Vibration Analysis Of Thick Rectengular Plate With Inline Support Using FEA

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Abstract: *In today's world, moderately thick plates are extensively used in modern structures. Analysis of these plates is of great importance for design engineers. The present study is focused towards the dynamic characterization of moderate thick rectangular plate with inline support at various location by finite element simulation. The FEA based Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes. The modal analysis performed for newly designed system to know the natural frequencies of system. It compared with working frequency of system to avoid resonance and failure corresponds to that resonance for safe design. So the influence of inline support location on frequency parameters was shown graphically.*

Keywords:free vibration, moderate thick plate, Mindlin plate theory, inline support.

I. INTRODUCTION

Vibration, in mechanics, is the to and fro motion of an object. Vibration can be exploited for useful tasks, such as the use of a vibrator to massage the body, to compact loose soil, to increase the workability of wet concrete and to shake sugar, pepper and salt from their containers. On the other hand, vibration can cause discomfort for people and problems for machines. Too much vibration can cause people to loose concentration and to fall sick. In machines, vibration causes wear and tear and can even cause the malfunctioning of the machine. So The vibration design aspect is even more important in micro machines such as electronic packaging, micro-robots, etc. Because of their enhanced sensitivities to vibration. Moderately thick plates are extensively used in modern structures. Analysis of these plates is of great importance for design engineers. The solution of the flexural vibration depends on the boundary conditions of the plate. Rectangular plates are commonly used as structural components in many branches of modern technology, namely, mechanical, aerospace, electronics, marine, optical, nuclear and structural engineering. Thus, the study of their free vibration behaviour is very important to the structural designers. [1]

In view of intensive use of plate made from metal alloy and understanding of their vibratory characteristics are very important for designers. The study of natural frequencies and mode shape of these plates is required from the thin plate applications to thick plate applications. A plate is typically considered to be thin when the ratio of its thickness to representative lateral dimension (e.g., circular plate diameter, square plate side length) is 1/20 or less. In fact, most plates used in practical applications satisfy this criterion. This usually permits one to use classical plate theory(CPT) or thin plate theory to obtain a fundamental (i.e., lowest) frequency with good accuracy. However, the second frequency of a plate with thickness ratio of more than or equal to 1/20, determined by classical plate theory(CPT), will not be accurate. It will be somewhat too high. And the higher frequencies will typically be much too high too high to be of practical value. The inaccuracies described above are largely eliminated by use of the Mindlin Plate Theory(MPT), for it does include the effects of additional plate flexibility due to shear deformation, and additional plate inertia due to rotations (supplementing the translational inertia). Both effects decrease the frequencies. There are still other effects not accounted for by the Mindlin Plate Theory (e.g., stretching in the thickness direction, warping of the normal to the midplane), but these are typically unimportant for the lower frequencies until very thick plates are encountered. For such situations a three-dimensional analysis should be used.The basic assumption of Mindlin plate theory is that a straight line originally normal to the plate middlesurface is constrained to remain straight but not generally normal to the middle surface after deformations. The plate having twenty one possible boundary conditions (a combination of clamped, simply supported and free boundary condition) for the vibration analysis using the same plate geometry. The present study is to determine the dynamic characterization of moderate thick plate for the case of C-F-F-F boundary condition with various inline support location.

II. ASSUMPTIONS FOR MODERATE THICK PLATE

Consider a flat, isotropic, rectangular thick plate of uniform thickness (h), length (a), width (b) ,modulus of elasticity (E), Poisson's ratio (v), shear modulus $(G = E/2(1+v))$, and density per unit volume, (ρ), oriented.so that its mid-plane surface contains the x_1 and x_2 axis of a Cartesian co-ordinate system, (x_1, x_2, x_3

), as shown in Figure 1. The frequency parameters which are calculated using the exact characteristic equations are obtained for this case, which can cover a wide range of plate aspect ratios η and relative thickness ratio δ. where $\eta = a/b$ and $\delta = h/a$ [3]

Mindlin's theory is derived based on the following assumptions:[2]

- Straight lines normal to the xy-plane before deformation remain straight and with unchanged length, but not compulsory normal to the mid-surface after deformation.
- Deformation of mid-plane linear elements is neglected;
- Stress component σ_z is too small regarding the other two components, so it can be neglected;
- Material is homogenous, isotropic ideally and elastic.
- Effect of rotary inertia is included.

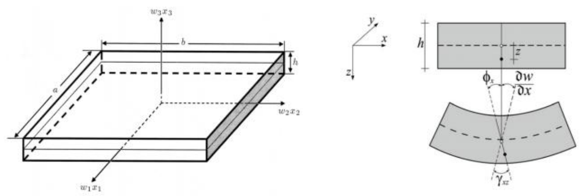

Fig 1 Deformation of a plate c/s according to Mindlin's s asumptions

The displacements along the x_1 and x_2 axes are respectively marked as W_1 and W_2 are the displacement in the direction perpendicular to plane of x_1 and x_2 is marked as W_3. W_3 Is the displacement in the direction perpendicular to the undeformed mid-plane surface.

According to Mindlin plate theory, the value of displacement components in these directions can be calculated by formulas:

$$W_1 = -x_3\psi_1(x_1, x_2, t)$$

$$W_2 = -x_3\psi_2(x_1, x_2, t)$$

$$W_3 = \psi_3(x_1, x_2, t)$$

Where t is the time, ψ_3 is the transverse displacement, ψ_1 and ψ_2 are the slope, due to bending alone in the respective planes.

$$\varepsilon_{11} = \frac{\partial W_1}{\partial x_1} = -x_3\psi_{1,1}$$

$$\varepsilon_{22} = \frac{\partial W_2}{\partial x_2} = -x_3\psi_{2,2}$$

$$\varepsilon_{33} = \frac{\partial W_3}{\partial x_3} = 0$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial W_1}{\partial x_2} + \frac{\partial W_2}{\partial x_1}\right) = -\frac{1}{2}(\psi_{1,2} + \psi_{2,1})x_3$$

$$\varepsilon_{13} = \frac{1}{2}\left(\frac{\partial W_1}{\partial x_3} + \frac{\partial W_3}{\partial x_1}\right) = -\frac{1}{2}(\psi_1 + \psi_{3,1})$$

$$\varepsilon_{23} = \frac{1}{2}\left(\frac{\partial W_3}{\partial x_2} + \frac{\partial W_2}{\partial x_3}\right) = \frac{1}{2}(\psi_2 + \psi_{3,2})$$

Using Hook's law, the components of the stress may be expressed as:

$$\sigma_{11} = \frac{E}{1-v^2}(\varepsilon_{11} + v\varepsilon_{22})$$

$$\sigma_{22} = \frac{E}{1-v^2}(\varepsilon_{22} + v\varepsilon_{11})$$

$$\sigma_{12} = \frac{E}{1+v}\varepsilon_{12} = 2G\varepsilon_{12}$$

$$\sigma_{13} = \frac{E}{1+v}\varepsilon_{13} = 2G\varepsilon_{13}$$

$$\sigma_{23} = \frac{E}{1+v}\varepsilon_{23} = 2G\varepsilon_{23}$$

The value of W_1, W_2 and W_3 for moderate thick plate is greatly depends upon the boundary conditions of thick plate. Boundary conditions of thick plate are a combination of free, clamped and simply supported boundary condition. There are different twenty one possible boundary conditions for the vibration analysis of thick plate shown in figure below.

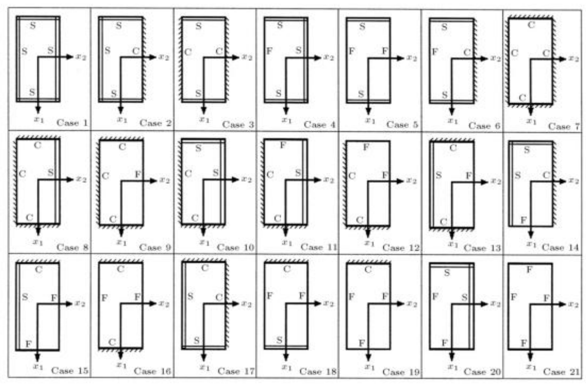

Fig 2 various boundary conditions for moderate thick plate[1]

III. APPLYING INLINE SUPPORT TO MODERATE THICK PLATE

It is considered that the plate, as shown in Figure consists of two spans that were divided at the locations of an internal line support in the X direction. For analysis,this plate was separated into two plates. The boundary conditions of each span contain classical boundary conditions along the edges and special boundary conditions along the inline supports. Along the interface between the spans, for example span (I) and span (II), the following essential and natural boundary conditions must hold to ensure the continuity of the plate and satisfaction of the inline support conditions:

Along the line support:

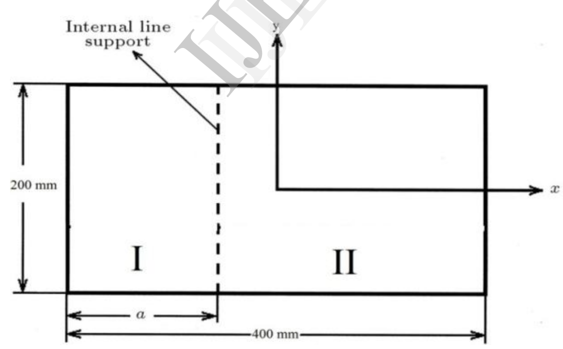

Fig 3 rectangular plate with inline support

Inthe inline support applies on the plate such that length of inline support is parallel to Y-axis. The location of inline support along X-axis considered as variables. The linear displacement in one direction is restricted by applying the inline support. So at that location of inline support thick plate having only 2 DOF.

IV. SIGNIFICANCE OF MODAL ANALYSIS

Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes.[4] The modal analysis performed for newly designed system to know the natural frequencies of system. It compared with working frequency of system to avoid resonance and failure corresponds to that resonance for safe design.

There are two methods of modal analysis one is experimental modal analysis and another is software based modal analysis. In experimental modal analysis we perform experiment on real specimen in actual working condition using the FFT analyser or any other Signal analysis devices. The device then gives the dynamic characteristics of the system (natural frequencies, mode shapes, damping factor). In software based modal analysis we can go for modelling of the actual component and then do its modal analysis by applying proper loading condition and constraints. The results show the set of natural frequencies and associated mode shapes at corresponding frequency. Modal analysis studies the dynamic properties or “structural characteristics” of a mechanical structure under dynamic excitation (resonant frequency, mode shapes, damping).The odd number of mode shapes shows the shape of deformation is a bending motion and even as shape of deformation is twisting motion. As shown in below figure 4.

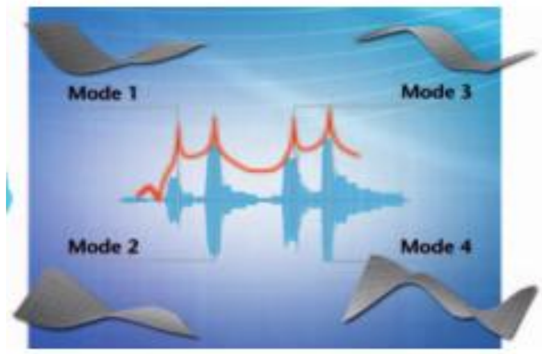

Fig 4 Single mode from mode shapes [4]

V. SOFTWARE BASED MODAL ANALYSIS IN ANSYS

The dynamic (modal) analysis can be performed by using the recently available software packages. The packages like ANSYS, ALGOR, ABAQUS, and PCTRAN etc. The finite element analysis codes usually used several mode extraction methods. The ANSYS provides several mode extraction methods like block Lanczos, subspace, power dynamics, reduced, asymmetric, damped, QR damped. Note that the default mode extraction method chosen is the Reduced Method. This is the fastest method as it reduces the system matrices to only consider the Master Degrees of Freedom. The Subspace Method extracts modes for all DOF's. It is therefore more exact but, it also takes longer to compute (especially when the complex geometries). This Lanczos is the recommended method for the medium to large models. In addition to its reliability and efficiency, the Lanczos method supports sparse matrix methods that significantly increase computational speed and reduce the storage space. This method also computes precisely the eigenvalues and eigenvectors. [5]

The thick plates considered in this and subsequent sections have following material characteristics:

- Structural material : stainless steel alloy 304/L/H
- Modulus of Elasticity: 193GPa
- Poisson's ratio: 0.28
- Density: 7800 kg/m^3

The dimensions of the plates which are considered for modal analysis are given below:

Length of the plate (a) = 400mm

Width of the plate (b) = 200mm

Thickness of the plate (h) =25.3mm

$$\eta = a/b = 2.0$$

$$\delta = h/a = 0.06325$$

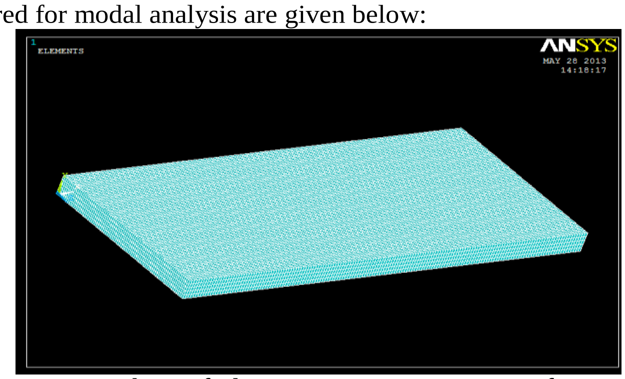

Fig 5 Meshing of Plate Geometry using FEA software

The finite element model of the thick rectangular plate with inline support at various locations is generated in ANSYS software using the SOLID185 element. SOLID185 is a higher order 3-D 8-node solid element that exhibits quadratic displacement behaviour. The element is defined by 8 nodes having three degrees of freedom per node: translations in the nodal x, y, and z directions. In the mashing the geometry divides in small elements. For accurate result obtained for FEA, thick plate having 64000 elements and 78248 nodes. The result obtained from the post processing module of ANSYS contains the information like natural frequency, mode shapes and damping coefficient. The natural frequencies for first 10 mode shapes for the thick plate using the boundary condition of CFFF (case 19 in figure 2) are listed as below in table 1.

Table 1 convergence study of natural frequency of CFFF rectangular plate without and with inline support at various location

Mode	Natural frequency(Hz)			
	CFFF	CFFF with inline support at 100mm from clamped edge	CFFF with inline support at 200mm from clamped edge	CFFF with inline support at 300mm from clamped edge
1	4.0616	6.1415	11.302	24.495
2	16.85	22.028	27.276	27.276
3	24.918	27.277	33.906	49.936
4	27.276	38.56	67.983	52.668
5	53.979	72.265	92.958	75.913
6	67.966	98.781	96.035	98.793
7	98.795	102.22	98.807	104.19
8	100.98	104.19	104.19	107.48
9	103.6	110.52	115.45	130
10	104.19	140.95	122.21	130.91

From above discussion, it is observed that addition of an inline support at any location increase the natural frequency of the plate using the same boundary condition without using an inline support. The graphical comparison of thick plate using CFFF boundary condition without inline support and with an inline support at 100mm, 200mm and 300mm with CFFF boundary condition shown in below fig 6.

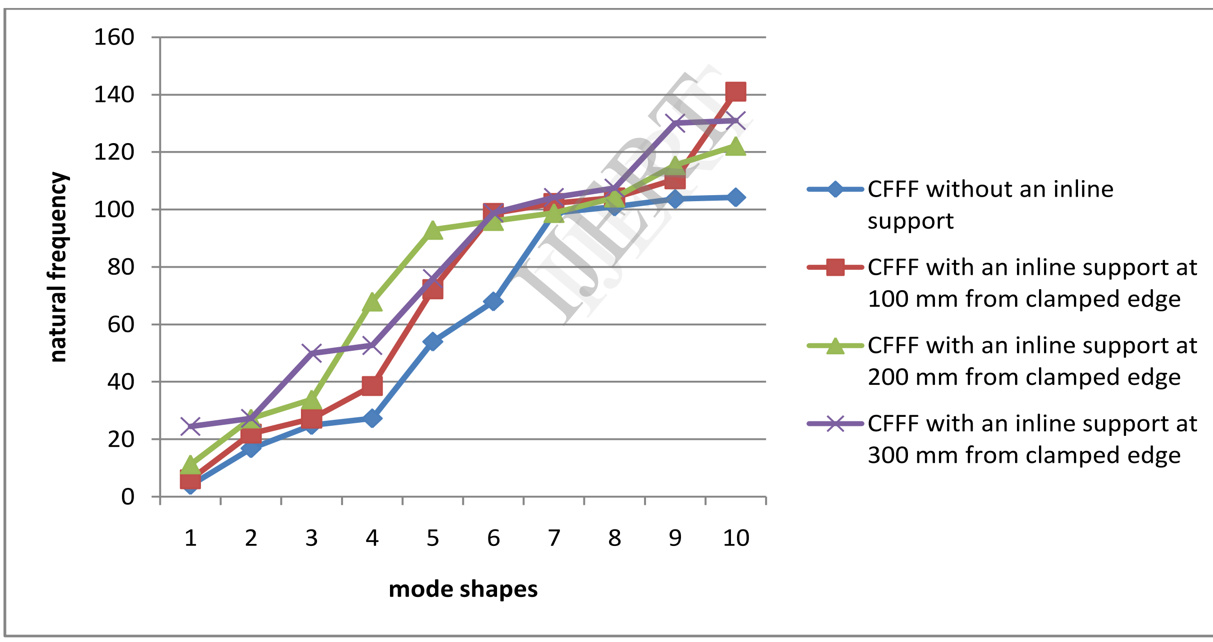

Figure 6 natural frequencies versus mode shapes of rectangular plate using CFFF boundary condition with different inline support location

VI. CONCLUSION

This paper presents a software based modal analysis for the dynamic characterization of multi-span moderately thick rectangular plates. A rectangular plate was divided into two spans in an X-direction and the result obtained from the post processing module of ANSYS contains the information like natural frequency, mode shapes and damping coefficient

- After performing FEA on both CFFF boundary condition and CFFF boundary condition with inline support, the result available having high range of natural frequency for CFFF boundary condition with inline support.
- The location of inline support having significant importance for the same boundary condition. Changing the location of inline support obtained change in the natural frequency even for same boundary condition.

- For CFFF boundary condition, the inline support located away from the clamped edge, the natural frequency is significantly increased.

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