Vibration Analysis of a Portable Crane

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***Abstract* - A Portable Crane, specifically designed for small scale Industries. In depth analysis including Bending, Shear, Impact, Fatigue, Torsion, etc. is generally carried out. Analysis of Vibration in a Crane like structure is often neglected as of being lesser importance. The presented paper focuses on the Vibration Analysis of Beam of a Portable Crane due to motion of the load at the end of the Beam.**

Keywords- Vibration Analysis, Portable Crane, Natural Frequency, Momentum

I – INTRODUCTION

Portable Cranes are being widely used in Small Industries, Automobile Garages, Workshops to lift and lower the load at any location with or without the help of electricity supply. Hydraulic and Mechanical lifting winches has been used in past years for the purpose. A brief analysis has been done over years on the Portable Cranes to make them more appropriate for users. Here in this paper, Vibration analysis of a Portable Crane has been carried out. The Load hanging at the end of a Beam of a Crane is subjected to Transverse and Longitudinal motion. These motion, even if the angle of motion is significantly small, results in momentum which becomes Varying Force condition consequently. However in most of the cases , the analysis is neglected but can't be neglected as these Varying Forces gives rise to the Vibrations in the Beam. The frequency of Vibration matching with the Natural Frequency of Vibration of a Beam-Mass Assembly causes Resonance Condition resulting in calamity. Theoretically infinite deflection and failure of Beam follows the Resonance. The paper includes Theoretical Work of deriving equation of vibration in given conditions, relating how it causes the resonance and finding out natural frequency of Vibration for given system. Apart from theoretical work, experimental setup has been done of a real portable crane. Simplified three dimensional diagram has been shown here in Fig 1.1

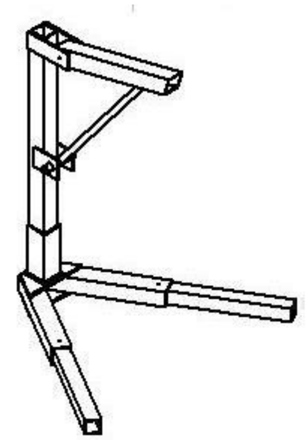

Fig 1.1 Typical Portable Crane used for Experiment

Portable Cranes are hardly to make vibrations in beams under small loads and with the use of a truss. But the equations given here are most useful for the conditions of Girder Cranes and Tower Cranes as the main equation of Deflection and Resonance remains same. The only change getting into it is the change of configuration and Natural Frequency of Vibration accordingly.

II –ANALYSIS

The Portable Crane used for practical analysis consist of many different components starting from a screw to mechanical winches. However, for the theoretical purpose here, Beam is only considered element with the Load hanging with it. Consider a situation shown in Fig- 2.1 , showing pendulum with a mass less string attached with the certain Mass (m). As the Pendulum Bob is given motion in any direction, it starts swinging in the a particular direction to and fro giving rise to fluctuating load as F_0cosø. As the value of ø changes with time the Net Load changes in the string and on the edge of the Beam. In general physics, the condition of a pendulum is considered such that the load is attached to a Fixed Point via string. But here in this case the Cantilever Beam, the point is not fixed, as it deflects and the variable load.

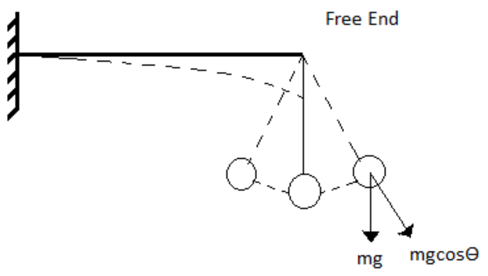

Fig 2.1 Deflection of Cantilever under varying load

Now the situation can be simplified by making an assumption that instead of deflection of the fixed point, the cantilever is fixed and a spring of certain stiffness k is attached to the mass m and given the force of $F_0\cos\o$ as shown in Fig 2.2

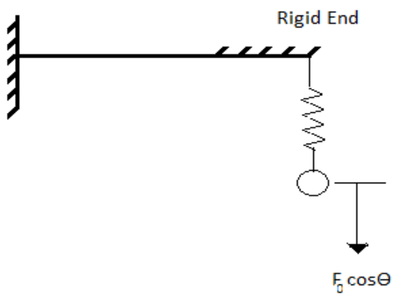

Fig 2.2 Forced Vibrations with Rigid End

The above mentioned problem can be solved as A Forced Vibration of a Single Degree of Freedom system . Assumption has been made that the motion of Pendulum Bob is in a single plane. The differential equation of motion with the force $F_0\cos\o$ can be written as

$m\ddot{x} + kx = F_0\cos\emptyset$ (Equation 1.1)

$$\ddot{x} + \left(\frac{k}{m}\right) = \frac{Fo}{m}\cos\emptyset$$

$$(D^2 + \left(\frac{k}{m}\right))x = \frac{Fo}{m}\cos\emptyset$$

; where F_0= Constant value of Force
K = Spring constant as a function of stiffness of Beam
$D^2 = \frac{\partial^2}{\partial_n^2}$ = Second order differential operator

The above equation can be solved in Complementary Functions and Particular Integrals. Complementary Function solution simply gives the summation of cosine and sine function with given value of Amplitudes A_1 and A_2 .

$$C.F = A_1\cos\sqrt{\frac{k}{m}}\,t + A_2\sin\sqrt{\frac{k}{m}}\,t$$

; where C.F = Complementary Function

However finding out Particular Integrals requires boundary conditions as the equation is second order differential. The boundary conditions can be tabulated as below.

ø (ωt)	Δ	$\ddot{x}$
0	$\left(\frac{Fo}{k}\right)$	0
$\o_{max}$	$\left(\frac{Fo}{k}\right) \pm Xmax$	$\omega^2 x_{max}$

Putting the boundary conditions in 1.1 we get,

$m(-\omega^2 X_{max}) + k\left(\frac{Fo}{k} \pm Xmax\right) = Fo\cos(\omega t)_{max}$

$-m\omega^2 x \pm kx = Fo\cos(\omega t) - Fo$

$$P.I = \frac{Fo(\cos(\omega t) - 1)}{-m\omega^2 \pm k}$$

$$P.I = \frac{\left(\frac{Fo}{m}\right)(\cos(wt) - 1)}{\omega_n^2 - \omega^2}$$

; where P.I = Particular Integral

Summation of C.F and P.I gives final solution of equation and can be written as follows.

$$x = A_1\cos\sqrt{\frac{k}{m}}\,t + A_2\sin\sqrt{\frac{k}{m}}\,t + \frac{\left(\frac{Fo}{m}\right)(\cos(wt) - 1)}{\omega_n^2 - \omega^2}$$

$$x = A_1\cos\omega_n t + A_2\sin\omega_n t + \frac{\left(\frac{Fo}{m}\right)(\cos(wt) - 1)}{\omega_n^2 - \omega^2}$$

;where x= Deflection of Beam
A_1 & A_2 = Arbitrary Constant

Now analyzing the above equation we can simply see that first two terms containing A_1 and A_2 gives normal behavior of sine cosine functions depending upon the natural frequency of vibration. They never take much part in resonance. However the last term has $\omega_n^2 - \omega^2$ in denominator which states that as the value of ω^2 starts approaching ω_n^2 , the deflection starts developing uncontrollably and theoretically reaches the value infinite. This fails the Beam under vibration. That means oscillatory motion frequency should never attain the value of Natural Frequency of a system.

Looking at the nature of the equation one can simply understand that even if the equation proven for Cantilever Portable Cranes, it can be used anyway for many other types of cranes. Take an example of a Girder Crane found in the normal industry to life the heavy equipments, instead of a portable crane. A simply supported beam with a load giving maximum value of bending in the center works on the same equation. Only think that will change in the values is the natural frequency of vibration of the system. With the use of Bending Theory one can estimate stiffness of the beam and furthermore with the use of Rayleigh's Effective Mass Concept, one can find out natural frequency of vibration of beam under given end conditions. Paper by Keith B Smith and William C Shust on Bounding Natural Frequency in Structures gives the value of Natural Frequency of vibration of various structures based on Rayleigh's End Conditions.

In our case of Portable Crane with a Cantilever Beam, the mass is at the end of the Beam giving maximum bending moment at the Joint of Column and Beam. Also depending upon the length, density and dimensions of the Beam, weight of the beam itself can be or can't be neglected (based on the value of mass attached to it). Taking mass of the beam into account we have the equation of natural frequency as follows.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{3EI}{(0.2235\ \rho L + m)L^3}} \quad \text{(Equation 1.2)}$$

;where f_n = Natural Frequency of Vibration
E = Young's Modulus
ρ = Density of Beam Material

If the mass of the beam is too less as compared to mass attached with it, one can simply neglect the term 0.2235ρL and still find near fundamental frequency. The practical had been carried out on the portable crane designed by the author himself. Simple diagram of the Crane Structure has been shown here. This simplifies the study for people to understand it.

For this Portable Crane, design data like cross section dimensions, material properties (Density, Young's Modulus etc), mass attached at the end and calculation had been carried out. Giving the value of Natural Frequency as 0.5955Hz with the mass of 200kg attached at the end. Now this natural frequency of vibration should not be coinciding with the frequency of vibration of the mass hanging with the Beam. From simple use of physics it can be said that frequency of oscillation of pendulum is independent of mass, area of the load, friction of air. (Damping due to string/steel cable has been neglected here) . However, the length of the string/steel cable joining the mass to the beam decides the value of frequency of oscillations. For small oscillations, we have the value of frequency as follows

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Equating the both equations we can easily find out for what value of length of the string phenomena of resonance is supposed to take place. For our practical case the length had been found out to be of 0.7m (for 200kg load). That means by any means it is preferable to stop the lifting the load before the hanging length of the string happens to be 70cms. Other means of that is the use of Truss with the beam as shown in Fig

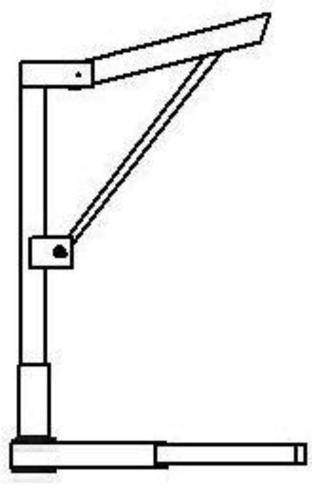

Fig 2.3 Use of Truss to change of Natural Frequency

Truss can be used to increase the load capacity as well as decrease the moment at the joint. A silent advantage of a truss turned out to be to restrict the phenomena of resonance. Use of truss decreases the value of Length of the beam as the load gets transferred to the base of a crane via truss, also decreases the value of numerator in the equation of natural frequency of vibration as given by the equation 1.2. Portable Cranes are not subjected to heavy loads and are hardly to take failure due to infinite deflection. But the analysis of vibration can't neglected in case of Girder or Tower Cranes. Tower Cranes are specifically subjected to wind loads giving possibility of failure by Resonance.

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