# Vibration Analysis Of Rotating Cantilever Beam By Simulating Foreign Object Damage

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## ABSTRACT

Gas turbine and aero engine components are cost intensive due to the complexity of design and fabrication and the exotic alloys used. Foreign Object Damage (FOD) during operation can significantly reduce the life of the critical parts like compressor and fan blade airfoils. In addition it can change the vibration characteristics, particularly natural frequencies of the component, which can result at high vibratory stress levels and reduce the high cycle fatigue life. As the blades operate at high pressure and temperature, the damage caused can significantly alter the overall stress and vibration pattern of the blades. The severity of FOD-induced notches can vary significantly, depending on their geometry and location on the aerofoil. Though the geometry of FOD's are irregular shape the Testing and analysis is carried out with a specific shape and size of the cut out on the blade aerofoil which can generate a known Stress Concentration Factors (SCF). The standards set by the industry stipulate an allowable stress concentration factor of 2.0 for the compressor blades. Simulating these SCF can be complicated issue on a specific aerofoil. Finite Element analysis is carried out on a rectangular cantilever plate for which closed form solution is available. Further the study is extended by idealizing the problem of cantilever rectangular plate with a turbine blade both in static and rotating conditions with various possibilities of notch parameters used for converting the nicks and dents to notch of known geometry and without the notches. Modal analysis is performed to determine structural characteristics in the form of modal parameters, resonant frequencies and associated mode shapes.



## INRODUCTION

FOD is a major source for generation of stress concentration on blade airfoils on jet engines. It can range from a scratch or dent to a deep gouge on the blades. Early stage compressor blades are prone to HCF failure initiating from FOD on or near the leading edges. FOD is usually distributed along the leading edges of the blades ranging from the platform toward the tip, with a higher concentration of FOD near the higher velocity tip. A complex and irregular distribution of minor indentations covers the surface, including leading edge impacts. The damage caused by foreign objects often in the form of a geometric discontinuity like a notch. However the presence of residual stresses and sub-structural damage in regions adjacent to the notch prohibit the use of simple notch analyses. FE analysis is used to estimate the stress concentration effect of the geometry of the notch. For this purpose, the compressor aerofoil blade is idealized into simple rectangular cantilever plate for FEM study. The complexity of the problem is reduced to a simplex problem by assuming the aerofoil section to a flat rectangular plate. Because of the non- presence of aerofoil section, analysis is so simple for finding the stress concentration effect of the geometrical notch made. To analyze the stress concentration effect of FOD, different notches are made by varying the notch dimensions viz. depth, radius of the notch and the location of the notch on the trailing edge of the rectangular plate.

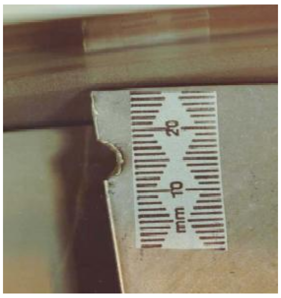


**Fig 1: Foreign object damaged compressor blade**

Foreign object damaged compressor blades with nicks, dents and cracks can be ground into the smooth curved cutouts to reduce the stress intensification and stress concentration factors with a high speed grinding wheel. In doing so, depending on the extent and type of damage, designer has a requirement to estimate the size and shape of the cutout to be made on the aerofoil that generates the known SCF. With the estimated values of SCF for different geometries of cut using FEM further a modal analysis has been conducted to assess the variation of frequencies for different geometry of cut outs with known values of SCF. To aster the correctness of the solution, the finite element analysis is carried out on a rectangular cantilever plate for which literature is available. Further the study is extended to an idealized problem of cantilever rectangular plate both in static and rotating conditions with and without the notches.

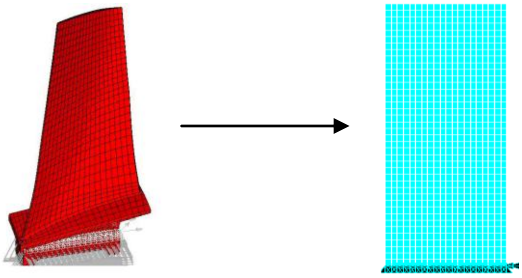


**Fig 2: Idealization of the compressor blade to rectangular cantilever beam**

On blade FOD is inspected by using the bore scope and measured using microscope as shown in figure-2. The maintenance engineer on occasions is required to take emergency decisions on operation of engines with FOD resulting in stress concentration on leading edge of compressor blades. In some situation where a quick fix solution is needed to suitable high speed grinding of FOD which generates high stress levels of stress intensities, such as nick, dent or crack to a reduced smoothened cutout of known geometries with contour shapes of semicircular or U-notch, which generates the known SCF. The typical allowable stress concentration factor (SCF) of the order of 2.0 for the compressor blades. Parametric studies of typical compressor blade with semicircular and U-notch cutouts along the leading edge from the root to the tip, by varying notch geometry as well as its location heights on the leading edge.

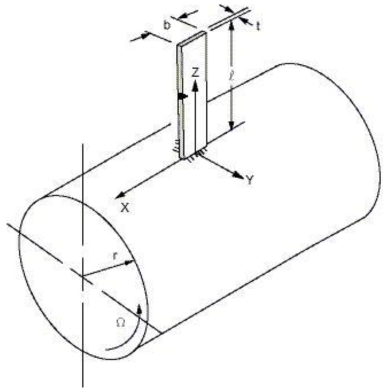


**Fig 3: Rotating cantilever plate with edge notch**

## MATHEMATICAL FORMULATION

In a normal mode, each element of the beam oscillates up and down at the same frequency. The amplitude of oscillation varies along the beam as shown below for each of the first four normal modes (with Y(x) exaggerated). Y(x) shows the shape of the beam at the extreme of the oscillation when all points on the beam are instantaneously at rest. All the points also go through zero displacement at the same time.

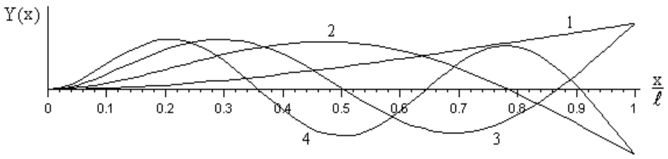


**Fig 4: First four normal modes for cantilever beam**

Y(x) is the vertical displacement relative to the fixed end, and the horizontal scale is expressed as a fraction of the full length $\ell$ of the beam. In formal terms, a cantilever is a beam that is constrained to have Y(x) = 0 and dY(x)/dx = 0 at x = 0 and the other end free. The parameters that determine the shape, Y(x), are as follows:

The theoretical expression for the displacement of the free end of a static cantilever is:

$$Y(\ell) = \frac{\rho A g}{8EI}\ell^4 \qquad \ldots (1)$$

If, in addition, a load of mass m is suspended from the free end the displacement is increased to

$$Y(\ell) = \frac{\rho A g}{8EI}\ell^4 + \frac{mg}{3EI}\ell^3 \qquad \ldots (2)$$

In mode 1, all parts of the beam move, except the fixed end. In mode 2 there is a stationary point or node away from the end (at $x/\ell = 0.784$). In mode 3 there are two nodes, and so on.

The Natural frequency of flexural modes is given by $\omega_n = \beta^2_n \sqrt{(EI / \rho L)}$
Where $\beta$ is a constant from the separation of variables technique, $\beta = \{(2n-1)/2\} * \pi$

The angular frequencies of the normal modes, are given by

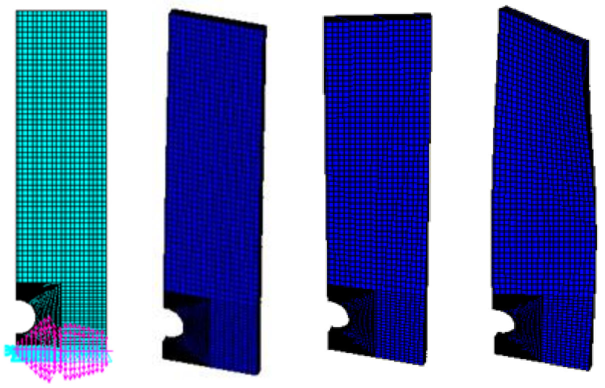
$$\omega_n = 2\pi\upsilon_n = \frac{\theta_n^2}{\ell^2}\sqrt{\frac{EI}{\rho A}} = \theta_n^2\sqrt{\frac{EI}{\rho A \ell^4}} \qquad \ldots (3)$$



## FINITE ELEMENTAL FORMULATION

The finite element analysis of the rectangular plate subjected to rotation at constant speed is analyzed to study modal characteristics having semicircular and U- notches from the root to the tip of the blade along the leading edge of the blade for first Flexural mode and first Torsion mode. A Block Lanczos Modal Analysis is applied to the model to solve for the frequencies and mode shapes.

The model of the cantilever geometry built up with solid 3D brick elements. The element is defined by eight nodes having three degrees of freedom at each node. The dimensions of the rectangular plate taken are length 200 mm, width 50 mm and thickness 5 mm. The radius of rotation of the plate is 250 mm from x axis. The sizes of the semicircular notches are of radius 1, 2, 3, 4, 5, 6, 8, 9and 10 mm at location heights of 50, 80, 100, 120, 150 and 180 mm from the root to the tip of the blade along the leading edge of the plate. The U-notches are analyzed for a depth b = 1 and 2 mm with mentioned radius of notches and from the location heights mentioned. The cantilever plate is rotating at a constant speed of 15000rpm about the x-axis and hence having an angular velocity of 1570.796 rad/sec i.e. N= 15000 rpm and the gravity is acting in the negative y direction. The cantilever plate is assumed to linear elastic isotropic material. The material taken for the cantilever plate is assumed to be made of steel of Young's Modulus 2.1 x $10^5$ MPa, Density 7850 Kg/mm$^3$ and Poisson's ratio 0.3. The plate is fixed in all the directions i.e. UX, UY and UZ at Length L=0 of the plate i.e. at 250 mm radius of the rotation as shown in fig-5.



(a) (b) (c) (d)

**Fig 5: (a).Meshing and B.Cs of the cantilever beam with notch, (b). 1F mode displacement, (c). 1T mode displacement, (d). 2T mode displacement**

## RESULTS AND DISCUSSIONS

### a. Rotating beam with Semicircular Notch

Finite Element Analysis is carried out on the rectangular cantilever beam in rotation with Semicircular notch of radius ‘r’ at a height of ‘H’ from root. Un-notched frequencies for 1F and 1T mode are 372.084Hz and 906.12Hz. It is observed that frequency is constant for smaller radius of notch from the root to the tip of the blade. As radius of the notch increases up to certain level of height 120mm from the root, frequency of the rotating cantilever beam decreases drastically as shown in fig-6&7. When the notch is nearer to the root, more the stress concentration will be developed due to the centrifugal effect of the rotating cantilever beam. This is the considerable problem to be discussed else it can destroy the whole engine and aircraft. In 1F mode, for maximum of 10mm notch radius and notch location H50, the percentage of decrease in frequency is of 0.17%. It slowly improves to 0.055% decrease at H120 and as the notch is away this position there will be increase in frequency of the cantilever beam by 0.65% more at H180 with respect to un-notched. But where in case of 1T mode the percentage of decrease in frequency is 4.41% for H50, 0.27% for H120 and there will be increase in frequency from H120 to H180 of 3.32% with respect to un-notched.

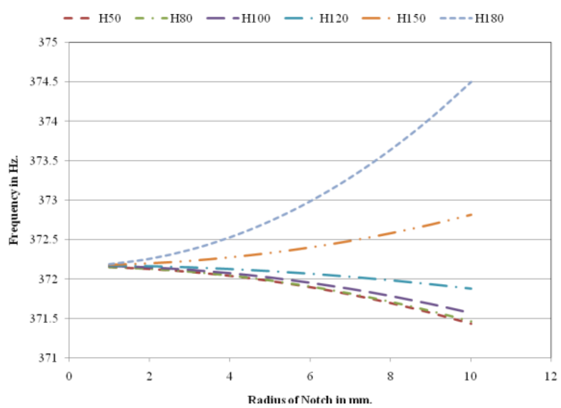



**Fig 6: 1F Modal Frequency for different Notch Radius at different Heights**

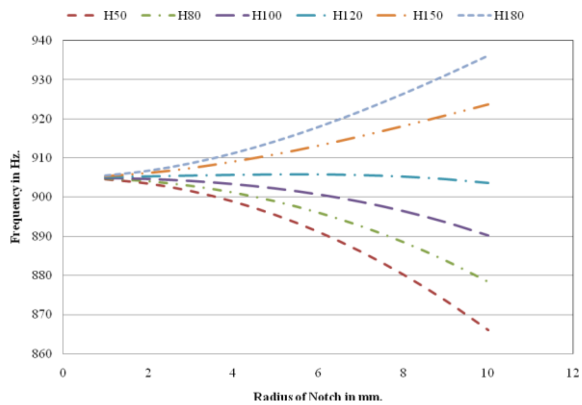



**Fig 7: 1T Modal Frequency for different Notch Radius at different Heights**

It is well sufficient to discuss the 1F and 1T mode as if 2F and 2T is considered the frequency of the cantilever beam is much ahead of its working. Normally turbine blades are subjected to centrifugal force and gas bending, the centrifugal force very much greater than the gas bending load. There won’t be much possibility of failure due to flexural mode because of it centrifugal force. As the turbine blades are in aerofoil shape in structure it majorly failure due to torsion effect. So it should be well defined to know the torsion frequency mode characteristics of the damaged turbine blade model to define the turbine blade is fit for working without affecting the operation of the engine.

### b. For U-type edge notches

Normally damaged portion of the turbine blades subjected to FOD are not in semicircular or U-shape, it is made of semicircular in case of small damage else it is framed to the shape of U-notches and more of the cases come across are U-notch shape standardized. The analysis is carried out for U-notches of radius ‘r’ by varying the depth of the notch for b = 1, 2 mm at various location of notch ‘H’ same as that of the semicircular notches and compared with the frequency of un-notched rectangular cantilever beam.

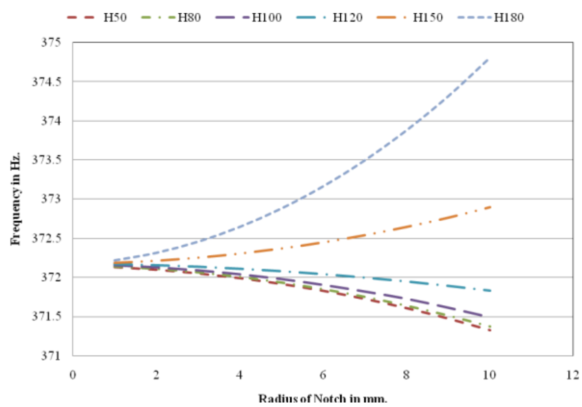



**Fig 8: 1F Modal Frequency for different notch radius at different Heights with b = 1mm**

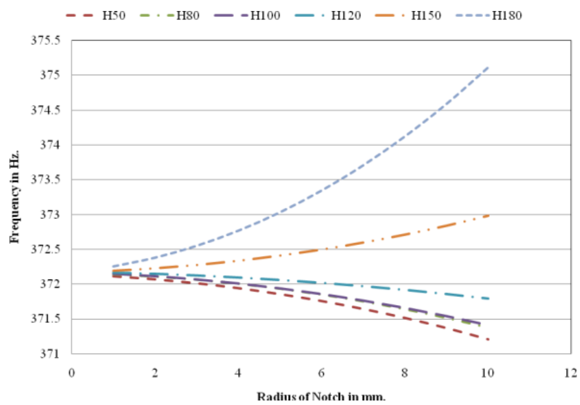



**Fig 9: 1F Modal Frequency for different notch radius at different location heights with b=2mm**

It is observed that the frequency of the U-notched rectangular cantilever beam in rotary motion is having a 1F frequency for maximum of 10 mm notch radius and depth 1mm of 0.2% less at a height of H50, but where in case of it slowly increases to 0.067% decrease and thereby increases by 0.73% more with respect to un-notched. Similarly for the depth of 2mm of 0.236% less at H50 and 0.078% less at H120 from their will increases by 0.815% more at H180 from its un-notched frequency. For the increase in depth of the notch and increasing the radius of the notch the frequency of the turbine blade will decrease when notch is near to the root and increase when it is away from the root of the turbine blade. Similarly for notch depth of b=1mm, 1T mode the frequency is of 5.27% less at H50, 0.58% less at H120 and from there increases to 3.52%more for at with respect to un-notched. But for 2mm notch depth 6.16% less at H50, leads to 0.88% less at H120 and from there after 3.77% more at H180 with respect to un-notched.

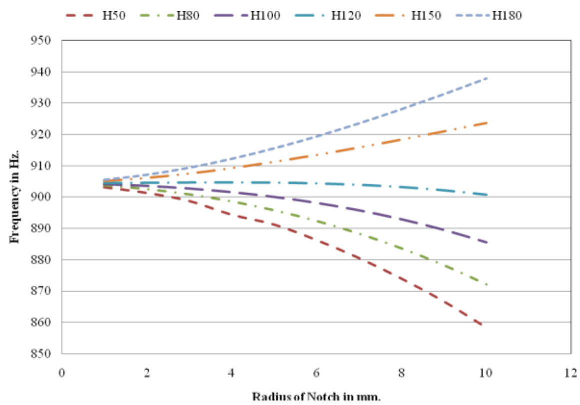



**Fig 10: 1T Modal Frequency for different notch radius at different location Heights with b=1 mm**

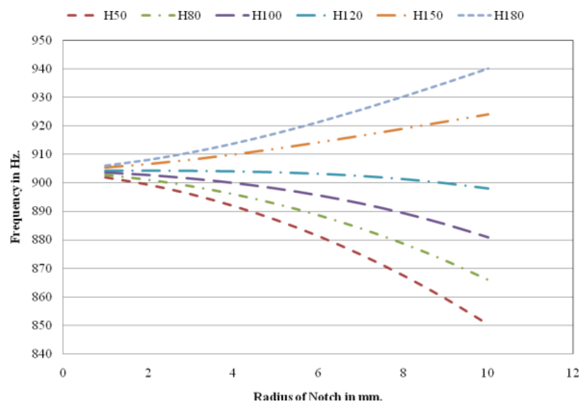



**Fig 11: 1T Modal Frequency for different notch radius at different location heights with b=2mm**

From the above discussion it can be observed that the variation of vibration characteristics of the turbine blade should be analyzed and tabulated or plotted for the region near to the root of the turbine blade where the sudden drop in frequency takes place for larger radius or depth of the notch due to stress concentration. But where in case of rise in frequency there won't be any problem and there won't be much affect on the working of the turbine blade. This will be helpful for the maintenance engineer to identify the life of the FOD damaged turbine blades, this is also cost effective.



## CONCLUSIONS

The Natural frequency and Mode shapes of rectangular cantilever beam rotating at constant speed (15000 rpm) has been carried out for various possibilities of notch parameters used for converting the nicks & dents to notch of known geometry and thereby predicting the modal values. An attempt has been made to address the effect of notches on cantilever beam which is an idealization of blades subjected to FOD. As the notch location moves from root to tip, the frequency increases. The frequency of the cantilever plate under rotation increases as notch diameter is increased, in case of increase in location height from the root frequency decreases. Larger notch sizes are not possible at the mid section and tip of the aerofoil blade because of the high increase in modal frequency. Large notches are not possible at root of the blade as the centrifugal field get altered drastically which can affect the overall strength of aerofoil adversely. This methodology can be extended to Aerofoil as the Finite Element Method results are in good agreement with the closed form solution.

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