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Vibration Analysis of Planetary Gear System

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Abstract

In this work a dynamic model of a planetary gear transmission is developed to study the sensitivity of the natural frequencies and vibration modes to system parameters in perturbed situation. Parameters under consideration include component masses ,moment of inertia , mesh and support stiff nesses .The model admits three planar degree of freedom for planets ,sun, ring, and carrier. Vibration modes are classified into translational, rotational and planet modes .Well-defined modal properties of tuned (cyclically symmetric) planetary gears for linear ,time-invariant case are used to calculate eigensensitivities and express them in simple formulae .These formulae provide efficient mean to determine the sensitivity to stiffness ,mass and inertia parameters in perturbed situation

1. Introduction

Planetary gearboxes are usually used in a wide variety of machinery such as automobiles, helicopters and aircraft engines .Their numerous advantages are high speed reductions in compact spaces, high torque/weight ratio , greater load sharing , diminished bearing loads and reduced noise and vibration. A typical simple planetary gear set consists of a sun gear, a ring gear and a number of identical planet gears (typically 3–6) meshing both with the sun and ring gears. They are well known for their symmetrical structure which allows an equal share of the total external torque applied between the planetary gears, the sun and the ring. However non stationary conditions of system such as overload conditions, torque fluctuation may affect the dynamic behavior of a planetary gear transmission. The inequality of the load distribution, however arises on each planet gear because of random errors of manufacture , assembly and operating conditions. This results in noise and vibration which are key concerns in their applications and drop in efficiency of planetary gear system. In some helicopters planetary gear vibration is the primary source of cabin noise that can exceed 100dB.

Before 1990, the literature on analytical planetary gear dynamics is scarce. Cunliffe et al.[1] studied the eigenvalue problem for a thirteen degree of freedom system and identified the natural frequencies and vibration modes. Botman[2] investigated the vibration modes for a spur planetary gear with eighteen degrees of freedom. The effect of carrier rotation and variation of planet bearing stiffness on the natural frequencies are studied numerically. Frater et al.[3] extended Botman's natural frequency analysis by including unequal planet stiffness. Kahraman[4,5,6,7] presented a series of papers on planetary gear dynamics. He derived a nonlinear, time-varying planar dynamic model[9] and subsequently extended it to three-dimensions and examined the influence of planet phasing on dynamic response[5]. Kahraman[7] reduced his model to a purely torsional one to predict natural frequencies and vibration modes. Parker[8] rigorously proved the effectiveness of using planet phasing schemes to suppress planetary gear vibration. Kahraman [6] showed the effects of mesh/support stiffnesses on the natural frequencies in his torsional model of planetary gears. Investigation of natural frequency and vibration mode sensitivity to system parameters for tuned and mistuned planetary gears was done by J.Lin and R.G.Parker [15].They also investigated the natural frequency spectra and vibration modes of planetary gears to avoid power train resonances.[7]

Analytical investigation of the sensitivities of natural frequencies and vibration modes to stiffness and inertia parameters of general compound planetary gears for both tuned and mistuned are studied in[12]. How the highly structured free-vibration properties of equally spaced planet systems change due to unequal planet spacing was studied in[5].Characterizing the effects of various errors on the dynamic properties of planetary gear system including tooth thickness ,runout and load sharing as the most common gear system manufacturing was done by Parker,R.G. and G.J.Cheon.[13].The effects of some important parameters ,such as the variation of mesh stiffness and static transmission errors on the nonlinear dynamics of a planetary gear system with multiple clearances was studied in[14].The objective of this paper is to analytically investigate the natural frequency and vibration modes of general planetary gears in perturbed situation and comparing them with unperturbed situation .This allows one to find the dominant parameter effecting the perturbation of the system .Investigating the natural frequency spectra one can tune the system to avoid resonance .

2 Modeling and equations

The analysis deals with planar vibration of single stage planetary gears .A Lumped-parameter model used in this work for dynamic analysis is shown in Fig 1. All gears are considered as rigid bodies and component supports are modeled by springs. Each of sun, ring, carrier and Z planets are treated as rigid bodies. Each component has three degrees of freedom: two translational and one rotational. The model is similar to that used by Parker [12], the planet deflections are described by radial and tangential coordinates and more naturally describe the vibration modes. The coordinates illustrated in Fig.1 are used. The carrier, ring and sun translations a_g , b_g, where g=c,r,s and planet translations β_L, γ_L, L = 1,...Z are measured with respect to a rotating frame fixed to the carrier with origin O. The$a_g's$ are directed towards the equilibrium position of planet 1, and β_L, γ_L are the radial and tangential deflections of the z-th planet. The rotating frame rotates with the constant carrier angular speed Ω_c. The rotational coordinates are$J_u = r_u\theta_u$,u = c,r,s,1,... Z, where θ_u is the component rotation; r_u is the base circle radius for the sun, ring and planet, and the radius of the circle passing through the planet centers for the carrier. Circumferential planet locations are specified by the fixed angles $\emptyset_z$, where $\emptyset_z$ is measured relative to the rotating basis vector i so that $\emptyset_1 = 0$. Fig.1 shows a sun planet mesh with masses m_s, m_p and moment of inertiaI_s, I_p. ρ_s is the pressure angle of sun/planet mesh and $\emptyset_{sz} = \emptyset_z - \rho_s$.

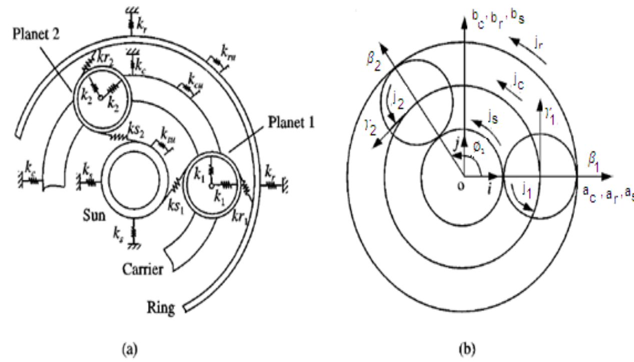


Figure 1. Lumped parameter model of planetary gears and system co-ordinates. (b) All translational co-ordinates a_g , b_g , g =c, r, s and β_L, γ_L, L = 1,...z are with respect to the frame {i, j, k} rotating at constant carrier speed Ω_c .

Free vibration ,time-invariant representation is considered and assumed identical. All planet bearing stiff nesses are equal $k_{pn}=k_p$, all sun-planet mesh stiffnesses are equal $k_{sn}= k_{sp}$ and all ring planet mesh stiff nesses are equal $k_{rn}=k_{rp}$ .The planets are equally spaced and cyclically symmetric structures, therefore can be divided into Z sector having a central angle$\emptyset = \frac{2\pi}{Z}$. Following the matrix derivation of Fox method [16, 17], the general eigenvalue equation for perturbed free vibration is: [19]

$$[M][X][\lambda] - [K][X] = 0 \qquad (1)$$

$$X = (\underbrace{a_c, b_c, j_c}_{carrier} \underbrace{a_r, b_r, j_r}_{sun}, \underbrace{a_s, b_s, j_s}_{ring}, \underbrace{\beta_1, \gamma_1, j_1}_{planet1}, \dots\dots\dots, \underbrace{\beta_n, \gamma_n, j_n}_{planetz})^T$$

de = perturbation in design variable 'e'
[I] = identity matrix
$[K_1]$ = unperturbed stiffness matrix
$[dK^e]$ = perturbation in $[K_1]$ due to change in design variable 'e'.
[K] = perturbed stiffness matrix
$[\lambda_1]$ = unperturbed eigenvalue matrix (diagonal)
$[d\lambda^e]$ = perturbation in $[\lambda_1]$ due to change in design variable 'e'
$[M_1]$ = unperturbed mass matrix
$[dM^e]$ = perturbation in $[M_1]$ due to change in design variable 'e'

[M] = perturbed mass matrix
$[X_1]$ = Unperturbed eigenvector matrix (mode shape)
$[dX^e]$ = perturbation in $[X_1]$ due to change in design variable 'e'
[X] = perturbed eigenvector matrix

M is the inertia matrix and K_b is the bearing stiffness matrix given in Appendix A.. To model the time-varying stiffness associated with changing numbers of teeth in contact at each mesh, K_m can be decomposed into mean and time-varying components. Tooth separation nonlinearity is implicitly included in $K_m(t)$. Manipulating equation (1) we obtain the first order equation

$$[d\lambda^e] + [\rho^e]\,[\lambda_1] - [\lambda_1][\rho^e] = [\xi^e] \qquad (2)$$

The diagonal terms give the eigenvalue perturbations

$$[d\lambda_i^e] = [\xi_{ii}^e]$$

Or the derivatives

$$\frac{\partial \lambda_i^e}{\partial e} \triangleq \frac{d\lambda_i^e}{de} = \frac{\xi_{ii}^e}{de}$$

Considering the case of equal alteration of all sun-planet mesh stiffness i.e., $k_{sz} = k_{sp}$, the perturbed system remains tuned. Eigenvector derivative can be written as [18]

$$dX = (I_n - X_1 e_j^T).(\lambda_1 I_n - M_1^{-1}).(M_1^{-1} - \frac{X_1 X_1^T}{X_1^T M_1 X_1})(dK - \lambda_1.dM^e)X_1$$

Considering the case of equal alteration of all sun-planet mesh stiffness i.e., $k_{sz} = k_{sp}$,the perturbed system remains tuned. The derivatives of the mass and stiffness matrices with respect to k_{sp} are:[5,12]

$$M' = 0 \qquad K' = \frac{1}{k_{sp}} \sum_{z=1}^{Z} \begin{bmatrix} \ddots & & & & \\ & K_{s1}^z & \cdots & K_{s2}^z & \\ & \vdots & \ddots & \vdots & \\ & (K_{s2}^z)^T & \cdots & K_{s3}^z & \\ & & & & \ddots \end{bmatrix} = \sum_{z=1}^{Z} \frac{\partial \boldsymbol{K}}{\partial k_{sz}}$$

All sub matrices of $\frac{\partial \boldsymbol{K}}{\partial k_{sz}}$ are zero except the four that involve k_{sz} . For rotational mode ,the Eigen sensitivies are obtained as below

$$\frac{\partial \lambda_i}{\partial k_{sp}} = \sum_1^z (\delta_{sz}^i)^2 \qquad (3)$$

$$\frac{\partial X_i}{\partial k_{sp}} = \sum_{k=1,k\neq i}^{N} \sum_{z=1}^{Z} \frac{\delta_{sz}^i \delta_{sz}^k}{\lambda_i - \lambda_k} X_k$$

Where δ_{sz}^i is the spring deformation of the sun-planet z mesh in mode X_1 given by

$$\delta_{sz}^i = y_s \cos(\emptyset_z - \rho_s) - x_s \sin(\emptyset_z - \rho_s) - \gamma_L cos\rho_s - \beta_L sin\rho_s + J_s + J_n$$

The rotational mode property implies that all sun-planet mesh deformations are equal i.e. $\delta_{sz}^j = \delta_{sz}^i$, so equation (3) becomes

$$\frac{\partial \lambda_i}{\partial k_{sp}} = \sum_1^z (\delta_{s1}^i)^2$$

The Eigen sensitivity to masses (m_s, m_r, m_c, m_p) and moment of inertia $(I_s, I_r, I_c, I_p)$for sun ,ring ,carrier and planets for a tuned perturbed system are considered ,the eigenvalue derivatives for the three types of modes are

$$\frac{\partial \lambda_i}{\partial m_g} = -\lambda_i(x_g^2 + y_g^2) = -\frac{2}{m_g} T_g$$

$$\frac{\partial \lambda_i}{\partial I_g} = -\frac{\lambda_i}{r_g^2} u_g^2 = -\frac{2}{I_g} T_{gj} \qquad g = r, s, c$$

$$\frac{\partial \lambda_i}{\partial m_p} = -\lambda_i \sum_{L=1}^{Z} (\beta_L^2 + \gamma_L^2) = -\frac{2}{m_p} \sum_{L=1}^{Z} T_L$$

$$\frac{\partial \lambda_i}{\partial I_g} = \frac{-\lambda_i}{r_p^2} \sum_{L=1}^{Z} u_L^2$$

T_g = modal translational kinetic energy of ring, sun and carrier

T_{gj} = modal rotational kinetic energy of ring, sun and carrier

Planet Mode

Characteristics of planet mode in perturbed situation are:

- Carrier,ring and sun have zero translation and rotation i.e. $F_g = [0,0,0]^T$
- Natural frequencies has multiplicity Z =1 in perturbed situation.

Therefore planet mode will get the form

$$\zeta = [0,0,0, f_1, f_1 F_1, f_2 F_1, \ldots\ldots\ldots\ldots\ldots, f_z F_1]^T$$

f_z is a multiple of $f_1 = 1$, then

$$\Sigma\omega_z \sin\emptyset_z = 0 \quad \Sigma\omega_z \cos\emptyset_z = 0 \quad \Sigma\omega_z = 0 \qquad (4)$$

In planet modes the sun ,carrier and ring have zero rotation and translation.The Eigen solution properties is illustrated using the nominal system parameters are listed in table 1.

Table1- Nominal system parameters of the planetary gear

	Sun	Ring	Carrier	Planet
Mass (kg)	0.4	2.35	5.43	0.66
I/r^2 (kg)	0.39	3.00	6.29	0.61
Base diameter (mm)	77.42	275.03	177.8	100.35
Teeth number	27	99		35
Mesh stiffness (N/m)	$k_{sp} = k_{rp} = k_m = 5x10^8$			
Bearing stiffness (N/m)	$k_p = k_s = k_r = k_c = 10^8$			
Torsional stiffness (N/m)	$k_{ru} = 10^9 \quad k_{su} = k_{cu} = 0$			
Pressure angle (°)	$\rho_s = \rho_r = \rho = 24.6$			

The variation of eigenvalues in perturbed situation for two different conditions are shown below:

1- *M* is assumed constant , K is assumed variable .(Figure- 2)

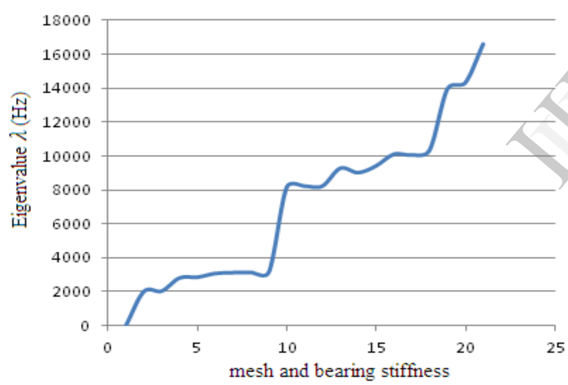


Figure - 2

2- M is assumed variable , K is assumed constant .(Figure – 3)

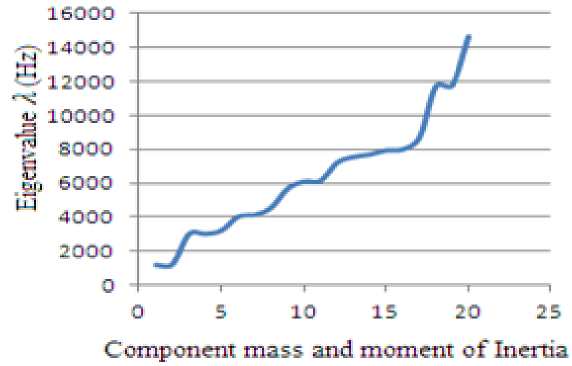
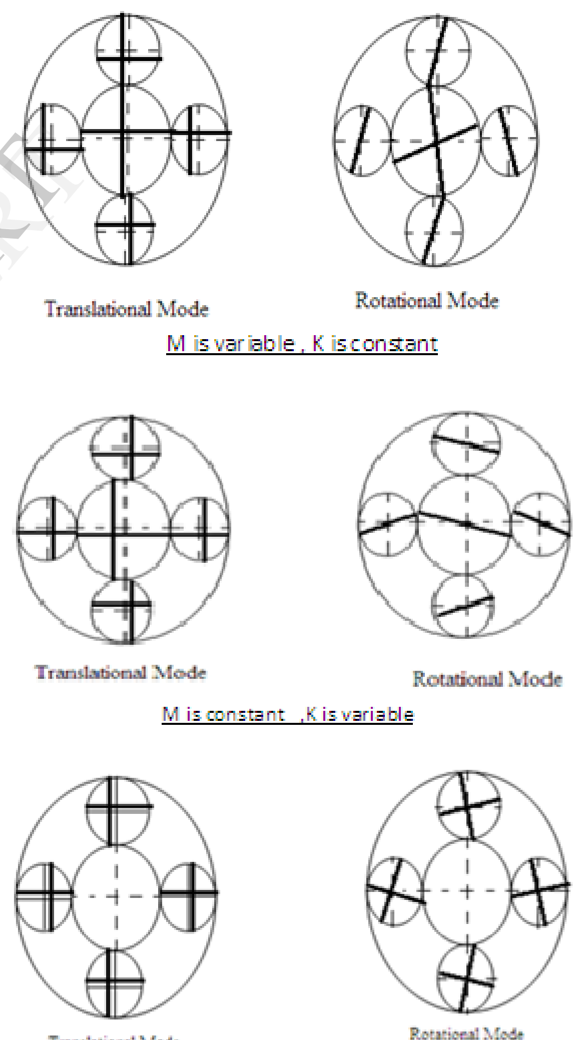


Figure - 3

Typical vibration modes for equally spaced planets are shown below. The movements of the ring and carrier are not shown in figure 4 .Solid lines are perturbed situation and dotted lines are unperturbed situation.



Planet

Figure4

Conclusion and future work

The present study identifies the parameters affecting the natural frequencies of a general planetary gear system. The results can be applied to specific configurations (fixed sun, carrier or ring).For a tuned system a well-defined structure to the natural frequencies are obtained. The results are:

Sensitivity to mass, moment of inertia and stiffness in perturbed situation is shown in Figure2,3. It is observed that fluctuation in all the natural frequencies presented in figure 2 are on higher side than those in figure 3.It is concluded that stiffness component dominates prominently than mass on the natural frequency From figures 2 and 3 it is observed that in perturbed tuned situation with degenerate natural frequency multiplicity m is 1.Figure 4 shows typical vibration modes for equally spaced planets. For planet modes ,motion of ring ,carrier and sun is zero. In gear systems satisfying relation (4),translation and rotation modes have structured properties. Planet motions are a multiple of the motion of first planet. For sun, carrier and ring the rotational modes have pure rotation. All planets move in identical form. Their translational modes have pure translation.For vibration modes reduced-order Eigen value problems are achieved. Sensitivity to mass, moment of inertia and stiffness are investigated for perturbed situation .

The scope for future work is to apply the method used in this paper for various mass components and stiffnesses using relevant materials for manufacturing gears and propose the one having less influence on the vibration characteristic of the system.

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APPENDIX – A

$$M=\mathrm{diag}(M_c,M_r,M_s,M_1,\ldots,M_Z)$$

$$M_g = \mathrm{diag}(m_h,m_h,I_h/r_h^2), \qquad L=c,r,s,1,\ldots,Z$$

$$K_b=\mathrm{diag}(K_{cb},K_{rb},K_{sb},0,\ldots,0)$$

$$K_{gb} = \mathrm{diag}(k_{gx},k_{gy},k_{gu}) \qquad g=c,r,s$$

$$K_m = \begin{bmatrix} \Sigma Kc1^Z & 0 & 0 & Kc2^1 & Kc2^2 & \ldots & Kc2^z \\ & \Sigma Kr1^Z & 0 & Kr2^1 & Kr2^2 & \ldots & Kr2^z \\ & & \Sigma Ks1^Z & Ks2^1 & Ks2^2 & \ldots & Ks2^z \\ & & & Kpp^1 & 0 & \ldots & 0 \\ & & & & \ldots & \vdots & \\ & \text{symmetric} & & & & & \\ & & & & & & Kpp^z \end{bmatrix}$$

$$K_{pp}^z = K_{c3}^z + K_{r3}^z + K_{s3}^z$$

$$K_{c1}^z = k_{pz}\begin{bmatrix} 1 & 0 & -\sin\emptyset_z \\ & 1 & \cos\emptyset_z \\ \text{symm.} & & 1 \end{bmatrix}$$

$$K_{c2}^z = k_{pz}\begin{bmatrix} -\cos\emptyset_z & \sin\emptyset_z & 0 \\ -\sin\emptyset_z & -\cos\emptyset_z & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$K_{c3}^z = \mathrm{diag}(k_{pz},k_{pz},0)$$

$$K_{r1}^z = k_{rz}\begin{bmatrix} \sin 2\emptyset_{rz} & -\cos\emptyset_{rz}\sin\emptyset_{rz} & -\sin\emptyset_{rz} \\ & \cos 2\emptyset_{rz} & \cos\emptyset_{rz} \\ \text{symmetric} & & 1 \end{bmatrix}$$

$$K_{r2}^z = k_{rz}\begin{bmatrix} -\sin\emptyset_{rz}\sin\rho_s & \sin\emptyset_{rz}\cos\rho_s & \sin\emptyset_{rz} \\ \cos\emptyset_{rz}\sin\rho_s & -\cos\emptyset_{rz}\cos\rho_s & -\cos\emptyset_{rz} \\ \sin\rho_s & -\cos\rho_s & -1 \end{bmatrix}$$

$$K_{r3}^z = k_{rz}\begin{bmatrix} \sin 2\rho_r & -\cos\rho_r\sin\rho_r & -\sin\rho_r \\ & \cos 2\rho_r & \cos\rho_r \\ \text{symmetric} & & 1 \end{bmatrix}$$

$$K_{s1}^z = k_{sz}\begin{bmatrix} \sin 2\emptyset_{sz} & -\cos\emptyset_{sz}\sin\emptyset_{sz} & -\sin\emptyset_{sz} \\ & \cos 2\emptyset_{sz} & \cos\emptyset_{sz} \\ \text{symmetric} & & 1 \end{bmatrix}$$

$$K_{s2}^z=k_{sz}\begin{bmatrix} \sin\emptyset_{sz}\sin\rho_s & \sin\emptyset_{sz}\cos\rho_s & -\sin\emptyset_{sz} \\ -\cos\emptyset_{sz}\sin\rho_s & -\cos\emptyset_{sz}\cos\rho_s & \cos\emptyset_{sz} \\ -\sin\rho_s & -\cos\rho_s & 1 \end{bmatrix}$$

$$K_{s3}^z=k_{sz}\begin{bmatrix} \sin 2\rho_s & \cos\rho_s\sin\rho_s & -\sin\rho_s \\ & \cos 2\rho_s & -\cos\rho_s \\ \text{symmetric} & & 1 \end{bmatrix}$$

$$\emptyset_{rz} = \emptyset_z - \rho_s \qquad \emptyset_{rz} = \emptyset_z + \rho_s$$