Voltage Control of Buck Converter using Sliding Mode Controller

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***Abstract*— DC-DC converters are required to be designed to provide a well stabilized output voltage for a wide range of input voltage and load current variations. This paper presents the output voltage control of buck converter under continuous conduction mode using sliding mode controller. State space model of the buck converter is developed with output voltage error as one of the state variables. A suitable sliding surface is designed to force the trajectory to track the sliding line and to reach the asymptotically stable origin. The voltage error is given as the input to the sliding mode controller. Output of the controller controls the gate of the power electronic switch and thus maintains the output voltage at desired value. Simulation results show that the output voltage remained constant for wide variations of load and input voltage conditions.**

Keywords*— *Buck converter, continuous conduction mode, output voltage, sliding mode, trajectory

I. INTRODUCTION

Buck converters are the most widely used dc-dc converter in many of the portable devices such as cellular phones and laptops. They are employed to obtain a regulated lower DC output voltage from a given input DC voltage. Design of a suitable controller will ensure regulated output voltage under the effect of input voltage and load variations.

Conventionally, the dc-dc converters have been controlled by linear voltage mode and current mode control methods. These controllers offer advantages such as fixed switching frequencies and zero steady-state error and give a better small-signal performance at the designed operating point. But their performance degrades under large parameter and load variation [1].

Sliding mode (SM) control techniques are well suited to dc-dc converters as they operate based on the variable switching strategy. These controllers are robust concerning the converter parameter variations, load and line disturbances [1] [2] [3]. In the sliding mode control method trajectories are forced to reach sliding surface in finite time and to stay on the surface for all future time [4] [5].

In this paper we use sliding mode controller to control the output voltage of buck converter. The output voltage of buck converter is compared with a constant reference value which is the desired output voltage and the error in that is fed to the sliding mode controller [6]. Depending upon the output voltage error the sliding mode controller generates the gate signal of MOSFET which thus regulates the output voltage. Design of the suitable sliding coefficient of the controller is important for the fast and reliable response.

II. SLIDING MODE CONTROL OF BUCK CONVERTER

A. State space model of buck converter

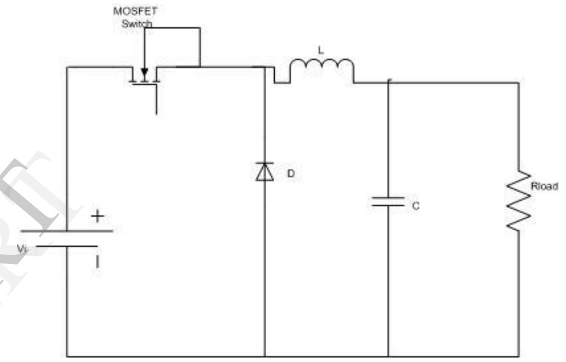

Figure 1 Buck Converter

A general representation of buck converter is shown in Figure 1. The two state variables are represented as x_1 and x_2 where x_1 is the voltage error and x_2 is the rate of change of voltage error [6].

Hence

$$x_1 = V_{ref} - V_o \qquad (1)$$

$$x_2 = \frac{dx_1}{dt} = -\frac{dV_o}{dt} = -\frac{i_C}{C} \qquad (2)$$

where i_c is the capacitor current and $i_c = C\frac{dV_o}{dt}$

When the switch is 'ON', L and C are connected in series with the source. Applying KVL and KCL in the circuit (3) and (4) respectively can be obtained

Applying KVL

$$Vin - L\frac{di_L}{dt} - V_c = 0 \qquad (3)$$

Applying KCL

$$i_C = i_L - i_R \qquad (4)$$

Differentiating equation (2) with respect to time

$$\dot{x_2} = \frac{1}{C}\frac{d(i_R - i_L)}{dt} \quad (5)$$

Substituting i_L and i_R values and rearranging

$$\dot{x_2} = -\frac{x_2}{R_L C} + \frac{V_{ref} - Vi}{LC} - \frac{x_1}{LC} \quad (6)$$

Therefore a state space model is derived as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{LC} & -\frac{1}{R_L C}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{LC}\end{bmatrix}\mathrm{V_i} + \begin{bmatrix}0\\ \frac{V_{ref}}{Vi*LC}\end{bmatrix}\mathrm{V_i} \quad (7)$$

When switch is 'OFF', diode is forward biased and L & C are in series. Then the following equations are obtained by applying KVL and KCL in the circuit

Applying KVL

$$L\frac{diL}{dt} + Vc = 0 \quad (8)$$

From (8) we get

$$i_L = \int \frac{-Vo}{L}dt \quad (9)$$

Applying KCL

$$i_C = i_R + i_L \quad (10)$$

Differentiating (2) with respect to time

$$\dot{x_2} = \frac{1}{C}\frac{d(i_R - i_L)}{dt} \quad (11)$$

Substituting i_L and i_R values and rearranging

$$\dot{x_2} = -\frac{x_2}{R_L C} + \frac{V_{ref}}{LC} - \frac{x_1}{LC} \quad (12)$$

Therefore a state space model is derived as

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{1}{LC} & -\frac{1}{R_L C}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix}\mathrm{V_i} + \begin{bmatrix}0\\ \frac{V_{ref}}{Vi*LC}\end{bmatrix}\mathrm{V_i} \quad (13)$$

B. *Trajectory*

Trajectories are plotted based on the state space model derived in the previous section and for switch is OFF (u=0) and switch is ON (u=1) are shown in figure 2 and figure 3 respectively.

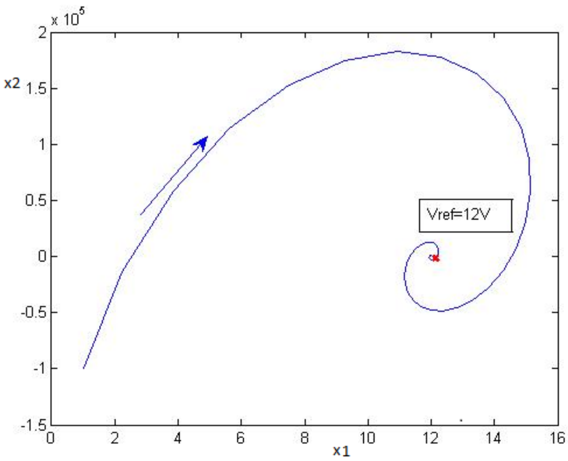

Figure 2. Trajectory for u=0

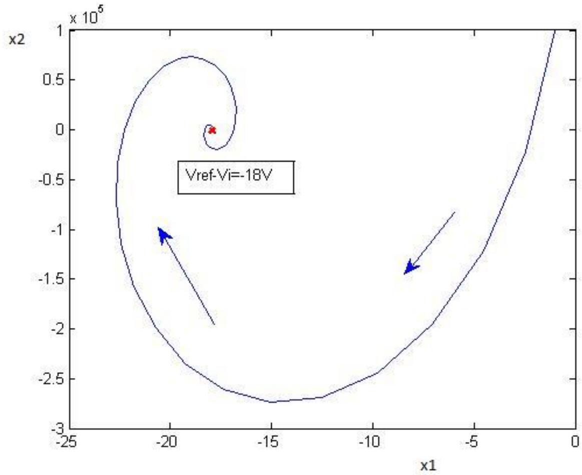

Figure 3. Trajectory for u=1

It can be observed that when u=0 the trajectory converges to equilibrium point at V_{ref} and when u=1 the trajectory converges to equilibrium point of V_{ref}-V_i. But since the requirement is to make the voltage error zero, x_1 and x_2 should converge at origin. This can be achieved by designing a suitable sliding surface.

C. *Design of Sliding Surface*

In the sliding mode control trajectories are forced to reach sliding surface in finite time and stay on the surface for all future time [4] [5]. Our aim is to make the trajectories reach and track the sliding line towards the origin, i.e. x_1=0 and x_2=0.

Sliding surface can be defined as $S=\alpha x_1+x_2=0$, where x_1 is the factor to be controlled and α is the control parameter (sliding coefficient). Thus in our problem it is designed as from (1) and (2)

$$S = \alpha(V_{ref} - V_o) - \frac{i_c}{C} \quad (14)$$

Trajectories for ON and OFF switch positions with the sliding surface are shown in figure 4 and 5.

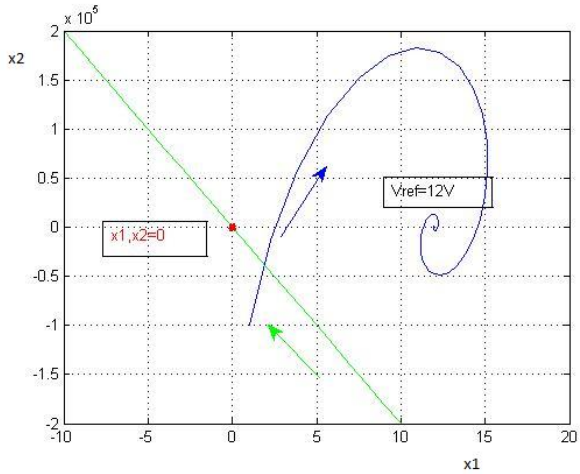

Figure 4. Sliding line for u=0

A suitable control law is to be designed for forcing the trajectories to reach the sliding surface in finite time. It is clear from figure 4 that any point below sliding line should follow the trajectory for u=0 to reach the sliding line and track towards origin.

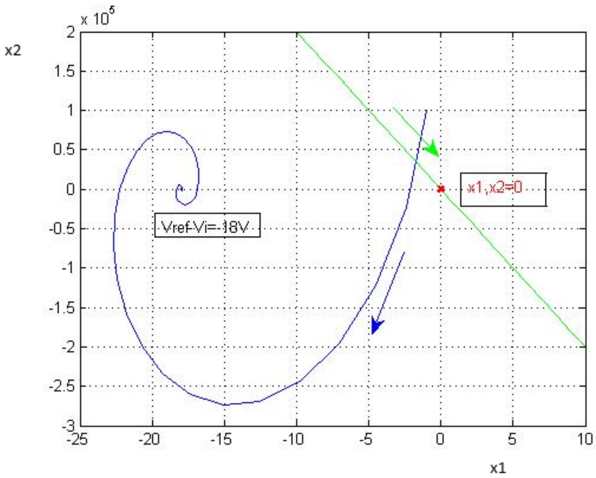

Figure 5. Sliding line for u=1

Similarly from figure 5 we can see that any point above the sliding line should follow the trajectory for u=1 to move towards origin.
From these observations the control law can be defined as,

$$\begin{bmatrix} u = 0 & S < 0 \\ u = 1 & S > 0 \end{bmatrix} \qquad (15)$$

But there is no assurance from the control law that the origin is asymptotically stable and trajectory will be maintained on sliding line for all future time. To ensure that origin is asymptotically stable a suitable Lyapunov function can be defined as $\frac{1}{2}s^2$.

According to Lyapunov's second method for asymptotic stability origin will be asymptotically stable if

$$\lim_{s\to 0} S\frac{dS}{dt}<0 \qquad (16)$$

Thus if S>0, then $\frac{ds}{dt}$ should be <0 and vice versa, where

$$\frac{dS}{dt} = \alpha\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0 \qquad (17)$$

According to the switch ON and OFF conditions, $\frac{dx_1}{dt}$ and $\frac{dx_2}{dt}$ can be calculated from (7) and (13).
But it is already established in the control law that when S>0 then u should be 1. So $\frac{ds}{dt}$ for <0 should use the state model for u=1. $\frac{ds}{dt}$ for u=1 is denoted by λ_1 and can be derived as
,

$$\lambda_1 = \left(\alpha - \frac{1}{R_LC}\right)x_2 - \frac{1}{LC}x_1 + \frac{V_{ref} - V_i}{LC} < 0 \qquad (18)$$

Similarly $\frac{ds}{dt}$ for >0 should use the state model for u=0 and $\frac{ds}{dt}$ for u=0 can be denoted as λ_2. λ_2 can be derived as,

$$\lambda_2 = \left(\alpha - \frac{1}{R_LC}\right)x_2 - \frac{1}{LC}x_1 + \frac{V_{ref}}{LC} > 0 \qquad (19)$$

Using the portion of the sliding line surface which is bounded by (18) and (19) will ensure that the origin is asymptotically stable.

D. Selection of alpha

In a buck converter output voltage will be always between 0 and Vi, thus x_1 should be between V_{ref} and V_{ref}-V_i respectively. By putting alpha equal to $\frac{1}{R_LC}$ in (18) and (19) this can be obtained. Thus alpha equal to $\frac{1}{R_LC}$ is one of the feasible selection of alpha.
The dynamic behavior of $x_1(t)$ can be derived from

$$S = \alpha x_1 + \frac{dx_1}{dt} = 0 \qquad (20)$$

Hence

$$x_1(t) = x_1(t_0)e^{-\alpha(t-t_0)} \qquad (21)$$

From (21) it is clear that alpha > $\frac{1}{R_LC}$ results in fast dynamic response. But this may leads to the overshoot in output voltage. λ_1 and λ_2 lines can be calculated from (18) and (19) with alpha > $\frac{1}{R_LC}$ and can be plotted along with sliding surface as shown in figure 6. It can be seen from figure 6 that the feasible region of sliding surface (between A&B) becomes small and results in the possibility of overshoot in output voltage.
When alpha< $\frac{1}{R_LC}$ from (21) it can be seen that system is slower and there is no improvement on feasible sliding region since x_1 should be between V_{ref} and V_{ref}-V_i. Thus in this paper we selected alpha equal to $\frac{1}{R_LC}$.

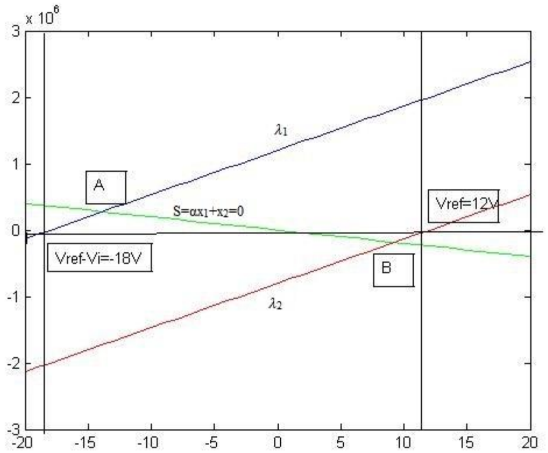

Figure 6. λ_1 and λ_2 lines with sliding surface for alpha > $\frac{1}{R_LC}$

So the sliding surface is modeled as,

$$S = k_1(V_{ref} - V_o) + k_2 i_c \qquad (22)$$

where k_1=$\frac{1}{R_LC}$ and k_2= -$\frac{1}{C}$ are the feedback gains.

According to this design feedback gains are very high since it depends upon the value of C, which makes the simulation very slow. Thus by keeping α=$\frac{1}{R_LC}$, multiplying throughout with C, the sliding line in (22) can be redefined as,

$$S = \frac{1}{R_L}\left(V_{ref} - V_o\right) - i_c \qquad (23)$$

E. Introduction of hysteresis band and relationship between switching frequency and hysteresis band

Ideally sliding line will work at infinite frequency but due to imperfections in switching devices there will be some delay leading to chattering. Chattering problem can be reduced by introducing hysteresis band K. Thus the control law can be modified as

$$\begin{bmatrix} u = 0\ '\mathrm{OFF}' & S < -K \\ u = 1\ '\mathrm{ON}' & S > K \end{bmatrix} \qquad (24)$$

where K is the hysteresis band and it can be between 0.1 to 0.2 [2]. It is important to limit hysteresis band to a small value to minimize error in output voltage but it should not be too small as K becomes too sensitive [2]. From [2] and [5] the switching frequency in terms of hysteresis band can be defined as,

$$f_s = V_o \frac{\left(1 - \frac{V_o}{V_i}\right)}{2KL} \qquad (25)$$

III. RESULTS

Buck converter with sliding mode controller has been implemented in Matlab. The specifications used for the design of buck converter are given below.

TABLE I. SPECIFICATIONS OF BUCK CONVERTER

Parameter	Value
Input voltage (V_i)	30V
Output voltage (desired)	12V
Switching frequency, f_s	100khz
R_{load} nominal	6Ω

Using equation (25) value of L is determined as 180μH and $\frac{\Delta I_o}{I_o}$ is calculated as <25% which is well within limits [7] using below equation

$$L = \frac{V_o(1-D)}{\Delta I_o f} \qquad (26)$$

C can be calculated as 8.334μF from (27) by assuming $\frac{\Delta Vo}{Vo} <$ 0.1% [7].

$$\frac{\Delta Vo}{Vo} = \frac{1}{8}\frac{Ts^2(1-D)}{LC} \qquad (27)$$

Fig 7 shows the Matlab implementation of buck converter with sliding mode controller

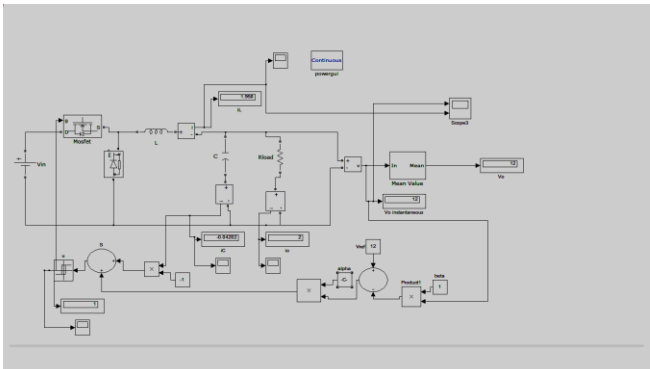

Figure 7 Buck converter with sliding mode controller.

Simulation has been performed with nominal load, with variations in input voltage and load current. Simulation results for output voltage and inductor current at nominal load are given in Figure 8 and it can be seen that output voltage is equal to the desired output voltage at nominal load. By keeping the input voltage same, load is varied between 1Ω and 15Ω and it has been observed that in this range the output voltage is maintained at the desired value.

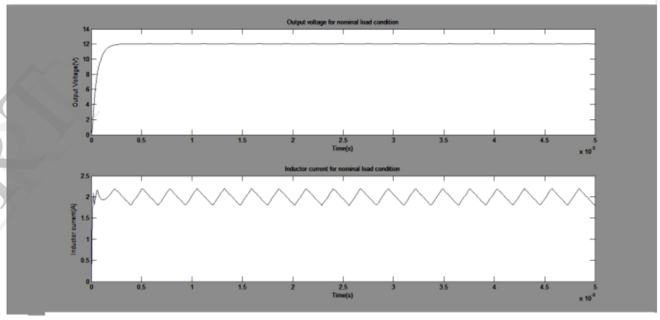

Figure 8 Output voltage and inductor current at nominal load

Also simulated the circuit with step variations in load for both overload and under load conditions. Figure 9 shows the output voltage and inductor current with a step change in load from 6Ω to 3Ω at 0.02s. It can be seen from figure 9 that output voltage takes some time to recover back to the desired output voltage. Similarly inductor current also takes some time before settling to the overloaded current value.

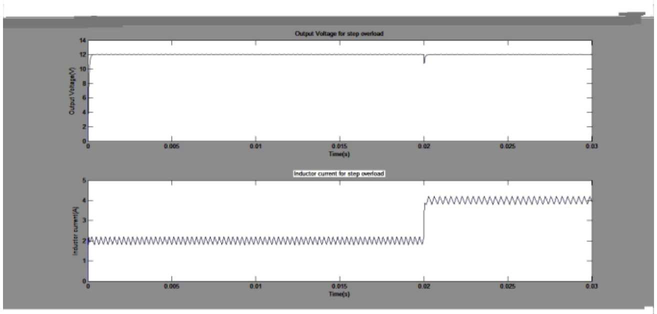

Figure 9. Output voltage and inductor current at step over loaded condition

Similarly step under loaded condition has been simulated by changing load from 6Ω to 18Ω at 0.02s and observations are shown in figure 10.

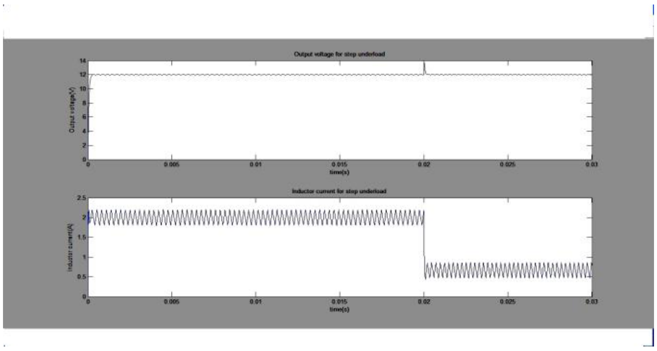

Figure 10. Output voltage and inductor current at step under loaded condition

Keeping the load at nominal value performed the simulations by varying the input voltage over a range of 20 V to 35 V. It is observed that under this input voltage variations output voltage remained constant. Figure 11 shows the output voltage with a step change in input from 30V to 25V at 0.02sec. It is clear that output voltage maintains the desired value with very less disturbance in spite of the step variation in input.

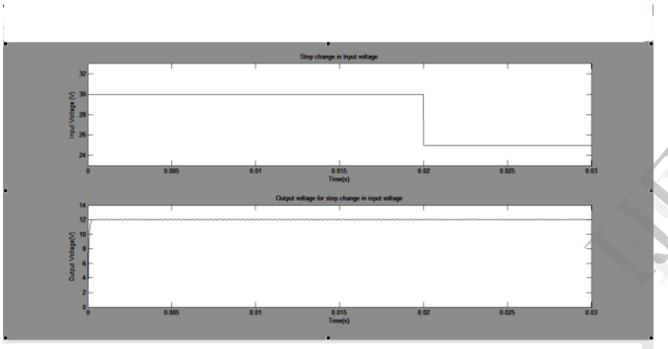

Figure11. Step variation in input voltage and the corresponding output voltage

IV. Conclusions

Buck converter is used in applications which demand regulated output voltage with fast transient response for wide variations in load and input voltage. The sliding mode control of buck converter is one of the best methods to control the output voltage under the above required conditions. In this paper buck converter with sliding mode controller is implemented in Matlab and simulated for two different conditions: by varying the input voltage between 20 to 35V with a nominal load and by varying the load between 1Ω to 15Ω with constant input voltage. It is observed that the output voltage is maintained to the desired output voltage under the variable conditions with fast dynamic response. Sliding mode controller is designed by plotting the trajectories from the developed state model and selecting a suitable value for the sliding coefficient.

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