Apply the Adomian decomposition method, equation (18) we obtain

$$\int_0^x u'(x)\,dx = \int_0^x u^2\,dx$$

This implies

$$u(x) = 1 + \int_0^x u^2\,dx$$

From here, we have

$$u_0 = 1$$

Second term

$$u_1 = L^{-1}(u_0^2) = x$$

Third term,

$$u_2 = L^{-1}(2u_0u_1) = x^2$$

Fourth term

$$u_3 = L^{-1}(u_1^2 + 2u_0u_1) = x^3$$

The solution is

$$u(x) = u_0 + u_1 + u_2 + u_3 + \cdots = 1 + x + x^2 + x^3 + \cdots$$

Which is equivalent to exact solution

111. Conclusion

The primary goal of this work is to solve the initial value problem. We note that the ADM is an powerful technique for solve nonlinear initial value problem. To show the applicability and efficiency of the proposed method, the method is applied to obtain the solution of several examples. It is worth mentioning that the proposed technique is capable of reducing the volume of the computational work as compared to the classical method.

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